Research progress survey on interactive evolutionary computation

Pei, Yan
Computer Science Division, Universally of Aizu

Takagi, Hideyuki
Faculty of Design, Kyushu University

Research Progress Survey on Interactive Evolutionary Computation

Yan Pei*, and Hideyuki Takagi†

*Computer Science Division, Universally of Aizu, Aizu-wakamatsu, Japan 965-8580

Email: peiyan@u-aizu.ac.jp

http://web-ext.u-aizu.ac.jp/~peiyan/

†Faculty of Design, Kyushu University, Fukuoka, Japan 815-8540

takagi@design.kyushu-u.ac.jp

http://www.design.kyushu-u.ac.jp/~takagi/

Abstract—In this paper, we evaluate the acceleration performance of evolutionary computation by using estimated convergence points. The obtained convergence points can be obtained by the evolutionary computation algorithms with the characteristic that there is a one-to-one correspondence between a parent individual and its offspring. We can obtain multiple evolutionary paths or moving vectors for estimating convergence points for evolutionary computation algorithms acceleration. We use the estimation method and its four clustering improvement method for bimodal task to design a differential evolution with elite strategy. From the evaluation results, our proposed method can significantly enhance optimization performance of differential evolution. We also discuss the analyse the open topics and future opportunities on the estimation methods, some discoveries are investigated and analysed.

Index Terms—interactive evolutionary computation; human model; acceleration of evolutionary computation; human science; optimization

I. INTRODUCTION

Interactive evolutionary computation (IEC) is a method framework of systemic optimization that uses a real human's subjective evaluation in its optimization process. There are many tasks whose performances are difficult to measure or almost impossible to be quantified by machine, but can be evaluated by human beings. In some practice problems of real-world systemic optimization, it is difficult to design a fitness function that simulates a real human's subjective evaluation, therefore, we use a real human to replace the fitness function in order to make the problem converge to subjective evaluation of a real human.

The IEC shown in Fig. 1 can optimize such tasks by involving a human user in an IEC optimization loop. The optimization framework of IEC has three parts, an IEC algorithm (including IEC interface), a real human, and an optimized target system. Research on the three parts of IEC correspondingly gives rise to three directions of IEC studies.

1) IEC algorithm study for reducing user fatigue;
2) human psychological and physiological study using IEC, i.e., IEC for human science research; and
3) real-world optimization application using IEC, i.e., application-oriented research of IEC.

The first study direction of IEC is the research on its algorithm. The main issue of IEC is the user fatigue problem when a real human gives fitness to an IEC algorithm. The objective of IEC algorithm research makes sure that IEC can be applied in real-world applications without serious user fatigue. The user fatigue problem occurs in the process of IEC optimization, when a real human needs to repeatedly give feedback to an IEC algorithm from his/her subjective evaluation. This does not occur in computer evaluation processes. This problem restricts the application of IEC optimization.

The second study direction is the human psychological and physiological research using IEC, i.e., IEC for human science research. The philosophy of this research lies in the subjective evaluation of IEC coming from a real human's psychological and/or physiological characteristics. When an IEC algorithm converges to the real human's characteristics in its optimization process, we can use IEC optimization data to analyse these psychological and/or physiological characteristics to obtain new and unknown knowledge or indexes. From this view, IEC can be considered as a data analysis and processing tool for human related research, especially for discovering psychological and/or physiological knowledge about humans.

The third study direction of IEC is regarding its real-world application research in practice. This research includes three aspects. One is the art innovation applications, these include computer graphics, music design, clothing design, and architectural design. Another is engineering applications, these include image and audio processing, robot control, data mining, and software design. The rest are various other IEC-based applications, these include education, games, etc. Reference [44] makes a summary of these IEC research projects.

From the later part of the 1980s until now, the research fields and application practices of IEC have been enriched and perfected. On conducting an investigation in SCOPUS database to find papers on IEC, we found that the number of IEC papers increase from 2000 to 2017 (see Fig. 2). We use the keywords [1] to investigate the number of IEC papers, the total number of which was 1267 during 1980 to 2017. The rate of increase of IEC papers from 2000 to 2017 is

---

[1] the key words are ("interactive evolution" OR "interactive genetic" OR "interactive evolutionary" OR "interactive differential evolution" OR "interactive swarm" OR "interactive PSO" OR "interactive ACO" OR "interactive ABC")
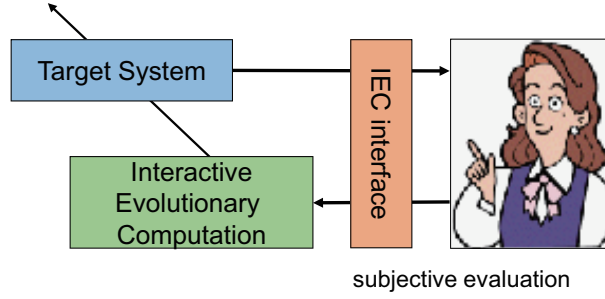
Fig. 1. An optimization framework of interactive evolutionary computation (IEC), it includes an IEC algorithm (including interface), a real human, and an optimized target system in general.

about 1.10 per year, which is as the same as that of papers on evolutionary computation (EC), whose rate of increase is about 1.12 (retrieved in Apr. 2018). This indicates that IEC is a perspective study topic in the evolutionary computation research community.

This paper firstly investigates the characteristics of IEC in section II, and the humans model used in IEC algorithm research and its research issues in section III. In addition, we briefly introduce the latest research philosophies and methodologies of three research directions and aspects of IEC. Based on these introductions and summaries, we present future research issues and concrete research methods. In IEC algorithm research, improving the convergence of the IEC algorithm is one of the solutions to the user fatigue problem. Besides improving the IEC user interface, allowing the user participation in IEC search, and using machine learning to design a better human model, we emphasize and introduce the three research directions and methods in section IV, section V, and section VI, respectively. The first of these is the accelerating algorithm convergence of IEC, second is developing a new IEC algorithm and its optimization framework, and the third is multi-objective IEC. This is a perspective research issue and method that uses IEC to investigate psychological and/or physiological characteristics of humans in human science. We present several research works on this topic, and introduce the research philosophy, methodology, and results on IEC for human science in section VII. Finally, we make a conclusion of the whole work in section VIII.

II. CHARACTERISTICS OF INTERACTIVE EVOLUTIONARY COMPUTATION

Optimization with human's subjective evaluation is a primary characteristic of IEC. From the optimization framework viewpoint, any EC algorithm has its IEC version when the fitness function is replaced with a real human's evaluation. Several EC techniques are used in IEC, such as interactive genetic algorithms (IGA) [2], interactive genetic programming [40], interactive evolution strategy [9], human based genetic algorithm [17], interactive particle swarm optimization [21], interactive differential evolution (IDE) [47], etc. It is necessary to understand the characteristics of IEC when studying IEC algorithms and/or applying IEC to a certain application, so that the proper IEC algorithm can be correctly applied. Concretely speaking, IEC has the following characteristics.

1) optimization using IEC algorithm needs less individuals and less generations to obtain the optimum;
2) it is hard to obtain the global optimum, and the global optimal solution in IEC is not unique;
3) the fitness of IEC is a relative and discrete value; and
4) the fitness of IEC has noise.

Because IEC optimization uses a real human's subjective evaluation, user fatigue will increase along with increasing individual number and evaluation generation times. It is necessary to design an IEC algorithm with less individuals and less generations. It is a promising research direction and solution to establish a human model using machine learning techniques, and use the relations of search variables and their fitness to replace a real human's evaluation.

If a real human cannot distinguish the difference between two individuals, the individuals are considered the same or almost same when we conduct an IEC evaluation. This is the reason that it is hard to obtain the global optimal solution in IEC optimization, and the global optimum is not unique.

The fitness of IEC optimization is a relative value because the fitness comes from the comparisons of individuals. If the fitness in IEC optimization is an absolute and continuous value, the fitness of an individual will have a worse value in the earlier generations, and will have a better value in the later generations. Absolute and continuous fitness value of IEC reduces the selective pressure of the IEC algorithm so as to influence convergence of the IEC algorithm.

The fitness value of IEC optimization is discrete, i.e., there are five or seven discrete values as input of IEC fitness value. This evaluation method ignores the real difference of solutions, so the fitness of IEC optimization has noise. Because IEC evaluation cannot avoid noise, the IEC algorithm, which is sensitive to noise, will lead to poor evaluation performance and result. Therefore, the IEC algorithm that is sensitive to the noise should have a noise filter processing to improve its optimization capability.
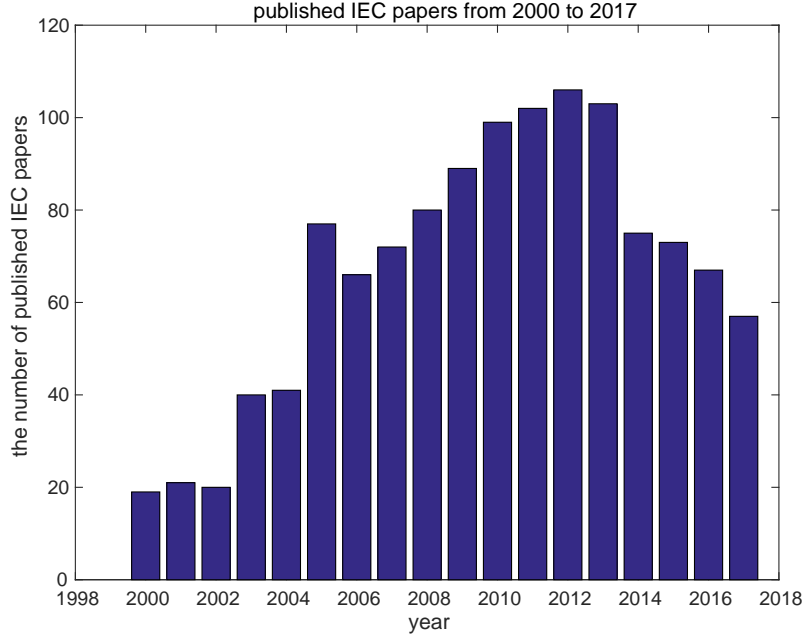
published IEC papers from 2000 to 2017

Fig. 2. IEC papers published in each year from 2000 to 2017, the rate of increase of IEC papers is about 1.10 per year. The data is retrieved from SCOPUS database in Apr. 2018.

III. Human Model Used in Algorithm Research of Interactive Evolutionary Computation

A. Design Philosophy of Human Model

It is very important to simulate IEC optimization in IEC algorithm research. Although the final stage of IEC research is its application to a specific, concrete problem, it is not suitable to use a real human to evaluate IEC algorithm in its research and investigation stage. When we need to multiply running results of IEC evaluation to obtain a statistical conclusion of the IEC algorithm, the real human's evaluation is not reliable and repeatable. The human model of IEC algorithm can replace the real human to obtain reliable and repeatable evaluations. With regard to statistical tests in IEC algorithm evaluations, the Wilcoxon signed-rank test [53] is usually used in two IEC algorithms' comparison, and the Friedman test [5] and the Bonferroni-Dunn procedure [3] are involved in multiple IEC algorithms' ranking and comparison.

The IEC algorithm research needs the simulation of human evaluation. These simulations are implemented by designing a human model. When we design a human model for IEC algorithm research, we need to consider the characteristics of IEC optimization, e.g., fitness being a relative and discrete value, etc. In a common EC algorithm, if we change the fitness function to that with the discrete value, it is a method for designing a human model to simulate IEC optimization. Concretely speaking, in the process of IEC optimization, we divide a continuous search range of fitness into n levels averagely to implement the relative and discrete fitness value. If IEC optimization requires an elite strategy, we can choose

one of individuals, whose fitness is in the first level of fitness range.

Designing a human model for IEC algorithm research, it demands some special considerations, such as in recurrent IEC research, where the Gaussian mixture model (GMM) has been used as a human model [36]. The design philosophy of GMM depends on the following.

1) model's fitness landscape should be simple;
2) model should be multi-modal;
3) model has a big valley structure in the whole scale; and
4) complexity and shape of the model should be controlled by tuning some parameters.
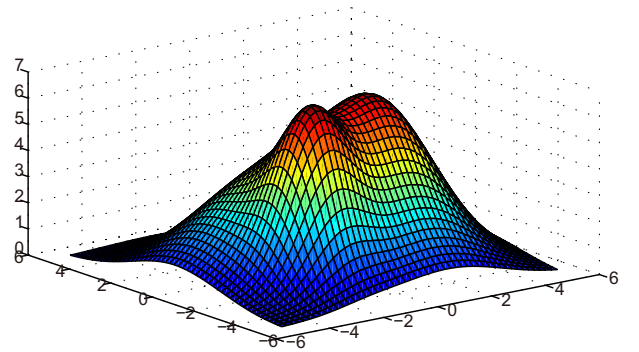

Fig. 3. A three dimensional view of a Gaussian mixture model.

In a certain range of evaluation for a human user, he/she cannot distinguish the better or worse of the solutions, so the human model should be relatively simple. The user of

IEC optimization will give the same evaluation to the solutions, such as image, audio, and other evaluation objects, although the design parameters (search variables) are different. Therefore, the human model should be designed as a multimodal function. The user of IEC optimization can obtain a satisfactory solution after a few generation evaluations, so the human model should be a big valley structure in the whole range view (see Fig. 3). For different applications, the fitness landscape needs to be tuned to be adaptive to a variety of such applications, so the complexity and shape of human model should be controlled by tuning some parameters. Fig. 3 gives an example of GMM, its parameter design is in Eq. (1). Fig. 3 presents a three dimensional GMM landscape. In the common condition, the designed parameters should be controlled less than ten, the parameter search variable range (x) of this GMM is $[-5.12, 5.12]$.

$$GMM(\mathbf{x}) = \sum_{i=0}^{k} a_i \exp(-\sum_{j=0}^{n} \frac{(x_{ij} - \mu_{ij})^2}{2\sigma_{ij}^2}). \quad (1)$$

where

$$\sigma_{\mathbf{ij}} = \begin{pmatrix} 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 & 1.5 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}.$$

$$\mu_{\mathbf{ij}} = \begin{pmatrix} -1 & 1.5 & -2 & 2.5 & -1 & 1.5 & -2 & 2.5 & -1 & 1.5 \\ 0 & -2 & 3 & 1 & 0 & -2 & 3 & 1 & 0 & -2 \\ -2.5 & -2 & 1.5 & 3.5 & -2.5 & -2 & 1.5 & 3.5 & -2.5 & -2 \\ -2 & 1 & -1 & 3 & -2 & 1 & -1 & 3 & -2 & 1 \end{pmatrix}.$$

$$a_i = \left\{ \begin{array}{cccc} 3.1, & 3.4, & 4.1, & 3.0 \end{array} \right\}.$$

It is the fundamental difference between common EC and IEC that the fitness of IEC has relative and discrete characteristics. The human model of IEC can support a relative fitness value rather than an absolute value. Its implementation lies in the separation of the fitness values into n level discrete values, e.g., from one to five levels. Compared with the common EC algorithm, it needs an accurate and continuous fitness value.

B. Training of Human Model

From the systemic viewpoint of the evaluation in IEC optimization, user input is the optimized parameters (in parameter space), and the output is subjective evaluations. We can establish this input-output relation with a mathematical model so as to analyse the human model to obtain the knowledge of a certain IEC user. At the same time, we can use such human model to help the IEC algorithm search to accelerate its search convergence. There are three primary modeling methods to establish a human model for IEC algorithm research. They are the case reasoning-based human model [20], neural network-based (NN) human model [26], and fuzzy system-based human model [16].

1) Case Reasoning-based Human Model: In the case reasoning-based human model, it is easy to calculate the distance between designed variables in parameter space so as to obtain the differences of a new solution and searched solutions [20]. However, in a specific IEC-based optimization application, the influences of designed parameters are not equal. Because the relation of fitness and designed parameters is not linear, the influences of designed parameters are also different. From these designed parameter sensitive and unsensitive applications, we should distinguish these conditions. These conventional distance-based human model cannot obtain a good evaluation result. A human model, which can distinguish parameter sensitive and parameter unsensitive cases, should be considered in such applications.

2) Neuron Networks-based Human Model: Because the neural network is a universal approximator [10], an NN-based human model can establish any relation of designed parameter and fitness using the non-linear modeling characteristic of NN [26]. The primary issue of NN-based human model lies in the training of the model. Because the IEC has the characteristics of less population size and less evaluation generations, it can only support a small number of input-output data. The trained NN by using such a small amount of data will lead to less accuracy of the human model, so that it restricts the application of the NN-based human model in IEC optimization. Therefore, only this NN-based model, which can be trained with less data sets and obtains high model accuracy, can be used in modeling the IEC human model.

3) Fuzzy System-based Human Model: Fuzzy system-based models have been widely studied since fuzzy sets, fuzzy logic and fuzzy system were found [56], and it was proven as a universal approximator as well [52]. The advantage of the fuzzy system-based human model lies in its training [16]. Because of the off-line training characteristic of the fuzzy system-based human model, it can be trained from the first generation of IEC optimization, rather than the NN-based model, which needs more training data to a certain level of accuracy to be used in IEC optimization. Further, the NN-based human model cannot be used from the first generation of IEC optimization. These are two differences of fuzzy system-based human model and NN-based human model.

4) Training Issues of Human Model: In order to obtain enough training data for the human model, it is necessary to collect evaluation data from the IEC user in the initial generations. After obtaining such a well trained human model, we can use it in the later generations of IEC optimization to replace the real human so that the IEC with large population size and more generations can enhance the IEC search capability to a fast convergence. This is a promising research topic for further investigation. We can also establish several human models in the IEC optimization, and find the best configured model in the specific application to replace the real human for accelerating the IEC search. This is another research subject regarding the IEC human model. In addition, how to find the difference of an IEC user and established human model, and how to reduce this difference are important research issues and topics in IEC human model study.

C. Use of Multiple Models of Past IEC Users

The human model of IEC evaluation is a promising research topic both to accelerate IEC search and to relieve IEC user fatigue. However, before using an IEC human model, the

Fig. 4. IEC optimization with a user model.

Fig. 5. IEC optimization with models of other persons and the user himself/herself.

model must be made using inputs/outputs data to/from the user during the IEC optimization process in some early generations. An IEC human model can help its real human user after a trained model is obtained (see Fig. 4). In the worst case, the IEC optimization may end before a human model is made.

Preparing multiple models of past IEC users in advance and using the one most similar to the current IEC user among them until his/her model is actually trained out is one of the solutions for this problem [8]. We collect several past IEC user models before we start IEC optimization. When inputs are given to an IEC user, the same input data are given to these models and their model outputs are calculated. Next, actual IEC user evaluation responses are compared to these model outputs, and one model whose outputs are the most similar to the actual user responses. This selected model is used as a temporal human model and is used to estimate user evaluation values for many individuals and the top best n individuals are given as offspring candidates in the next generation. During this temporal model's use, the inputs/outputs to/from the actual IEC user are used to train his/her actual human model. This process is continued until the generation when his/her human model is made, and the trained human model is used to estimate his/her evaluation values since then (see Fig. 5).

IV. ACCELERATION OF SEARCH IN INTERACTIVE EVOLUTIONARY COMPUTATION

A. Function Approximation of a Fitness Landscape

One of the EC acceleration methods is towards approximating a fitness landscape with a simpler shape and quickly reach towards the global area [14]. Unlike the original complex fitness landscapes, it is possible to reach to the global optimum of the approximated simple function quickly without converging to a local optimum. This approach is based on an assumption that the global optimum of the approximated function is located near that of the original fitness landscape.

1) Fitness Landscape Approximation in Original Search Space: Our first proposal was to approximate a fitness landscape with a uni-modal (single peak) function and replace the worst individual with the peak point of the approximated function as an elite individual [11] (see Fig. 6). The roughest approximation of any fitness landscape would be a uni-modal function, and we can obtain its peak location analytically, i.e., without EC search, when we use a certain kind of uni-modal function. We approximated an n-dimensional (n-D) fitness landscape with an n-D quadratic function and obtained an elite individual by solving n pluralistic simultaneous equations in our experiments.
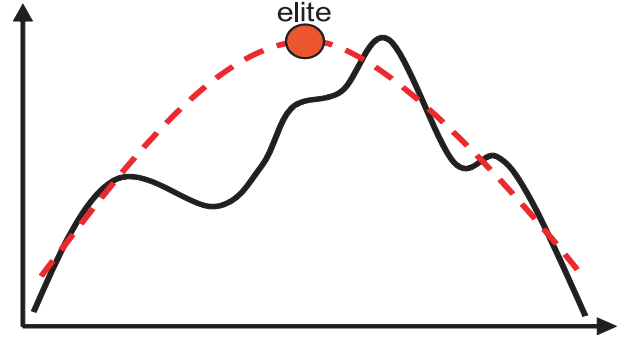

Fig. 6. Function approximation of n-D parameter space with a uni-modal function. The apex is used for search as an elite. We can use the elite to accelerate IEC search by replacing it with the worst individual. It is a low risk and high return method.

2) Fitness Landscape Approximation in Dimensionality Reduction Search Space: Our second proposal was to simplify our first proposal and reduce its computational cost for solving simultaneous equations [33]. Its idea is to project an n-D fitness landscape onto each parameter axis, approximate each projected one-dimensional (1-D) fitness landscape with a 1-D uni-modal function, and estimate the elite by synthesizing the peaks of n 1-D uni-modal functions instead of approximating an n-D fitness landscape with one n-D uni-modal function directly (see Fig. 7). Experimental results showed the computational cost of the second method became 1/3 - 1/2 of that of the first method while their acceleration performance are almost the same [36].
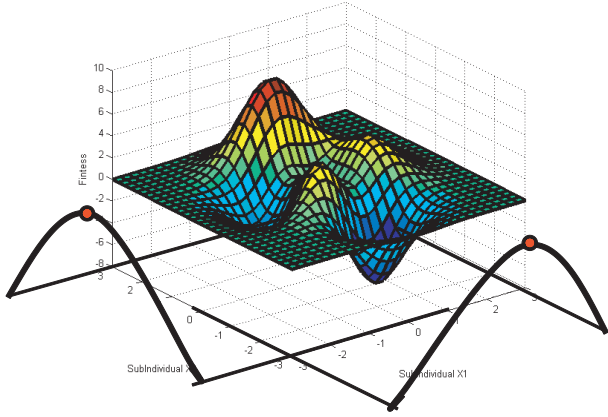
Fig. 7. 1-D function approximation and its obtained elite location in an projected 1-D parameter space. We estimate the elite by synthesizing the peaks of n 1-D uni-modal functions to accelerate IEC search.

B. Complexity Analysis of Fitness Landscape Using Fourier Transform

The novelty of the Fourier transform approach is to accelerate EC search using frequency information of a fitness landscape [35]. We obtain fitness values of multiple search points at even intervals in a parameter space, apply fast Fourier transform [7] to them, and obtain the frequency with the biggest amplitude, denoting this frequency as a principal frequency. Our idea is to approximate a fitness landscape with a sine or cosine curve obtained by the principal frequency and its phase information and use the peak point of the approximated sine or cosine curve as the elite with the same method as described above (see Fig. 8). Experimental results with eight benchmark functions showed that this method is efficient in accelerating most of them [34].

We also proposed a Fourier Niche method by modifying the method mentioned to discover better multiple local optima. While the Fourier approach mentioned in the above uses only one major frequency, the Fourier Niche method uses not only it but also the second, third, and other major frequency components and approximates a fitness landscape to estimate the second, third, ... local optima. Experimental results with six one dimensional and two dimensional benchmark functions showed that this method was efficient in finding local optima and accelerating most of them.

C. Interactive Differential Evolution with (gravity + Moving) Vector

IEC user fatigue is a serious issue for practice of IEC applications as we described. However, many accelerating methods for differential evolution (DE) proposed so far are not always applicable to interactive differential evolution (IDE) [47]. Since IEC users cannot continue to evaluate IEC tasks for a long time, only methods which have acceleration effects that appear in early search generations with smaller population size without increasing additional human evaluations can be used for IEC. Our development of IEC acceleration methods must satisfy these restrictions.

IEC can reach towards the global optimum area even when searching with a small population size in a few generations. This means that fitness landscapes of IEC tasks are simple. As the DE/best approach works better than the DE/rand approach for a simple fitness landscape in general [42], we want to use the IDE/best to make IDE converge quickly. However, an IDE user must compare all individuals to find the best individual when he/she uses IDE/best, and it loses the good feature of DE, a paired comparison nature, and increases IDE user fatigue.

We proposed the DE/gravity approach that uses the center of population gravity as a base vector [6]. It does not increase the number of user evaluations, but its performance is close to the DE/best approach. We also proposed to use moving vector information from parent individuals to their offspring. The good news is that these two methods work complementary. When the global optimum locates around a center point in a parameter space, the DE/best works well but moving vectors cancel each other and their average moving vector works ineffectively. On the other hand, when the global optimum locates around the edge of a search space, the average moving vector works well, while DE/gravity does not work well. Simulation results showed that the convergence speed of the (IDE/gravity + moving vector) is faster than the IDE/rand and close to the IDE/best thanks to their complementary effect without increasing the numbers of human evaluations.

D. Triple or Quadruple Comparisons-based Interactive Differential Evolution

We proposed a triple or quadruple comparisons-based mechanism to enhance differential evolution (DE), especially IDE, without increasing IDE user's fatigue to a large extent [37]. Not only a target vector and a trial vector of canonical DE but also their opposite vector(s) generated by opposition-based learning [51] are compared, and the best vector among them becomes an offspring in the next generation.

The biggest feature of the paired comparison-based IDE is "less user fatigue", and the triple or quadruple comparisons-based IDE weaken this feature. However, there are many IEC tasks that are shown to an IEC user time-sequentially but we can memorize/recall three or four task outputs easily [23]. These kinds of tasks do not increase IDE user fatigue seriously unlike interactive genetic algorithms (IGA) [2]. If the increased number of human comparisons in each generation results in the acceleration of IDE convergence and decreases the total number of user evaluations, it may be better than canonical IDE.

We evaluated the proposed method by comparing it with canonical IDE and conventional opposition-based IDE using a Gaussian mixture model for simulating IDE. We also compare them using 24 benchmark functions for evaluating DE. The experiments showed that our proposed methods could enhance IDE and DE search efficiently from several evaluation indexes including the converged fitness values at the same generation numbers and the same number of fitness calculations, fitness
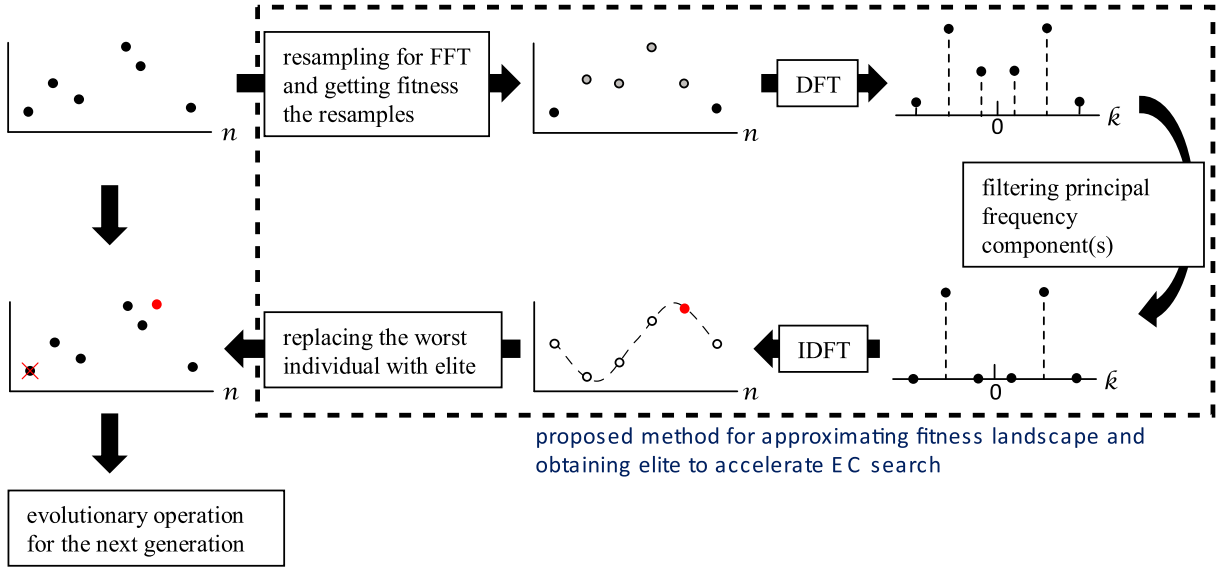
Fig. 8. Obtaining frequency information by applying fast Fourier analysis and using an elite from a trigonometric function ($\sin(x)$ or $\cos(x)$) to accelerate IEC search.

calculation cost, success rates of convergence, and acceleration rates [37].

E. Triple Comparisons-based Interactive Differential Evolution with Memetic Search

Another implementation of multiple comparison-based IDE algorithm uses memetic search to construct its optimization framework [38]. In conventional IDE, the comparison of target vector and trial vector supports a local fitness landscape where is the promising search area in parameter space. If the fitness of target vector is better than that of the trial vector, we can apply a memetic search around the target to obtain a third vector, and vice versa. We compare the target vector, trial vector, and the third vector to implement another form of triple comparison-based IDE. The evaluation results presented a better optimization capability comparing with conventional IDE.

F. Prediction of an Estimated Convergence Point

IEC is a stochastic optimization algorithm. From the fundamental point of view, it can be explained by the probability theory. If we embed a deterministic search method in IEC or EC algorithms, its optimization should be enhanced. A moving vector from the last generation to current generation supports fitness landscape information where the global optimum is located. If we use such information to estimate the convergence point, IEC and EC searches should be enhanced.

$$
\begin{aligned}
\hat{\mathbf{x}} &= \left(\sum_{i=1}^{n} H_i\right)^{-1}\left(\sum_{i=1}^{n} H_i\mathbf{a}_i\right). \qquad (2)\\
&= \left\{\sum_{i=1}^{n}\left(I_d - \mathbf{b}_{0i}\mathbf{b}_{0i}^{\mathbf{T}}\right)\right\}^{-1}\left\{\sum_{i=1}^{n}\left(I_d - \mathbf{b}_{0i}\mathbf{b}_{0i}^{\mathbf{T}}\right)\mathbf{a}_i\right\}.
\end{aligned}
$$

We introduce a method to estimate a convergence point analytically to accelerate an IEC or EC search [24]. When we search a d-dimensional parameter space using one of the IEC or EC algorithms with n individuals, let the i-th parent individual, its offspring individual, and moving vector be $\mathbf{a}_i$, $\mathbf{c}_i$, $\mathbf{b}_i = \mathbf{c}_i - \mathbf{a}_i$, respectively; $\{(\mathbf{a}_i, \mathbf{c}_i),\ i = 1, 2, ..., n;\ \mathbf{a}_i, \mathbf{c}_i \in \mathbf{R}^d\}$ (see Fig. 9). The unit directional vector of $\mathbf{b}_i$ is also defined as $\mathbf{b}_{0i} = \mathbf{b}_i/\|\mathbf{b}_i\|$ $(\mathbf{b}_{0i}^{\mathrm{T}}\mathbf{b}_{0i} = 1)$. In Fig. 9, when we refer to a vector we mean a column vector. Given n vectors, $\mathbf{b}_i$, that extend from n vectors, $\mathbf{a}_i$, let $\mathbf{x} \in \mathbf{R}^d$ be the point that is nearest to the lines made by extending the line segments $\mathbf{b}_i$ ($\mathbf{x}$ is indicated by the $\star$ mark in Fig. 9.). Eq. (3) shows how to estimate $\mathbf{x}$, where I_d is an identity matrix, and $H_i = \mathbf{b}_{0i}\mathbf{b}_{0i}^{\mathrm{T}} - I_d$. We can use this elite point to accelerate EC and IEC search [54], and perspectively it can be extended into multi-modal [55] and multi-objective optimization tasks.

V. NEW ALGORITHM FRAMEWORK OF INTERACTIVE EVOLUTIONARY COMPUTATION

A. Paired Comparison-based Interactive Differential Evolution

It is hard for an IGA user to evaluate tasks displayed time-sequentially, such as sounds or movies, because he or she cannot compare them at once and must compare them in his/her memory, which increase human fatigues. IGA is not suitable for tasks for which individuals cannot be given to an IGA user at once.

Tournament GA is one of its solutions, and a tournament IGA user chooses a better individual or gives fitness to paired individuals [22]. Although pair-based comparisons can reduce IGA user's fatigue, the tournament method does not compare whole individuals, and therefore fitness values must include noise, which reduces their search capability.
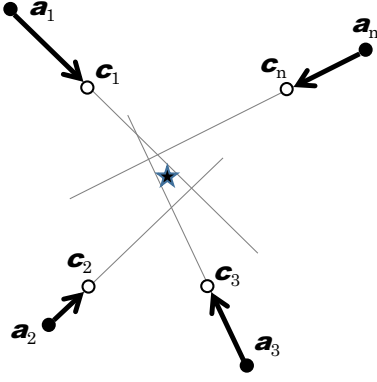
Fig. 9. The convergence point ($\star$) estimated by the moving vectors between individuals ($\mathbf{a}_i$, $i = 1, 2, ..., n$) in the d-dimensional parameter space in the k-th generation and those ($\mathbf{c}_i$, $i = 1, 2, ..., n$) in the $(k + 1)$-th generation. We can use this estimated point as an elite individual to accelerate EC or IEC search. This method also can be extended into multi-modal and multi-objective optimization tasks.

We pointed out that DE includes paired comparison in its algorithm and is suitable for IEC, and we proposed the paired comparison-based IDE [47]. The final stage of DE operation is the comparison of the target vector and a trial vector, and the better one becomes an offspring individual in the next generation. The IDE user compares pairs of sounds, movies, and other tasks without any modification of the DE algorithm unlike the tournament IGA.
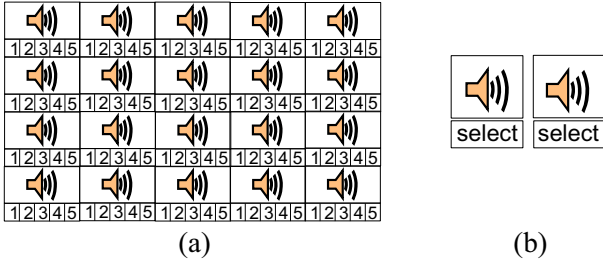

Fig. 10. Graphical user interfaces of (a) IGA and (b) IDE. The IGA user compares all individuals simultaneously and gives a relative evaluation value to each of them. The IDE user compares a pair of individuals and chooses the better one.

Graphical user interfaces of IGA and IDE are shown in Fig. 10. The IGA user compares all individuals and gives a relative evaluation value to each of them. When the tasks are sounds or movies, it is hard for IGA user to memorize them and evaluate them. The IDE user compares a pair of individuals and chooses the better one. It is not hard to compare two individuals even if they are sounds or movies. We confirmed that the IDE converges faster than IGA through simulation experiments [47] and are evaluating user fatigue through human subjective test.

B. Interactive Particle Swarm Optimization

Particle swarm optimization (PSO) is a meta-heuristic population-based algorithm inspired from the particle behaviour of animals [4]. The primary mechanism of it optimization follows the principle of evolution using the search positions of the particle itself, the best local particle, and the best global particle.

From the current comparison study of PSO, the optimization performance of PSO is better than that of genetic algorithm (GA) for these problems with the simple fitness landscape, and on the contrary, GA is better than PSO for these problems with complex fitness landscape. Interactive PSO can converge due to simple landscape of human evaluation space, when the interactive PSO algorithm has less generations and less population size. So the framework of PSO can be used in interactive application. However, when the interactive problem has evaluation noise, the interactive PSO is worse than IGA [25]. How to reduce noise from the human subjective evaluation, it is a promising research subject for further investigation in interactive PSO.

C. Interactive Fireworks Algorithm

The fireworks algorithm is an optimization algorithm that simulates the fireworks explosion phenomenon in parameter search space [50]. In its algorithm framework, the search range and the number of generated offspring are decided dynamically by considering global exploration and local exploitation. We have studied approximation sampling issues of firework algorithms, and found it only needs a few of individuals in its optimization process [39]. Because there are smaller population sizes (usually less than ten individuals) in the fireworks algorithm, it should be a great search algorithm in the IEC framework. The development of interactive fireworks algorithm and application of this algorithm is also a potential research subject for further study.

D. Interactive Chaotic Evolution

Determinism, probability, and chaos are three fundamental philosophies and methodologies in scientific research [30]. In the optimization field, determinative and stochastic optimization algorithms are well studied with theoretical supports from determinism and probability. However, there is little research that mentions chaotic optimization supported by chaos theory. Chaotic evolution is a new type of evolutionary computation algorithm that fuses the ergodicity of chaos and the iteration of evaluation [29]. It should be one of implementations of the chaotic optimization method.

The primary operation of chaotic evolution simulates chaotic motion in a parameter space when chaos occurs in the system. Because there is a paired-comparison mechanism in its optimization framework, and its convergent speed is better than DE, interactive chaotic evolution should be better than IDE, however, it also needs experimental evaluation to verify this hypothesis. Developing the interactive chaotic evolution algorithm, multi-modal chaotic evolution algorithm, and multi-objective chaotic evolution algorithm [32] are three promising research subjects in chaotic evolution.

VI. MULTI-OBJECTIVE INTERACTIVE EVOLUTIONARY COMPUTATION

The new framework of IEC research is the combination of IEC with evolutionary multi-objective optimization (EMO).

Since EMO algorithms handle solutions in an objective space, the coordinates of the solutions must be specified by numerical objective values. However, there are multi-objective tasks with objects that are not clearly specified numerically. For example, let's consider apartment search. EMO is applicable to finding less expensive and wider apartments located near a train station as these three objects have numerical values. However, it does not work for finding apartments with object(s) that have to be evaluated subjectively, e.g., "*greater view from a room window*". The framework of IEC+EMO is developed for EMO tasks with objects that are evaluated not only objectively but also subjectively.

A. *Micro-Electro-Mechanical System Design*

Micro-Electro-Mechanical Systems (MEMS) have been designed manually by using a computer-aided design system. To automate this design, EMO was applied together with an MEMS simulation tool calculating fitness values [57]. However, it is not easy to automate the design completely. One of the reasons is that we cannot describe all design knowledge into the system, and optimization parameters in MEMS design do not cover all design aspects influencing MEMS performance.

IEC can overcome this problem and increase the performance of MEMS design by cooperating with designers' subjective evaluations. MEMS design experts can evaluate how designed MEMSs are good by just glancing at whole designs based on their experience and knowledge.

We compared the EMO+IEC-based approach and EMO-only approach by applying it to designing an MEMS resonator. The objectives of EMO are MEMS area size, a resonant frequency, and stiffness, and IEC users evaluated designed MEMS circuits visually. Sign test result showed that the evaluations of the EMO+IEC-based approach was significantly higher than that of the EMO-only approach [15].

B. *Architectural Room Floor Planning*

Room floor planning includes multi-restrictions and multi-objectives such as room sizes, room shapes, moving line to any room without passing through other private rooms, and window sizes requested by architectural laws. Architects design floor layouts taking account of these points. If there is an automated room floor planning support system, architects can save their design time, and house owners can enjoy designing their houses before asking professional architects. This support system should be based on EMO to find solutions taking account of the above objectives. However, this task has further object; this is the *satisfaction* of architects or house owners with regard to the design plans. The EMO algorithm cannot handle qualitative objectives, and EMO+IEC is necessary for the support system. We developed an algorithm for generating room floor layouts [12], combined it with EMO and IEC, and evaluated the designed layouts [13]. A professional architect used the system and discovered a new type of room layout that he usually does not consider in his manual designing. His

evaluation was that the system is useful as an inspiration-assist system.

VII. Psychological and Physiological Research Using Interactive Evolutionary Computation

A. *Measuring Perceived Emotional Expression*

We applied IEC to measure a *happy–sad* range in the human mind and compared the ranges of schizophrenics and mentally healthy people. Some therapists feel that the emotional impressions shown on the faces of schizophrenic patients are fewer than those of mentally healthy people based on their experience. However, there was no way to measure this range. We asked three schizophrenics and five mentally healthy students to design 3-D computer graphics lighting of the *happy* impression and the *sad* impression using our IEC-based 3-D computer graphic lighting design support system [1], and asked 33 human subjects to evaluate 28 pairs ($=_{(3+5)}C_2$) of designed lighting images.

Fig. 11 is the psychological scale of *happy* constructed using the Scheffé's method of paired comparison. The *happy–sad* ranges obtained from the experimental results imply that it is hard for schizophrenic patients to identify especially a *happy* impression in lighting, and it is expected that this IEC approach may provide new data that are helpful for psychiatric diagnostics [48].
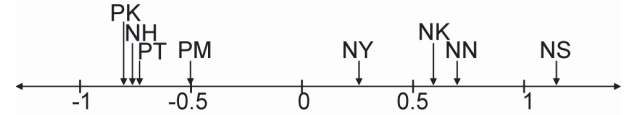

Fig. 11. Psychological scale constructed using the Scheffé's method of paired comparison and impression levels of the eight best lightings designed by three schizophrenics (PK, PT, and PM) and five mental healthy students (NH, NY, NK, NN, and ND). The bigger measure values, the higher evaluation of *happy*.

B. *Hearing-aid Fitting and Discovering Unknown Knowledge*

IEC is the best way to carry out hearing-aid fitting because sound qualities for a certain hearing-aid users cannot be measured. Another advantage is that it allows us to fit a hearing aid using any daily-life sounds, while the conventional fitting method has to use only pure tones and narrow band noise. Thanks to this feature, we could find several unknown facts [46]. They are:

1) the characteristics of hearing aids optimized using speech sounds that were different from those optimized using pure tones or band pass noise;
2) those optimized using speech sounds of speaker i with/without noise were almost similar ($i = 1, 2, ...$); and
3) those optimized using speech sounds were different from those optimized using music.

Nobody had recognized the fact that the best characteristics of hearing aids depend on the kinds of sounds used for fitting. This implies that an audible range in human sense level is

not the final cue for the best hearing. We could make these observations thanks to the IEC technique.

C. Cochlea Implant Fitting and Discovering Unknown Knowledge

Cochlea-implant fitting is a similar task to a hearing-aid fitting and has been conducted based on two hypotheses for better fitting: (1) the more electric channels of a cochlea-implant, the better and (2) the wider dynamic range of each channel, the better. As frequency resolution increase according to the number of electric channels, the hypothesis (1) means that higher frequency resolution helps to distinguish the difference of frequency characteristics of phonemes; this hypothesis sounds natural. The hypothesis (2) means that enabling a user to hear sounds from the minimum level to the maximum comfortable level is helpful to distinguish sounds; this hypothesis also sounds natural.

IGA was used to tune the fitting parameters of cochlea implants [19]. Their experimental result was that the dynamic ranges of all 15 channels were almost 0 except for three or four channels, and the dynamic ranges of the exceptional three or four channels are narrower than the maximum ranges (see Fig. 12). Nevertheless, its recognition rate with IGA fitting was higher than that of manual fitting. This result did not match the two hypotheses mentioned. This implies that there must be unknown audio-psychophysiological facts. It is a valuable subject to discover these unknown facts using IEC techniques. We will continue researching this subject in the future.
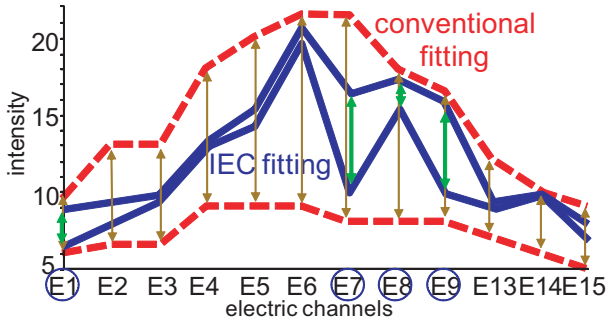

Fig. 12. Fitting characteristics of conventional cochlear implant fitting and that using IEC. Horizontal axis means electric channels and a vertical axis means electric voltage of each channel. This figure was remade based on an image in [19].

D. Interactive Evolutionary Computation Framework Using Physiological Measurement

The fitness of a conventional IEC algorithm comes from the subjective evaluation of humans, and these evaluations are representations of human's physiological space. IEC framework using physiological measurement is an extension of conventional IEC. There are further potential research directions for IEC framework using physiological measurement. The first direction uses physiological measurement as the IEC fitness function to tune the human's physiological condition to a certain level. We can use this framework to influence and

study human's physiological condition. The fitness of IEC framework using physiological measurement can use blood pressure, frequency of heart beat, frequency of respiration, etc as the physiological measurement. Reference [49] proposed an extension framework of IEC using physiological measurement, and conducted the experimental study using a simulated physiological signal. Fig. 13 presents an extension IEC framework.
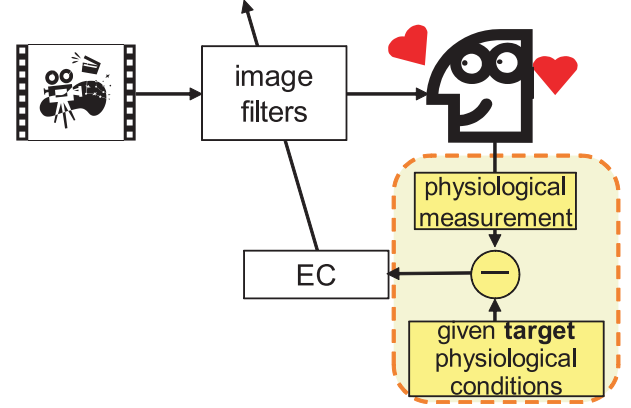

Fig. 13. Framework of extended IEC. IEC optimizes physical features of movies or music using a fitness value that is the difference between the target physiological responses and real one of an IEC user.

The second new type of IEC research is adapting physiological data. Usually, IEC optimizes a target system based on an IEC user's subjective evaluation, i.e., psychological evaluation. We may extend the evaluation from a psychological one to a physiological one. When the outputs of an IEC target system are given to an IEC user, such as listening to music, watching movies, or enjoying vibrations, some physical stimuli influence his/her physiology. Then, we may be able to drive his/her physiology by controlling the physical stimuli. Suppose that physiologists advise us on the ideal physiological conditions for relaxation, excitement, or other target mental conditions. IEC framework may be able to direct the human physiological conditions to the ideal by minimizing the error between measured physiological data and this ideal. Trials have been held in applying the extended IEC to control eight parameters of a vibration chair using physiological data of an extended IEC user on the chair and to direct emotions of video movie viewers [43] though they have not been completed. Another trial using physiological data is to guess IEC fitness by using the eye-tracking of an IEC user [27]. Other physiological data such as electroencephalogram can be used to guess IEC user's evaluation for the selection of better individuals.

E. Aesthetic Selection Analysis Using Subjective Evaluation from Interactive Evolutionary Computation

The machine learning algorithm attempts to find knowledge from data, and most machine learning research uses deterministic data analysis techniques to achieve this objective. There is some research that tries to apply human related computing techniques for assisting machine learning algorithms to discover the aesthetic selection of humans.

Reference [31] presents a work that uses IEC to select principal components of compressed images and tries to find human visual perception knowledge from human's subjective evaluation. The work compresses an image using principal component analysis [28], and in the process of resorting the image, pared comparison-based IDE algorithm is applied to display to the IEC user two restored images for judging which one is the clearer one. The selection results indicate that:

1) Human's subjective selection can obtain more detailed information from the visual perception;
2) Humans can perceive more detailed visual information that is presented by the principal components with the small eigenvalues;
3) Subjects' aesthetic characteristics can be obtained by analyzing the data of principal components; and
4) Human visual perception is sensitive to the shape and frame information from images' comparisons.

This work is an attempt to apply IEC techniques in machine learning algorithms to find the aesthetic characteristics of humans. Although some of the conclusions and analysis results need to be seriously verified by psychological and physiological fundamentals of human, hopefully, there will be an opportunity to apply IEC in such human related knowledge discovery.

F. Awareness Computing Using Interactive Evolutionary Computation

IEC can be used as a tool to analyse human's psychological and physiological characteristics. In awareness computing, IEC can also be used to analyse human's awareness mechanism [45]. Much research has different definitions on awareness computing from a variety of viewpoints. Although there are many studies on awareness computing, there is a need to further deepen the research on the human awareness mechanism, and establish the fundamentals of awareness computing.

Reference [45] defines awareness computing as follows. Awareness computing is a process that helps us to understand unknown or potential facts and concepts. This can use IEC to achieve such an objective. For example, when we evaluate a student with a mathematic credit, literature credit, and physics credit, and with his/her height, and weight, we can determine academic capability and physical condition, these two concepts, from this data. The process of obtaining these two concepts is a process of awareness computing, and these two concepts are the definitions of awareness of this process.

The model of awareness uses human's input as the model's input, and output after a human recognizes it as the model's output. The model helps us to find relations between this input and output. We can use input and output of IEC optimization to establish a model using machine learning methods, and then, separate this model into two parts to find the potential factors and concepts. It is a method to establish awareness model and implement awareness computing. For example, variables (x, z, y) present input, output, and potential concepts, respectively. We can establish an evaluation model $z = f(x)$, and awareness computing separates $f(x)$ into

Fig. 14. Finding out latent variables inside the human model in awareness computing. Variables (x, z, y) present input, output, and potential concepts, respectively. We can establish an evaluation model $z = f(x)$, an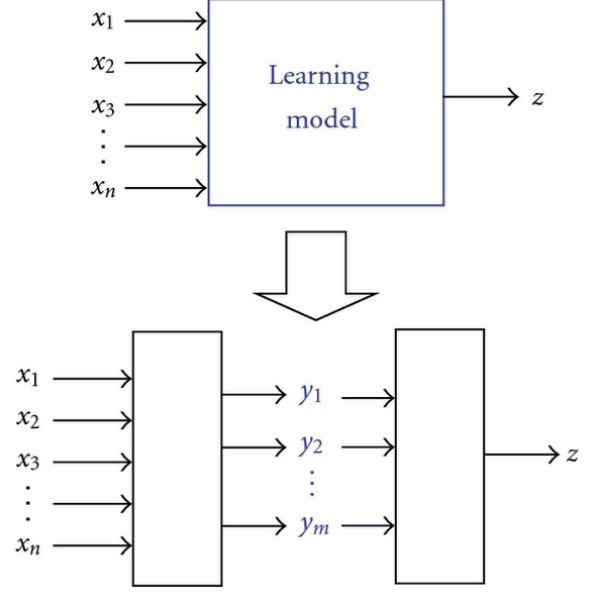d awareness computing separates $f(x)$ into $y_1 = g_1(x), y_1 = g_1(x), ...$ to find $y_i, i = 1, 2, ...$ that presents the potential concepts

$y_1 = g_1(x), y_1 = g_1(x), ...$ to find $y_i, i = 1, 2, ...$ that presents the potential concepts (see Fig. 14). The primary issues of this process try to find effective methods to separate the model. Principal component analysis [28], factor analysis [41], and non-negative matrix factorization [18], these feature selection methods, can be applied in model separation for implementing awareness computing. This is a promising research issue and topic in awareness computing with IEC.

VIII. CONCLUSION

IEC was born when Richard Dawkins showed that complex figures could be generated based on visual evaluation, and showed evolved 2-D line figures (biomorph) as is seen in examples in his book [2]. Since the 1990's, IEC application areas have spread from computer graphic applications to engineering, entertainment, and many other areas. We did not mention this major IEC research in this paper but explained our research on accelerating IEC and developing new IEC frameworks on the second research direction. Almost all IEC research works are categorized into either of these research directions.

As a new feature, we introduced our research, IEC for human science, as the third direction of IEC research. Artificial intelligence (AI) has become a hot keyword not only in computer science, but also in common scientific research. IEC can act as an interface for the design of a system that is more friendly between human and human, human and computer, human and machine, human and robot pet, etc. It also can be an effective AI tool in human-centered computing,

such as home-assisted robot design and pet robot design, to contribute to an aging society in some countries. The analysis and understanding of human characteristics is an important issue to implement these designs, and IEC is also a great tool to contribute to this field. We hope that many people realize that the capability of IEC is not only optimization from these efforts and IEC can make a much wider contribution.

REFERENCES

[1] Ken Aoki and Hideyuki Takagi. 3-d cg lighting with an interactive ga. In *1st International Conference on Knowledge-Based Intelligent Electronic Systems (KES1997)*, volume 1, pages 296–301. IEEE, 1997.

[2] Richard Dawkins. *The blind watchmaker: Why the evidence of evolution reveals a universe without design*. W.W. Norton & Company, 1986.

[3] Olive Jean Dunn. Estimation of the means of dependent variables. *The Annals of Mathematical Statistics*, pages 1095–1111, 1958.

[4] Russell Eberhart and James Kennedy. A new optimizer using particle swarm theory. In *Proceedings of the Sixth International Symposium on Micro Machine and Human Science (MHS1995)*, pages 39–43. IEEE, 1995.

[5] Milton Friedman. The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *Journal of the american statistical association*, 32(200):675–701, 1937.

[6] Ryohei Funaki and Hideyuki Takagi. Application of gravity vectors and moving vectors for the acceleration of both differential evolution and interactive differential evolution. In *2011 Fifth International Conference on Genetic and Evolutionary Computing (ICGEC2011)*, pages 287–290. IEEE, 2011.

[7] Michael T Heideman, Don H Johnson, and C Sidney Burrus. Gauss and the history of the fast fourier transform. *Archive for history of exact sciences*, 34(3):265–277, 1985.

[8] Shinta Henmi, Shino Iwashita, and Hideyuki Takagi. Interactive evolutionary computation with evaluation characteristics of multi-iec users. In *2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC2006)*, pages 3475–3480. IEEE, 2006.

[9] Michael Herdy. Evolutionary optimization based on subjective selection-evolving blends of coffee. In *5th European Congress on Intelligent Techniques and Soft Computing*, pages 640–644, 1997.

[10] Kurt Hornik, Maxwell Stinchcombe, and Halbert White. Multilayer feedforward networks are universal approximators. *Neural networks*, 2(5):359–366, 1989.

[11] Takeo Ingu and Hideyuki Takagi. Accelerating a ga convergence by fitting a single-peak function. In *1999 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE1999)*, volume 3, pages 1415–1420. IEEE, 1999.

[12] Makoto Inoue and Hideyuki Takagi. Layout algorithm for an ec-based room layout planning support system. In *IEEE Conference on Soft Computing in Industrial Applications (SMCia2008)*, pages 165–170. IEEE, 2008.

[13] Makoto Inoue and Hideyuki Takagi. Emo-based architectural room floor planning. In *2009 IEEE International Conference on Systems, Man and Cybernetics (SMC2009)*, pages 518–523. IEEE, 2009.

[14] Yaochu Jin. A comprehensive survey of fitness approximation in evolutionary computation. *Soft Computing*, 9(1):3–12, 2005.

[15] Raffi Kamalian, Hideyuki Takagi, and Alice M Agogino. Optimized design of mems by evolutionary multi-objective optimization with interactive evolutionary computation. In *Genetic and Evolutionary Computation Conference (GECCO2004)*, pages 1030–1041. Springer, 2004.

[16] Raffi Kamalian, Eric Yeh, Ying Zhang, Alice M Agogino, and Hideyuki Takagi. Reducing human fatigue in interactive evolutionary computation through fuzzy systems and machine learning systems. In *2006 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE2006)*, pages 678–684. IEEE, 2006.

[17] Alex Kosorukoff. Human based genetic algorithm. In *2001 IEEE International Conference on Systems, Man, and Cybernetics (SMC2001)*, volume 5, pages 3464–3469. IEEE, 2001.

[18] Daniel D Lee and H Sebastian Seung. Learning the parts of objects by non-negative matrix factorization. *Nature*, 401(6755):788, 1999.

[19] Pierrick Legrand, Vincent Péan, Esther Harboun-Cohen, Jacques Levy-Vehel, Bruno Frachet, Evelyne Lutton, Pierre Collet, et al. Interactive evolution for cochlear implants fitting. *Genetic Programming and Evolvable Machines*, 8(4):319–354, 2007.

[20] Azahar T Machwe and Ian C Parmee. Reducing user fatigue within an interactive evolutionary design system using clustering and case-based reasoning. *Engineering Optimization*, 41(9):871–887, 2009.

[21] Janos Madar, Janos Abonyi, and Ferenc Szeifert. Interactive particle swarm optimization. In *5th International Conference on Intelligent Systems Design and Applications (ISDA2005)*, pages 314–319. IEEE, 2005.

[22] Brad L Miller, David E Goldberg, et al. Genetic algorithms, tournament selection, and the effects of noise. *Complex systems*, 9(3):193–212, 1995.

[23] George A Miller. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological review*, 63(2):81–97, 1956.

[24] Noboru Murata, Ryuei Nishii, Hideyuki Takagi, and Yan Pei. Analytical estimation of the convergence point of populations. In *2015 IEEE Congress on Evolutionary Computation (CEC2015)*, pages 2619–2624. IEEE, 2015.

[25] Yu Nakano and Hideyuki Takagi. Influence of fitness quantization noise on the performance of interactive pso. In *2009 IEEE Congress on Evolutionary Computation (CEC2009)*, pages 2416–2422. IEEE, 2009.

[26] Miho Ohsaki and Hideyuki Takagi. Reduction of the fatigue of human interactive ec operators-improvement of present interface by prediction of evaluation order. *Jounal of Japan Society for Artificial Intelligence*, 13:712–719, 1998. (in Japanese).

[27] Denis Pallez, Philippe Collard, Thierry Baccino, and Laurent Dumercy. Eye-tracking evolutionary algorithm to minimize user fatigue in iec applied to interactive one-max problem. In *9th Annual Conference Companion on Genetic and Evolutionary Computation*, pages 2883–2886. ACM, 2007.

[28] Karl Pearson. On lines and planes of closest fit to systems of points in space. *Philosophical Magazine*, 2(6):559–572, 1901.

[29] Yan Pei. Chaotic evolution: fusion of chaotic ergodicity and evolutionary iteration for optimization. *Natural Computing*, 13(1):79–96, 2014.

[30] Yan Pei. From determinism and probability to chaos: chaotic evolution towards philosophy and methodology of chaotic optimization. *The Scientific World Journal*, 2015, 2015.

[31] Yan Pei. Principal component selection using interactive evolutionary computation. *The Journal of Supercomputing*, 73(7):3002–3020, 2017.

[32] Yan Pei and Jia Hao. Non-dominated sorting and crowding distance based multi-objective chaotic evolution. In *International Conference in Swarm Intelligence (ICSI2017)*, pages 15–22. Springer, 2017.

[33] Yan Pei and Hideyuki Takagi. Accelerating evolutionary computation with elite obtained in projected one-dimensional spaces. In *5th International Conference on Genetic and Evolutionary Computing (ICGEC2011)*, pages 89–92. IEEE, 2011.

[34] Yan Pei and Hideyuki Takagi. Comparative study on fitness landscape approximation with fourier transform. In *2012 Sixth International Conference on Genetic and Evolutionary Computing (ICGEC2012)*, pages 400–403. IEEE, 2012.

[35] Yan Pei and Hideyuki Takagi. Fourier analysis of the fitness landscape for evolutionary search acceleration. In *2012 IEEE Congress on Evolutionary Computation (CEC2012)*, pages 1–7. IEEE, 2012.

[36] Yan Pei and Hideyuki Takagi. Accelerating iec and ec searches with elite obtained by dimensionality reduction in regression spaces. *Evolutionary Intelligence*, 6(1):27–40, 2013.

[37] Yan Pei and Hideyuki Takagi. Triple and quadruple comparison-based interactive differential evolution and differential evolution. In *12th Workshop on Foundations of Genetic Algorithms XII (FOGA2013)*, pages 173–182. ACM, 2013.

[38] Yan Pei and Hideyuki Takagi. Local fitness landscape from paired comparison-based memetic search in interactive differential evolution and differential evolution. *International Journal of Ad Hoc and Ubiquitous Computing*, 25(1-2):17–30, 2017.

[39] Yan Pei, Shaoqiu Zheng, Ying Tan, and Hideyuki Takagi. An empirical study on influence of approximation approaches on enhancing fireworks algorithm. In *2012 IEEE International Conference on Systems, Man, and Cybernetics (SMC2012)*, pages 1322–1327. IEEE, 2012.

[40] Karl Sims. Artificial evolution for computer graphics. In *18th Annual Conference on Computer Graphics and Interactive Techniques (SIG-GRAPG1991)*, pages 319–328. ACM, 1991.

[41] Charles Spearman. " general intelligence," objectively determined and measured. *The American Journal of Psychology*, 15(2):201–292, 1904.

[42] Rainer Storn and Kenneth Price. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11(4):341–359, 1997.

[43] H Takagi. Directing the emotions of video movie viewers using interactive evolutionary computation based on physiological data. In *2nd International Symposium on Design of Artificial Environments*, pages 244–247, 2007.

[44] Hideyuki Takagi. Interactive evolutionary computation: Fusion of the capabilities of ec optimization and human evaluation. *Proceedings of the IEEE*, 89(9):1275–1296, 2001.

[45] Hideyuki Takagi. Interactive evolutionary computation for analyzing human awareness mechanisms. *Applied Computational Intelligence and Soft Computing*, 2012:3, 2012.

[46] Hideyuki Takagi and Miho Ohsaki. Interactive evolutionary computation-based hearing aid fitting. *IEEE Transactions on Evolutionary Computation*, 11(3):414–427, 2007.

[47] Hideyuki Takagi and Denis Pallez. Paired comparison-based interactive differential evolution. In *World Congress on Nature & Biologically Inspired Computing (NaBIC 2009)*, pages 475–480. IEEE, 2009.

[48] Hideyuki Takagi, Tomohiro Takahashi, and Ken Aoki. Applicability of interactive evolutionary computation to mental health measurement. In *2004 IEEE International Conference on Systems, Man and Cybernetics (SMC2004)*, volume 6, pages 5714–5718. IEEE, 2004.

[49] Hideyuki Takagi, Shangfei Wang, and Shota Nakano. Proposal for a framework for optimizing artificial environments based on physiological feedback. *Journal of Physiological Anthropology and Applied Human Science*, 24(1):77–80, 2005.

[50] Ying Tan and Yuanchun Zhu. Fireworks algorithm for optimization. In *International Conference in Swarm Intelligence*, pages 355–364. Springer, 2010.

[51] Hamid R Tizhoosh. Opposition-based learning: a new scheme for machine intelligence. In *International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce*, volume 1, pages 695–701. IEEE, 2005.

[52] Li-Xin Wang and Jerry M Mendel. Fuzzy basis functions, universal approximation, and orthogonal least-squares learning. *IEEE transactions on Neural Networks*, 3(5):807–814, 1992.

[53] Frank Wilcoxon. Individual comparisons by ranking methods. *Biometrics bulletin*, 1(6):80–83, 1945.

[54] Jun Yu, Yan Pei, and Hideyuki Takagi. Accelerating evolutionary computation using estimated convergence points. In *2016 IEEE Congress on Evolutionary Computation (CEC2016)*, pages 1438–1444. IEEE, 2016.

[55] Jun Yu and Hideyuki Takagi. Clustering of moving vectors for evolutionary computation. In *2015 7th International Conference on Soft Computing and Pattern Recognition (SoCPaR2015)*, pages 169–174. IEEE, 2015.

[56] Lotfi A Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965.

[57] Ningning Zhou, Alice M Agogino, and Kristofer SJ Pister. Automated design synthesis for micro-electro-mechanical systems (mems). In *ASME 2002 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, volume 2, pages 267–273, 2002.