Study on the spectrum of the asymmetric quantum Rabi model

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(非対称量子ラビ模型の固有値に関する研究)

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論　文　内　容　の　要　旨

The quantum Rabi model is a model in quantum optics used to describe the interaction between a two-level atom and a single-level photon field. This model, described in 1963 by Jaynes and Cummings, is the fully quantized version of the original semi-classical model proposed in 1936 by Rabi. In previous years, the study of the properties of Hamiltonian of the quantum Rabi model and its energy levels (eigenvalues of the Hamiltonian) has been relegated mainly for two reasons. The first one was the belief that the quantum Rabi model was not an exactly solvable model, that is, that its energy levels could not be described analytically. The second reason is that in the parameter regime achievable at the time in experiments and applications, the quantum Rabi model could be approximated successfully by the Jaynes-Cummings model, a simpler model known to be exactly solvable.

Both situations changed in recent years. In 2011, Daniel Braak proved the exact-solvability of the quantum Rabi model by constructing analytical solutions and defining a transcendental function, the G-function, whose zeros determine all the spectrum (with the exception of a at most countable set of exceptional eigenvalues). This pioneering technique has since then been successfully applied to show the exact-solvability of several models in quantum optics. On the other hand, due to the advances in experimental physics, starting from the first decade of this century, experiments have been steadily reaching parameter regimes where the Jaynes-Cummings model becomes unsuitable for approximation and in 2014, the experiments by Maissen et al reached regimes where it is imperative to consider the full Hamiltonian of the quantum Rabi model. Adding to this, recently there has also been proposals for applications of the quantum Rabi model in quantum information theory and quantum computing. For these reasons, there has been a large amount of research done in theoretical and experimental physics, and, more recently, mathematics on the subject of the quantum Rabi model, its generalizations and applications.

In this thesis, we study the asymmetric quantum Rabi model, one of the generalizations of the quantum Rabi model. In this generalization, an additional non-trivial interaction term is introduced in the Hamiltonian, losing the $\mathbb{Z}/2\mathbb{Z}$ -symmetry of the quantum Rabi Hamiltonian. The presence of this symmetry in the quantum Rabi model explains the presence of crossings in

the energy levels (degeneracies in the spectrum), so a priori there was no reason to expect degeneracies in the spectrum of the asymmetric version. In this work, we show that when the coefficient of the symmetry-breaking term is half-integer, degeneracies appear in the spectrum of the asymmetric quantum Rabi model. This is done by studying the properties of the constraint polynomials, polynomials appearing in certain conditions that the parameters of the model must satisfy in order to have certain eigenvalues, called Juddian eigenvalues.

The spectrum of the asymmetric quantum Rabi model is divided into different sets, an infinite set of regular eigenvalues, the zeros of the G-function and the exceptional eigenvalues, not captured as zeros of the G-function. The exceptional spectrum is further classified into Juddian and non-Juddian, according to the type of solution in the Segal-Bargmann space realization. These solutions are also described in terms of the □□(2)-picture of the Hamiltonian in terms of irreducible representations of □□(2). The case for Juddian and regular eigenvalues were already known, but in this thesis we complete the picture for the non-Juddian eigenvalues. Another main result of this work is to characterize the degeneracies of the asymmetric quantum Rabi model in terms of the types of eigenvalues and the parameters. Furthermore, by carefully studying the poles of the G-function, we define a new G-function whose zeros give the full spectrum of the asymmetric quantum Rabi model.

Finally, we present a study on continued fractions expansions of integer powers of the Napier constant e with a nice representation and good convergence properties. This study is a byproduct of the study of certain orthogonal polynomial families related to the constraint polynomials of the AQRM.