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An Application of the Golden Ratio and Fibonacci Numbers to Analyses of Canopy Structure of Forage Plants

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This study was designed to conduct analyses of canopy structure of forage plants by applying the golden ratio and the Fibonacci sequence to the infinite series of exponential function with base *e* used for analyses of production structures in forage plants. The results obtained were as follows. (1) The product of light extinction coefficient (K) and leaf area index (F) in the plant canopy took the value of golden ratio, where K and F were in inverse proportion. (2) The combination of large K and low F suggested the canopy leaves showing a horizontal inclination. (3) The small K and high F suggested the canopy leaves showing a vertical inclination. (4) These phenomena depended on the leaf inclination angle with which upper leaves shaded lower leaves, provided that other factors were the same. Therefore, the case (3) might give the canopy structure that was higher in the production of forage plants compared with the case (2). It was suggested that the golden ratio and the Fibonacci sequence, which appeared in the infinite series of exponential function with base *e*, were related to the analysis of canopy structure of forage plants.

INTRODUCTION

There are relationships between functional beauty of natural phenomena and theoretical beauty of mathematical phenomena (Adam, 2003). The golden ratio is considered one of the most beautiful ratios and is observed in the functional beauty that plants show (Adam, 2003). The golden ratio is also given by the limiting value of the ratio between the two adjacent numbers in the Fibonacci sequence. The Fibonacci sequence is observed in functional properties of plants, for example, a phyllotaxy that is the arrangements of leaves on a plant stem for efficiently receiving light by leaves (Adam, 2003). The Fibonacci sequence also appears in coefficients and constant terms in the geometric sequence of the golden ratio. It is known that the geometric sequence appears in the infinite series of exponential function with base *e* that is used for analyses of production structures in forage plants.

The present study was designed to investigate the analysis of canopy structure of forage plants by applying the golden ratio and Fibonacci numbers to the infinite series of exponential function with base *e*.

GOLDEN RATIO AND FIBONACCI NUMBERS IN THE INFINITE SERIES OF EXPONENTIAL FUNCTION AND APPLICATION TO PLANT CANOPY ANALYSIS

Infinite series of exponential function with base *e*

The infinite series of exponential function with base *e* is given by

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \frac{x^9}{9!} + \frac{x^{10}}{10!} + \cdots . \quad (1)$$

If x in function (1) is a solution to equation (2),

$$x^2 - x - 1 = 0, \quad (2)$$

then

$$x = \frac{1+\sqrt{5}}{2}, \qquad \because x > 0. \quad (3)$$

The solution (3) for x is called the golden ratio, and the following geometric sequence is known,

$$x = x,\quad x^2 = x+1,\quad x^3 = 2x+1,\quad x^4 = 3x+2,$$
$$x^5 = 5x+3,\quad x^6 = 8x+5,\quad x^7 = 13x+8,$$
$$x^8 = 21x+13,\quad x^9 = 34x+21,\quad x^{10} = 55x+34, \cdots . \quad (4)$$

Substituting the geometric sequence (4) for powers of x in function (1) gives

$$\exp(x) = 1 + \frac{x}{1!} + \frac{x+1}{2!} + \frac{2x+1}{3!} + \frac{3x+2}{4!} + \frac{5x+3}{5!} + \frac{8x+5}{6!} + \frac{13x+8}{7!} + \frac{21x+13}{8!} + \frac{34x+21}{9!} + \frac{55x+34}{10!} + \cdots . \quad (5)$$

The numerators in function (5) show that coefficients, constant terms or a mixture of them form the

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Fibonacci sequence, provided that x is golden ratio.

Applying function (5) to the function for analyzing the light extinction in the canopy of forage plants

The light extinction in the canopy of forage plants (Monsi und Saeki, 1953) is given by

$$I_j = 100 \cdot \exp(-K \cdot F_j)$$

$$= 100/\exp(K \cdot F_j), \quad (6)$$

where F_j = cumulative leaf area index from the top to the jth layer of leaves, 100 = relative light intensity above the canopy, I_j = relative light intensity below the jth layer of leaves, K = light extinction coefficient of the canopy.

If function (5) is applied to function (6), then $K \cdot F_j$ is given by the golden ratio,

$$K \cdot F_j = \frac{1+\sqrt{5}}{2}. \quad (7)$$

The solution (7) shows that K and F_j are in inverse proportion, suggesting the following phenomena. (i) The combination of large K and low F_j suggests that the canopy leaves show a horizontal inclination related to large extinction coefficient of light with low leaf area index. (ii) The combination of small K and high F_j suggests that the canopy leaves show a vertical inclination related to small extinction coefficient of light with high leaf area index. (iii) These phenomena depend on the leaf inclination angle with which upper leaves shade lower leaves, provided that other factors are the same. Therefore, the case (ii) may give the canopy structure that is higher in the production of forage plants compared with the case (i).

Conclusions

It is suggested from the present study that the golden ratio and the Fibonacci sequence, which appear in the infinite series of exponential function with base e, are related to the analysis of canopy structure of forage plants.

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