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https://doi.org/10.5109/19643

出版情報：九州大学大学院農学研究院紀要. 56 (1), pp.79-81, 2011-02. Faculty of Agriculture, Kyushu University

J. Fac. Agr., Kyushu Univ., **56** (1), 79–81 (2011)

Symmetry in Motion in Euler's Formula and its Breakdown in Hyperbolic Function and Growth Function

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(*Received October 29, 2010 and accepted November 8, 2010*)

This study was designed to investigate the symmetry in motion in Euler's formula and its breakdown in hyperbolic function and growth function. The results obtained were as follows. (1) The symmetry in motion between θ and $\exp(\boldsymbol{i}\theta)$ was shown when θ was kept in motion. The symmetry breakdown in motion between θ and $\cosh(\theta)$, between θ and $\sinh(\theta)$, and between θ and $\exp(\theta)$ was shown when θ was kept in motion. (2) There were some properties that were common to these three functions. (3) If hyperbolic function related exponential function with Lorentz transformation, then the Bondi–k factor was an exponential function. (4) Mathematical relationships between the Bondi–k factor and the imaginary unit were suggested. (5) Mathematical relationships between Euler's formula and growth function were discussed by relating qualitative differences in complex numbers with quantitative differences in real numbers. It was suggested that the breakdown of symmetry in motion in Euler's formula gave hyperbolic function and growth function, where some properties common to these three functions were also observed.

INTRODUCTION

Exponential function with base e is used to analyze the growth of a forage plant (Blackman, 1919). Shimojo *et al*. (2003) reported that exponential function with base e was derived from Euler's formula by the rotation of axis of time, where hyperbolic function appeared between them. Investigating this process from the viewpoint of symmetry is considered an issue of interest.

The present study was designed to investigate the symmetry in motion in Euler's formula and its breakdown in hyperbolic function and growth function.

EULER'S FORMULA, HYPERBOLIC FUNCTION AND GROWTH FUNCTION AS INVESTIGATED FROM SYMMETRY IN MOTION

Euler's formula

Euler's formula ($E(\theta)$) is given by

$$E(\theta) = \exp(\boldsymbol{i}\theta) = \cos(\theta) + \boldsymbol{i}\sin(\theta), \qquad (1)$$

where $\boldsymbol{i}$ = imaginary unit.
The symmetry in motion between θ and $E(\theta)$ is shown on the complex plane when θ is kept in motion. If this is valid for the relationship between time (t) and $\exp(\boldsymbol{i}\cdot t)$, then time is kept in passing.

Hyperbolic function

The symmetry in motion in Euler's formula is broken by the following mathematical operation, which gives hyperbolic function ($H(\theta)$),

$$H(\theta) = E(-\boldsymbol{i}\cdot\theta) = \cosh(\theta) + \sinh(\theta). \qquad (2)$$

The symmetry breakdown in motion between θ and $\cosh(\theta)$ or between θ and $\sinh(\theta)$ on the real plane is shown when θ is kept in motion. If this is valid for the relationship between time (t) and $\cosh(t)$ and between t and $\sinh(t)$, then time is kept in passing.

Growth function

$H(\theta)$ leads consequently to growth function ($G(\theta)$) that shows an exponential increase,

$$\cosh(\theta) + \sinh(\theta) = \exp(\theta) = G(\theta). \qquad (3)$$

The symmetry breakdown in motion between θ and $G(\theta)$ on the real plane is shown when θ is kept in motion. Since growth function is actually a function of time (t), $G(t)$, time is kept in passing.

Properties common to Euler's formula, hyperbolic function and growth function

The first property is that although the symmetry in motion in $E(\theta)$ is broken in $H(\theta)$ and $G(\theta)$, they all are solutions to the common differential equation (4),

$$\left(\frac{dC(\theta)}{d\theta}\right)^2 = C(\theta)\cdot\frac{d^2C(\theta)}{d\theta^2}. \qquad (4)$$

The second property that is common to these three functions is given by

$$E(\theta)\cdot E(-\theta) = \exp(\boldsymbol{i}\theta)\cdot\exp(\boldsymbol{i}(-\theta)) = 1, \qquad (5)$$

$$H(\theta)\cdot H(-\theta)$$
$$= (\cosh(\theta) + \sinh(\theta))\cdot(\cosh(-\theta) + \sinh(-\theta))$$
$$= 1, \qquad (6)$$

$$G(\theta)\cdot G(-\theta) = \exp(\theta)\cdot\exp(-\theta) = 1. \qquad (7)$$

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Functions (5) ~ (7) show that not only positive θ but also negative θ is required when these three functions are connected by the sign of equality.

Features of hyperbola

If $\cosh(\theta)$ and $\sinh(\theta)$ are described as follows,

$$\cosh(\theta) = 1/\sqrt{1-(v/c)^2}\,, \tag{8–1}$$

$$\sinh(\theta) = (v/c)/\sqrt{1-(v/c)^2}\,, \tag{8–2}$$

then they give Lorentz transformation for 4 dimensional space–time,

$$c \cdot t' = (c \cdot t) \cdot \cosh(\theta) - x \cdot \sinh(\theta), \tag{9–1}$$

$$x' = x \cdot \cosh(\theta) - (c \cdot t) \cdot \sinh(\theta), \tag{9–2}$$

where v = speed of an object, c = speed of light, x = x coordinate of 4 dimensional space–time.

If functions in (8) are combined with functions in (10), respectively,

$$\cosh(\theta) = (\exp(\theta) + \exp(-\theta))/2, \tag{10–1}$$

$$\sinh(\theta) = (\exp(\theta) - \exp(-\theta))/2, \tag{10–2}$$

then functions in (11) and functions in (12) are given,

$$\exp(\theta) = \sqrt{(1 + v/c)/(1 - v/c)}\,, \tag{11–1}$$

$$\theta = \log_e\left(\sqrt{(1 + v/c)/(1 - v/c)}\right), \tag{11–2}$$

$$\exp(-\theta) = \sqrt{(1 - v/c)/(1 + v/c)}\,, \tag{12–1}$$

$$-\theta = \log_e\left(\sqrt{(1 - v/c)/(1 + v/c)}\right), \tag{12–2}$$

where $0 \leq v < c$, $0 \leq \theta < \infty$, $1 \leq \exp(\theta) < \infty$, $-\infty < -\theta \leq 0$, $0 < \exp(-\theta) \leq 1$.

Functions (11) and (12) show the following. (i) If hyperbolic function relates exponential function with Lorentz transformation, then the Bondi–k factor is an exponential function. (ii) The rate at which θ increases is accelerated with the increase in v. (iii) Do phenomena (i) and (ii) imply the feature of space?

In addition, boldly writing at the risk of making mistakes, mathematical phenomena are suggested as follows.

(i) If $v \to \infty$ mathematically, then

$$\lim_{v\to\infty} \sqrt{(1 - v/c)/(1 + v/c)} = \lim_{v\to\infty} \sqrt{(c/v - 1)/(c/v + 1)} = \sqrt{-1} = \boldsymbol{i}\,, \tag{13–1}$$

$$\lim_{v\to\infty} \sqrt{(1 + v/c)/(1 - v/c)} = \lim_{v\to\infty} \sqrt{(c/v + 1)/(c/v - 1)} = \sqrt{-1} = \boldsymbol{i}\,, \tag{13–2}$$

where $\boldsymbol{i}$ = imaginary unit.

Although $v < c$ physically, one of the routes to the imaginary unit is suggested by (13–1) and (13–2) in which $v \to \infty$ plays the key role.

The imaginary unit is also given when $c = 0$ and $v \neq 0$,

$$\sqrt{(0/v - 1)/(0/v + 1)} = \sqrt{-1} = \boldsymbol{i}, \tag{14–1}$$

$$\sqrt{(0/v + 1)/(0/v - 1)} = \sqrt{-1} = \boldsymbol{i}. \tag{14–2}$$

What is the difference between calculation (13) and calculation (14)? What is the physical meaning of the imaginary unit?

(ii) If $v = c$ mathematically, then

$$\sqrt{(1 - v/c)/(1 + v/c)} = \sqrt{(1 - c/c)/(1 + c/c)} = 0 = \lim_{\theta\to\infty} \exp(-\theta). \tag{15}$$

Calculation (15) shows that the physical impossibility of $v = c$ might be related to infinitesimal.

(iii) When $0 \leq v < c$, functions (11–1) and (12–1) are given as already shown,

$$\exp(\theta) = \sqrt{(1 + v/c)/(1 - v/c)}\,, \tag{11–1}$$

$$\exp(-\theta) = \sqrt{(1 - v/c)/(1 + v/c)}\,. \tag{12–1}$$

This suggests that exponential functions appear when $v < c$.

(iv) If $v = 0$, then $\theta = 0$ as follows,

$$\exp(0) = \sqrt{(1 + 0/c)/(1 - 0/c)} = 1, \tag{16–1}$$

$$\exp(0) = \sqrt{(1 - 0/c)/(1 + 0/c)} = 1. \tag{16–2}$$

This shows that there is neither an increase nor a decrease when $v = 0$.

(v) If $v < 0$, then

$$\exp(-\theta) = \sqrt{(1 - v/c)/(1 + v/c)} \to \sqrt{(1 - (-v)/c)/(1 + (-v)/c)} = \exp(\theta), \tag{17–1}$$

$$\exp(\theta) = \sqrt{(1 + v/c)/(1 - v/c)} \to \sqrt{(1 + (-v)/c)/(1 - (-v)/c)} = \exp(-\theta). \tag{17–2}$$

These show that changing the sign of v causes the exchange between exponential increase and exponential decrease. This exchange is also given by changing the sign of c.

(vi) With regard to $\cosh(\theta)$ and $\sinh(\theta)$, there is another description as follows,

$$\cosh(\theta) = \left(\sqrt{(1 + v/c)/(1 - v/c)} + \sqrt{(1 - v/c)/(1 + v/c)}\right)/2, \tag{18–1}$$

$$\sinh(\theta) = \left(\sqrt{(1 + v/c)/(1 - v/c)} - \sqrt{(1 - v/c)/(1 + v/c)}\right)/2.$$

(18–2)

Functions (18–1) and (18–2) are equal to functions (8–1) and (8–2), respectively.

(vii) If $c = 0$ and $v = 0$, then

$$\sqrt{(1-v/c)/(1+v/c)}$$

$$=\sqrt{(1-0/0)/(1+0/0)} = \text{indefiniteness}, \quad (19\text{–}1)$$

$$\sqrt{(1+v/c)/(1-v/c)}$$

$$=\sqrt{(1+0/0)/(1-0/0)} = \text{indefiniteness}. \quad (19\text{–}2)$$

What does the indefiniteness in real and imaginary numbers caused by this motionlessness suggest?

Relationships between Euler's formula and growth function

If θ in functions (5) and (7) is replaced with $E \cdot t$ (E = energy, t = time), then transformations (20) and (21) are given,

$$\exp(-\boldsymbol{i}(E\cdot t)) \to \exp(\boldsymbol{i}\cdot(-\boldsymbol{i})(E\cdot t)) \to \exp(E\cdot t), \quad (20\text{–}1)$$

$$\exp(\boldsymbol{i}(E\cdot t)) \to \exp(\boldsymbol{i}\cdot\boldsymbol{i}(E\cdot t)) \to \exp(-E\cdot t), \quad (20\text{–}2)$$

$$\exp(\boldsymbol{i}(E\cdot t)) \to \exp((-\boldsymbol{i})\cdot\boldsymbol{i}(E\cdot t)) \to \exp(E\cdot t), \quad (21\text{–}1)$$

$$\exp(-\boldsymbol{i}(E\cdot t)) \to \exp((-\boldsymbol{i})\cdot(-\boldsymbol{i})(E\cdot t)) \to \exp(-E\cdot t). \quad (21\text{–}2)$$

By transformations (20) and (21), qualitative differences in complex numbers are related to quantitative differences in real numbers. What do these relationships suggest?

The feature of these transformations (20) and (21) is the square of the imaginary unit. Let us suppose that Euler's formula includes $\boldsymbol{i}$ that is given by (13) and (14) as already shown. Since $v < c$, one of the ways to avoid $v \to \infty$ in (13) or $c = 0$ and $v \neq 0$ in (14) might be introducing the square of the imaginary unit ($\boldsymbol{i}\cdot\boldsymbol{i} = -1$), which transforms Euler's formula into exponential functions including growth function. However, this is just a hypothesis that requires further investigation.

Conclusions

It is suggested from the present study that the breakdown of symmetry in motion in Euler's formula gives hyperbolic function and growth function, where some properties common to these three functions are also observed.

ACKNOWLEDGEMENTS

This study was supported by a Grant–in–Aid for Challenging Exploratory Research from Japan Society for the Promotion of Science (No. 21658089).

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