Note on Information Loss: Local Quantum Physics Perspective

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Abstract

This note supports the arguments of information loss by Unruh and Wald (`arXiv:1703.02140`) in terms of algebraic quantum theory.

1 Introduction

Last year Unruh and Wald published the arguments for the information loss in the case of the black hole evaporation [1]. Most of the arguments, however, had already appeared in Wald's monograph [2] in 1994. In the past two decades the arguments in the monograph did not have strong impact on the study of the information paradox of the black hole. We suspect that the reason is the lack of our understanding of the value of the algebraic quantum theory.

Recently the value has been recognized widely [3] so that we feel that the acceptance of their arguments[1] has become easier than before.

In this note we review the known results of the algebraic quantum theory relevant to discuss the information loss.

In the section 2 a vacuum is introduced as an entangled object. However, we can always obtain a local description as discussed in the section 3. Such a local description is consistent with the Unruh effect which is a manifestation of an entanglement as discussed in the section 4. In the final section the conclusion for the information loss is stated.

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[1] We do not mention their global argument on the basis of the Cauchy surface. We only comment on their local argument that can be supported by local quantum physics.

2 Entanglement

2.1 GNS construction

After [4]-§III.2.2 we briefly summarize the GNS construction.

First we assume the existence of a state ω which is a normalized positive linear form for a *-algebra $\mathcal{A}$. The state ω defines a Hilbert space $\mathcal{H}_\omega$ and the representation π_ω of $\mathcal{A}$ in $\mathcal{H}_\omega$. $\mathcal{A}$ itself is a linear space and we define a vector $|A\rangle$ as an equivalence class of $A \in \mathcal{A}$:

$$|A\rangle = \{A + N \mid N \in \mathcal{N}_\omega\} \tag{1}$$

where $\mathcal{N}_\omega$ is a left ideal of $\mathcal{A}$ ($AN \in \mathcal{N}_\omega$) with $\mathcal{N}_\omega = \{A \in \mathcal{A} \mid \omega(A^*A) = 0\}$. The scalar product is given by $\langle A|B\rangle = \omega(A^*B)$. After the completion the space of these vectors becomes a Hilbert space $\mathcal{H}_\omega$ ([6]-§A.2).

The representation $\pi_\omega(A)$ of A is given by

$$\pi_\omega(A)|B\rangle = |AB\rangle. \tag{2}$$

We assume that $\mathbb{I} \in \mathcal{A}$ where $\mathbb{I}$ is the unit element. The vector $|\Omega\rangle = |\mathbb{I}\rangle$ is cyclic[2] and the state is expressed[3] as

$$\omega(A) = \langle\Omega|\pi_\omega(A)|\Omega\rangle \tag{3}$$

where the normalization is given as $\langle\Omega|\Omega\rangle = \omega(\mathbb{I}) = 1$.

Roughly $\mathcal{H}_\omega = \pi_\omega(\mathcal{A})|\Omega\rangle$ where $\pi_\omega(\mathcal{A})|\Omega\rangle \equiv \{\pi_\omega(A)|\Omega\rangle \mid A \in \mathcal{A}\}$ ([5]-p.36). Precisely $\mathcal{H}_\omega = \overline{\pi_\omega(\mathcal{A})|\Omega\rangle}$ where the overline means closure ([6]-p.39). In this case we call $\pi_\omega(\mathcal{A})|\Omega\rangle$ <u>dense</u> in $\mathcal{H}_\omega$. $|\Omega\rangle$ is called a <u>cyclic</u> vector of the representation π_ω ([6]-§2.3).

2.2 Reeh-Schlieder Property

"The set of vectors $\mathcal{A}(\mathcal{O})\Omega$ generated from the vacuum by the polynomial algebra of any open region is dense in $\mathcal{H}$ ([4]-Theorem 5.3.1)".

This Reeh-Schlieder property[4] is obvious from the GNS construction[5].

[2]A representation π is called cyclic if there exists a vector $|\Omega\rangle$ in the representation space $\mathcal{H}$ such that $\pi(\mathcal{A})|\Omega\rangle$ is dense in $\mathcal{H}$. In this case $|\Omega\rangle$ is called a cyclic vector. It is obvious that $|\mathbb{I}\rangle$ is a cyclic vector.

[3]Any vector $|\Psi\rangle$ defines a state $\tilde{\omega}$ as $\tilde{\omega}(A) = \langle\Psi|\pi_\omega(A)|\Psi\rangle$ if we put $\tilde{\omega}(A) = \omega(B^*AB)$ and $|\Psi\rangle = \pi_\omega(B)|\Omega\rangle$.

[4]The vacuum in the theorem may be replaced by any vector with bounded energy ([4]-Theorem 5.3.1-Remark(ii)). The vacuum vector Ω is not only <u>cyclic</u> but also <u>separating</u>: if $A\Omega = B\Omega$, then $A = B$ for $A, B \in \mathcal{A}(\mathcal{O})$ ([6]-Theorem 4.14).

[5]The GNS construction has nothing to do with the space-time. Thus the Reeh-Schlieder property is understood as the relation of the algebra.

2.3 Summary

The Reeh-Schlieder property tells us that a vacuum cannot be decomposed into a product of space-like separated contributions [7]. Thus entanglement properties[6] do exist in local quantum physics.

3 Local Description

3.1 Local Quantum Physics

In this note we identify the algebraic quantum theory with the local quantum physics [4].

The local quantum physics starts from the local measurement: "A physical theory starts from a description of the information about a physical system obtained by measurements." ([6]-p.14)

"Physical measurements are accompanied by errors." ([6]-p.13) Thus we have to consider the 'physical topology' of the states. ([6]-§1.6)

The state $\omega(A)$ is introduced as the expectation value of the observable A: "The concept of states has been introduced in order to describe the results of a measurement of a physical system." ([6]-p.14)

The above measurement process is well described by von Neumann algebra[7] which is closed in the weak operator topology. ([6]-§2.2)

3.2 Quantum Mechanics

The local quantum physics is described by the type-III von Neumann algebra,[8] while usual quantum mechanics is described by the type-I von Neumann algebra. The difference is significant as shown in the following.

For comparison we clarify the status of usual quantum mechanics.
Finite degrees of freedom: $\rightarrow$ The unitary evolution is meaningful.
Lack of causal structure: $\leftarrow$ The speed of light is infinite.

[6]As noted in the previous footnote the GNS construction has nothing to do with the space-time. Thus the entanglement properties are also understood as the relations of the algebra.

[7]It is not known a priori so that we have to choose an algebra which describes the situation of our local measurements appropriately. It is the type-III von Neumann algebra.

[8]The minimum review of von Neumann algebra necessary for our discussion is found in [7, 8].

3.3 Relativistic Quantum Fields

The above two insufficiencies are closely related: "an upper bound on the propagation velocity of effects leads naturally to systems with an infinite number of freedom (relativistic quantum fields)" ([7]-p.325).

The system with finite degrees of freedom exhibits no spontaneous symmetry breaking (SSB) which is the heart of the field theory of the standard model. The generalized sector structure [9] of the type-III von Neumann algebra gives a natural framework to describe the SSB.

3.4 Open System

3.4.1 Extrinsic Mixing

In usual quantum mechanics the total system is a closed system. If we divide the total system into the inner system and its environment, the inner system becomes an open system. The density operator ρ for the inner system evolves as

$$\rho' = \sum_i V_i \rho V_i^\dagger \tag{4}$$

where V_i is the Kraus operator [10]. Here i stands for the environmental degrees of freedom. In the derivation of this evolution rule we need to trace out the information of the environment.

The unitary evolution

$$\rho' = U \rho U^\dagger \tag{5}$$

is expected for a closed system with finite degrees of freedom.

A closed system is described as a pure state and an open system is described as a mixed state.

3.4.2 Intrinsic Mixing

At the beginning of its description local quantum physics defines a local region $\mathcal{O}$ of space-time. Then it considers the algebra $\mathcal{A}(\mathcal{O})$ of local observables. Finally it considers the net of local algebras. The net describes the collection of open systems [4].

The state for the type-III von Neumann algebra is an intrinsically[9] mixed state [7]. In this case the label i distinguishes different 'generalized' sectors [9]. It should be noted that the 'generalized' sector is defined in terms of the factor.

[9]In this case we do not have to trace out some degrees of freedom. The 'generalized' sector structure is an inner structure of the states for the type-III von Neumann algebra.

Usual quantum mechanics is the theory for a single sector. On the other hand, a spontaneous symmetry breaking (SSB) occurs among different sectors [9]. Thus local quantum physics can naturally describe SSB but usual quantum mechanics cannot.

3.4.3 Living Room

Unruh and Wald [1] use an illustration of 'living room' which is an open system and described as a mixed state. The evolution of it is not unitary so that the information of it is not conserved. If we say that the information is lost in this case as Unruh and Wald do, the information loss is a daily event regardless of the presence of a black hole. In local quantum physics the state for a local space-time region $\mathcal{O}$ corresponds to the living room.

The discussions aiming to prove the conservation of the information in the process of black hole evaporation seem to implicitly assume the following:
- The whole universe is in a pure state.
- The pure state evolves unitarily.

Both are irrelevant to the situation of local quantum physics.

3.5 Statistical Independence

The local algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ are statistically independent [7], if there is a state ω on $\mathcal{A}_1 \vee \mathcal{A}_2$ such that

$$\omega(AB) = \omega_1(A)\omega_2(B) \tag{6}$$

for every pair of states ω_1 on $\mathcal{A}_1$ and ω_2 on $\mathcal{A}_2$ with $A \in \mathcal{A}_1$ and $B \in \mathcal{A}_2$. Namely, states can be independently prescribed on each local algebra and extended to a common uncorrelated state on the joint algebra [7].

This statistical independence is equivalent to the split property in the following.

3.6 Factor

Here we introduce a factor. The factor plays a vital role to perform the disentanglement.

If the von Neumann algebra $\mathcal{M}$ is a factor, the algebra $\mathcal{B}(\mathcal{H})$ of bounded operators on $\mathcal{H}$ is "factorized" [8] into $\mathcal{M}$ and its commutant $\mathcal{M}'$. Here the total Hilbert space $\mathcal{H}$ is separable: $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. $\mathcal{M}$ and $\mathcal{M}'$ are separable as $\mathcal{M} = \mathcal{B}(\mathcal{H}_1) \otimes \mathbb{I}$ and $\mathcal{M}' = \mathbb{I} \otimes \mathcal{B}(\mathcal{H}_2)$ so that $\mathcal{B}(\mathcal{H}) = \mathcal{B}(\mathcal{H}_1) \otimes \mathcal{B}(\mathcal{H}_2)$.

3.7 Split Property

In the following we assume the split property that follows from the nuclearity requirement ([4]-§5.1).

For simplicity we employ the situation where $\mathcal{O}_1 \subset \mathcal{O}_2$. The split property [11] means that there is a type-I factor $\mathcal{N}$ such that

$$\mathcal{A}(\mathcal{O}_1) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{O}_2) \tag{7}$$

where $\mathcal{A}(\mathcal{O})$ is the local algebra[10] on $\mathcal{O}$. We can set $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{N} = \mathcal{B}(\mathcal{H}_1)\otimes\mathbb{I}$.

Next we consider $\mathcal{O}_3$ which is separated from $\mathcal{O}_2$ in a space-like manner. Then we can set $\mathcal{A}(\mathcal{O}_3) \subset \mathcal{N}' = \mathbb{I} \otimes \mathcal{B}(\mathcal{H}_2)$.

Thus the split property leads to the statistical independence between $\mathcal{A}(\mathcal{O}_1)$ and $\mathcal{A}(\mathcal{O}_3)$.

3.8 Local Preparability

The discussion in this subsection is a mixture of [7] and [12].

Let us start from an arbitrary input state ω. The target state is a normal state φ.

To exploit the split property we introduce a local space-time region $\mathcal{O}_\varepsilon$ which includes $\mathcal{O}$ and slightly larger than $\mathcal{O}$. In this way we can exploit the split property for any $\mathcal{O}$. The split inclusion is written as $\mathcal{A}(\mathcal{O}) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{O}_\varepsilon)$. The split property leads to $\mathcal{A}(\mathcal{O}) \subset \mathcal{B}(\mathcal{H}_1)\otimes\mathbb{I}$ and $\mathcal{A}(\mathcal{O}') \subset \mathbb{I}\otimes\mathcal{B}(\mathcal{H}_2)$ where $\mathcal{O}'$ is separated from $\mathcal{O}_\varepsilon$ in a space-like manner.

For the type-I local algebra $\mathcal{N}$ there is a normal state $\varphi(A) = \mathrm{Tr}\{\rho A\}$ with a density operator ρ. The density operator is written as $\rho = \sum_i \mu_i P_i$ in terms of the projection P_i where $\mu_i \geq 0$ and $\sum_i \mu_i = 1$.

For $A \in \mathcal{A}(\mathcal{O}) \subset \mathcal{N}$ there is a spectral decomposition $A = \sum_j a_j P_j$ with a number a_j. Namely, for the algebra embedded in $\mathcal{N}$ such a description is almost exact approximation when the difference between $\mathcal{O}$ and $\mathcal{O}_\varepsilon$ is small.

For every $P_i \in \mathcal{N}$ we can find $W_i \in \mathcal{A}(\mathcal{O}_\varepsilon)$ such that $W_i W_i^* = P_i$ and $W_i^* W_i = \mathbb{I}$ by the type-III property of $\mathcal{A}(\mathcal{O}_\varepsilon)$.

The preparation operation of the state can be implemented by a local map T for $A \in \mathcal{A}(\mathcal{O})$. Such a map is expressed in terms of the Kraus operator $V_i \equiv \sqrt{\mu_i} W_i$ as $T(A) = \sum_i V_i^* A V_i$.

The transformation of $\omega(AB)$ via the local map T can be factorized as[11]

$$\tilde{\omega}(AB) = \varphi(A)\omega(B) \tag{8}$$

[10] When $\mathcal{O}_1 \subset \mathcal{O}_2$, $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$ is obvious.

[11] Since $A = \sum_j a_j P_j$, the relation $P_i A P_i = a_i P_i$ holds. By multiplying this relation from left with W_i^* and right with W_i we obtain $W_i^* A W_i = a_i \mathbb{I}$. Then the map T works as $T(A) = \sum_i \mu_i W_i^* A W_i = \sum_i \mu_i a_i \mathbb{I} = \varphi(A)\mathbb{I}$. Therefore $\tilde{\omega}(AB) = \omega(T(A)B) = \varphi(A)\omega(B)$.

where $\tilde{\omega}(AB) \equiv \omega(T(A)B)$. Thus the map T embodies the statistical independence (6).

Consequently "any state on $\mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O}')$ can be disentangled[12] by a local operation in $\mathcal{A}(\mathcal{O}_\varepsilon)$." ([7]-Theorem 5)

3.9 Quantum Channel

In this subsection we consider the isometry in a quantum channel [13]. Its construction is parallel to the procedure in the last subsection.

We start from a channel $\mathcal{N}$ which is a map $\mathcal{N}\colon\ \mathcal{L}(\mathcal{H}_A) \to \mathcal{L}(\mathcal{H}_B)$ where $\mathcal{L}(\mathcal{H})$ is the set of all bounded linear maps of the Hilbert space $\mathcal{H}$.

Then we introduce an isometric extension (Stinespring dilation) $W\colon\ \mathcal{H}_A \to \mathcal{H}_B \otimes \mathcal{H}_E$ of the channel $\mathcal{N}$ under the condition for $X_A \in \mathcal{L}(\mathcal{H}_A)$

$$\mathrm{Tr}_E\{W X_A W^\dagger\} = \mathcal{N}(X_A). \tag{9}$$

This isometry behaves as

$$W^\dagger W = \mathbb{I}_A, \qquad W W^\dagger = P_{BE} \tag{10}$$

where P_{BE} is a projection of the tensor-product Hilbert space $\mathcal{H}_B \otimes \mathcal{H}_E$. Such a behavior is expected if we consider the isometry as a rectangular matrix.

The isometry is explicitly constructed as [13]

$$W = \sum_j N_j \otimes |j\rangle \tag{11}$$

where N_j is the Kraus operator such that

$$\mathcal{N}(\rho_A) = \sum_j N_j \rho_A N_j^\dagger. \tag{12}$$

3.10 Entanglement-Breaking Channel

The procedure of the disentanglement can be easily demonstrated in the channel theory. ([13]-§4.6.7)

Roughly the disentanglement is possible if the channel has a Kraus representation. In usual quantum mechanics the Kraus representation is introduced extrinsically by the trace-out of the environmental degrees of freedom. On the other hand, in local quantum physics the Kraus representation is intrinsic because of the 'generalized' sector structure [9].

[12]Given a state ω on $\mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O}')$ there is an isometry $\{W_i\} \in \mathcal{A}(\mathcal{O}_\varepsilon)$ such that

$$\tilde{\omega}(AB) = \omega(A)\omega(B)$$

for all $A \in \mathcal{A}(\mathcal{O})$ and $B \in \mathcal{A}(\mathcal{O}')$. ([7]-p.343)

3.11 Local Filter

In this subsection we collect the conclusions reached by researchers of local quantum physics.

They needs a device to gain the data of the local measurement. The local map T which we employed describes the status of the device: "this operation T is used to represent the measuring devise itself." ([14]-pp.213-214)

In order to introduce the isometry we expand the local region of measurements slightly. Under such a situation the disentanglement is possible. "The existence of the local filter for a state in a certain domain means that the state in that domain can be prepared by using some apparatus in a domain which is a little bigger than the original one." ([6]-p.191)

The local filter[13] is introduced by Buchholz, Doplicher and Longo [15]. In the following we quote several sentences appeared in [15]-pp.5-6:

- All local observables can also be grounded on the basic experimental fact that it is possible to fix locally certain specific physical situations irrespective of the given initial conditions of the world.

- Pure filters are familiar from systems with a finite numbers of degrees of freedom. In quantum field theory, however, a pure filter cannot be a (local) observable, because it affects in a sharp way all states at arbitrarily large space-like distances. On the other hand, one never attempts to measure pure filters. In practice one is content with the possibility of fixing states within limited space-time regions. It is an important empirical fact that this can be achieved with an experimental set-up where only the parameters of the states in question enter. Phrased differently: by suitable monitoring experiments one can establish a definite state within a given region, irrespective of the unknown and complicated details of the rest of the world. So locally, such experiments have the same effect as a pure filter. Translating these facts into the setting of quantum field theory one is led to introduce the concept of a local filter for a given state.

- The empirical situation just described then suggests that all physically reasonable theories have to admit such local filters. We shall demonstrate now that this condition, which expresses a principle of experimental definiteness, implies the split property.

[13] Our isometry is a local filter revised in [12]. Yngvason's review [7] is written based on [12] entitled "Local Preparability of States and the Split Property in Quantum Field Theory." This title summarizes our strategy for local description.

3.12 Summary

We can perform a disentanglement so that we can make our measurements in local laboratories meaningful. It is a local description without knowing the other regions of the universe.

4 Unruh Effect

4.1 KMS Property

The Gibbs state ω for a finite system is represented by the trace $\omega(A) = \mathrm{Tr}\{\rho A\}$ where $\rho = e^{-\beta H}/\mathrm{Tr}\{e^{-\beta H}\}$ is the density operator. The equilibrium state has the KMS property

$$\omega(A(t)B) = \omega(BA(t+i\beta)) \tag{13}$$

where $A(z) \equiv e^{iHz}Ae^{-iHz}$ with complex z. This property is easily shown[14] only using the cyclic property of the trace.

This property also holds for infinite systems.

4.2 HHW Property

Haag, Hugenholtz and Winnink found a modular structure in the Gibbs state[15].

Here we consider the algebra $\mathcal{A}$ of $n \times n$ matrices. By giving the scalar product $\langle A|B\rangle \equiv \mathrm{Tr}\{A^*B\}$ the algebra $\mathcal{A}$ can be identified with a Hilbert space $\mathcal{H}$. In $\mathcal{H}$ we distinguish the left representation π: $\pi(A)B \equiv AB$ and the right representation π': $\pi'(A)B \equiv BA^*$. $\pi'(\mathcal{A})$ becomes the commutant of $\pi(\mathcal{A})$: $\pi'(\mathcal{A}) = \pi(\mathcal{A})'$.

We introduce an involution J by $JA = A^*$ where J is anti-unitary and $J^2 = \mathbb{I}$. This involution J maps $\pi(\mathcal{A})$ onto its commutant:

$$J\pi(\mathcal{A})J = \pi(\mathcal{A})'. \tag{14}$$

The ‘vector’ Ω defined[16] by $\Omega = \rho^{1/2}$ is cyclic and separating. The Gibbs

[14] See [4]-§V.1.1.

[15] Although they also discussed the case of infinite degrees of freedom, we only discuss the finite case after [16]. In [16] all the proofs of the statements are given.

[16] The factor $\rho^{1/2}$ also defines important quantities in other theories:

- the vacuum in the thermo field dynamics [17]
- the Kraus operator in the quantum fluctuation theorem [18].

state ω is written as $\omega(A) = \langle\Omega|\pi(A)|\Omega\rangle = (\langle\Omega|\pi'(A)|\Omega\rangle)^*$ which is consistent with the GNS representation. Within this subsection we put $\beta = 1$.

Next we introduce other involutions $S\pi(A)|\Omega\rangle = \pi(A)^*|\Omega\rangle$ and $F\pi'(A)|\Omega\rangle = \pi'(A)^*|\Omega\rangle$ where S and F do not conserve norms in general. S and F are related to J by a positive operator Δ as[17] $S = J\Delta^{1/2} = \Delta^{-1/2}J$ and $F = J\Delta^{-1/2} = \Delta^{1/2}J$. This positive operator Δ is related to the time-evolution operator $U(z)$ as follows.

The time-evolution operator is defined by $U(t) = \pi(e^{iHt})\pi'(e^{iHt})$ which acts as $U(t)A = e^{iHt}Ae^{-iHt}$. In terms of the positive operator $U(t) = \Delta^{it}$.

Symbolically we define the modular Hamiltonian K by $K = \pi(H) - \pi'(H)$. In terms of the modular Hamiltonian $\Delta = e^{-K}$ and $U(t) = e^{-iKt}$.

4.3 Tomita Property

We consider the von Neumann algebra $\mathcal{M} \in \mathcal{B}(\mathcal{H})$ in the standard form[18].

The modular conjugation J brings $\mathcal{M}$ into its commutant $\mathcal{M}'$:

$$J\mathcal{M}J = \mathcal{M}'. \tag{15}$$

The modular operator Δ forms an automorphism:

$$\Delta^{it}\mathcal{M}\Delta^{-it} = \mathcal{M}. \tag{16}$$

These are the Tomita properties ([5]-C.23). The HHW properties shown in the last subsection are carried over to the case of infinite degrees of freedom.

Mathematically the most primitive modular structure is seen in the representation of the locally compact group. In its left representation $\pi(t)|s\rangle = |t^{-1}s\rangle$. On the other hand, in its right representation $\pi'(t)|s\rangle = \Delta(t)^{1/2}|st\rangle$ where $\Delta(t)$ is the modular function.

Physically the modular conjugation J leads to a significant consequence: the modular Hamiltonian K is not positive. Its negative part is suppressed[19] with respect to $\mathcal{M}$ and its positive part is suppressed with respect to $\mathcal{M}'$.

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[17] $S = J\Delta^{1/2}$ is an essential relation in the Gibbs state. The derivation of it given in [4]-§V.1.4 is as follows. It is crucial to notice that $|\Omega\rangle = |e^{-\beta H/2}\rangle/Z^{1/2}$ explicitly. Here $Z \equiv \mathrm{Tr}\{e^{-\beta H}\}$. In GNS representation $\pi(X)|\Omega\rangle = |X\Omega\rangle$ and $\pi'(Y)|\Omega\rangle = |\Omega Y^*\rangle$. Thus $e^{-\beta K/2}|A\Omega\rangle = |\Omega A\rangle$. On the other hand, $J|A^*\Omega\rangle = |\Omega A\rangle$, since $J|X\rangle = |X^*\rangle$. Using $J^{-1} = J$ we obtain $Je^{-\beta K/2}|A\Omega\rangle = |A^*\Omega\rangle$. This is equal to $Je^{-\beta K/2}\pi(A)|\Omega\rangle = \pi(A^*)|\Omega\rangle$.

[18] In this Hilbert space $\mathcal{H}$ there is a unit vector Ω that is cyclic and separating for $\mathcal{M}$.

[19] The property $J\Delta J = \Delta^{-1}$ and $JE_\kappa^{(+)}J = E_\kappa^{(-)}$ plays an important role. Here $E_\kappa^{(\pm)}$ is the spectral projection of K. ([4]-V.2.1)

4.4 Takesaki Property

We consider the vacuum vector Ω that is the eigenvector of the modular Hamiltonian with zero eigenvalue.

The vacuum expectation value shows the KMS property[20]:

$$\langle\Omega|A(t)B\Omega\rangle = \langle\Omega|BA(t-i)\Omega\rangle \tag{17}$$

for $A, B \in \mathcal{M}$.

This is the Takesaki property. In [19]-p.79: "A characteristic feature of relativistic quantum field theory is the existence of vacuum fluctuations for all local observables. Actually, the vacuum state, restricted to a bounded region $\mathcal{O}$, has many features of an equilibrium state at nonzero temperature."

The property (17) is confirmed by a direct calculation ([4]-§V.2.1). The calculation shows that the modular operator Δ represents the vacuum fluctuation.

In relation to the space-time the von Neumann algebra $\mathcal{M}$ in the last subsection is the local algebra $\mathcal{A}(\mathcal{O})$. A and B have support on $\mathcal{O}$ so that the vacuum expectation value in (17) reduces to the expectation value by the local measurement at $\mathcal{O}$. The state for the local measurement is a mixed one as discussed in the subsection of open system.

4.5 Bisognano-Wichmann Property

In the last paragraph of §4.5 in [2]: "The Reeh-Schlieder theorem implies that the restriction of the ordinary vacuum state to $\mathcal{O}$ defines a mixed state. The Unruh effect provides an excellent illustration of this phenomenon."

Bisognano and Wichmann did this illustration in terms of modular structure. They found[21] that the modular operator is

$$\Delta_{\mathcal{O}}^{it} = U(\Lambda_1(2\pi t)) \tag{18}$$

and the modular conjugation is

$$J_{\mathcal{O}} = \Theta U(R_1(\pi)) \tag{19}$$

for the wedge $\mathcal{O} = \{x \in M | x^1 > |x^0|\}$ in the Minkowski space M. Here U is the representation of the Lorentz group and Θ is the PCT operator. The operation Λ_1 is the boost in x^1-direction and R_1 is the rotation around the x^1-direction.

For a uniformly accelerated observer in this wedge the vacuum state looks like a thermal state ([4]-§V.4.1).

[20] The KMS property is equal to $\langle\Omega|AB\Omega\rangle = \langle\Omega|B\Delta^{-1}A\Omega\rangle$ as written in (15.7)-[7].

[21] See [4]-§V.4.1 and [7]-§15.3.4.

4.6 Thermo Field Dynamics

The HHW properties are also implemented [20] in the framework of the thermo field dynamics [17].

The vacuum $|\Omega\rangle$ that is the eigenvector of the modular Hamiltonian K with zero energy is described as the coherent state in terms of the thermal pairs. Such a coherent state shows the Unruh effect [21].

4.7 Summary

The local quantum physics is compatible with the entangled vacuum. An entanglement leads to the Unruh effect. Such an effect results from the fact that our observation can reach only restricted part of the universe.

5 Conclusions

In the discussion of the information paradox of the black hole the conservation of the information of the universe became a central issue. However, such information is not a local observable.

The information might be meaningful if we consider a closed system with finite degrees of freedom described as a pure state. The evolution of such a system is unitary and the information is conserved.

On the other hand, the situation of the measurements in the local quantum physics is completely different. We measure an open system with infinite degrees of freedom described as a mixed state. The evolution of such a system is not unitary.

Although the local quantum physics cannot describe the global structure of the space-time (gravity itself), it can describe the Unruh effect which is an essential observable effect of gravity.

Consequently the paradox has been lost.

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