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Convergence Acceleration of the PinT Integration of Advection Equation using Accurate Phase Calculation Method

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Issue and Challenge

Issue of the parareal method for hyperbolic PDEs:



*M. Gander and M. Petcu, Analysis of a Krylov subspace enhanced parareal algorithm for linear problems, ESAIM Proc, 25, (2008), 114-129.

Hyperbolic PDEs: Waves \rightarrow

There is Phase Difference between fine and coarse solver's result \rightarrow Oscillations \rightarrow damages the convergence of the parareal method

■Challenge:

• Improvement of the convergence

 \rightarrow We must reduce the phase difference between fine and coarse solvers (\Rightarrow phase error).

(We just focus on the convergence.)

How do we perform reducing phase error?

Expected methods to reduce phase error in coarse solver with low cost

		To reduce the cost			
		Coarsening		Coarsenin g time step width	
		Space	Time integrator	<u> </u>	> δ t
To increase the phase accuracy	Using fine δt as δT coarse solver time step width	Using coarse space modes → High accurate convergence is not available by lack of high order space modes.	Using low cost time integrator → Available for only the case that fine solver cost is very high.	CAAP method	
	Using a high accuracy phase calculation method	calcu	lation	Using high integrator -> CIP so + STRS	order time Not enough theme scheme

We focus on mainly CIP scheme in this presentation.

Method overview

Methods overview of advection term calculation



We focus on the CIP in this study.

What is CIP(constrained Interpolation Profile) Scheme?

proposed by T. Yabe eta(1987)*

* H. Takewaki and T. Yabe, The cubic-interpolated pseudo particle (CIP) method: Application to nonlinear and multi-dimensional hyperbolic equations, J. Comput. Phys., 70 (1987), 353372.

•performs numerical advection process by the semi-Lagrange method using the cubic interpolation function of **both variable and its gradient**.

The CIP scheme gives the accurate phase.

Why?

The method using a space differential of the equation gives generally highly precise solutions even on coarse grids.



For advection equation, point is the use of **phase information (=gradient)**.

Ex. of FIC results



(b)1D Helmholtz problem



FIC gives the very accurate solution on the very coarse grids.

Let's set a reference to the high accurate phase calculation method.

In this time, we use the conventional methods for advection : (Upwind) differencing scheme

Conventional Discretization method of advection term



Semi-Lagrangian scheme

Considering the equation on the grid i



We choose the TVD scheme as conventional method and set Parareal_CN-TVD as reference.

Code of the coarse solver is same as one of the fine solver.

This part is different only.



δt: time step width of fine solver:
δT: time step width of coarse solver:

set by the CFL condition: $\Delta x/v > \delta t$ $\delta T >> \delta t$

Impact of CIP Method on the parareal convegence.

CIP method

Up stream calculation and time integration is performed by back-trace and shift operation (CFL free formula is used here.)

* Considering the equation on the grid i



id = grid that is near i grid of cell (id, id-1)

$$id = i - INT\left(\frac{x_u p}{\Delta x}\right) = i - INT\left(\frac{c\delta t}{\Delta x}\right)$$

Back-trace points finding

$$\xi_F = x - x_{id} = x_i - c\delta t - x_{id} = -c\delta t - (x_i - x_{id}) = -c\delta t - \Delta x(i - id) = -c\delta t + \Delta x \text{INT}\left(\frac{c\delta t}{\Delta x}\right)$$

Upstream finding
$$D_F = -s(c)\Delta x, \quad s(c) = \text{SIGN}(1.0, c)$$

1-5+1

Detail of Formula

CIP 3rd method

CIP 5th method (We developed it as more accurate CIP at this time.)

$$\begin{array}{l} \partial_t \phi + c \partial_x \phi = 0 \\ \partial_t g + c \partial_x g = 0 \\ g = \partial_x \phi \quad (\text{gradient}) \end{array} \\ & \bullet \quad \text{Space discretization: by the cubic interpolation function} \\ \phi(x) = F_{id}(x) = a_{id}(x - x_{id})^3 + b_{id}(x - x_{id})^2 + g_{id}(x - x_{id}) + \phi_{id} \\ g(x) = \partial_x F_{id}(x) = 3a_{id}(x - x_{id})^3 + b_{id}(x - x_{id})^2 + g_{id}(x - x_{id}) + \phi_{id} \\ g(x) = \partial_x F_{id}(x) = 3a_{id}(x - x_{id})^2 + 2b_{id}(x - x_{id}) + g_{id}(x - x_{id})^4 + c_{id}(x - x_{id})^3 + \chi_{id}/2(x - x_{id})^2 + g_{id}(x - x_{id}) + \phi_{id} \\ g(x) = \partial_x F_{id}(x) = 3a_{id}(x - x_{id})^2 + 2b_{id}(x - x_{id}) + g_{id}(x - x_{id})^4 + c_{id}(x - x_{id})^3 + \chi_{id}/2(x - x_{id})^2 + g_{id}(x - x_{id}) + \phi_{id} \\ g(x) = \partial_x F_{id}(x) = 3a_{id}(x - x_{id})^2 + 2b_{id}(x - x_{id}) + g_{id}(x - x_{id})^3 + 3c_{id}(x - x_{id})^3 + \chi_{id}/2(x - x_{id})^2 + g_{id}(x - x_{id}) + \phi_{id} \\ g(x) = \partial_x F_{id}(x) = 3a_{id}(x - x_{id})^2 + 2b_{id}(x - x_{id}) + g_{id}(x - x_{id})^3 + 3c_{id}(x - x_{id})^3 + 3c_{id}(x - x_{id})^2 + g_{id}(x - x_{id}) + \phi_{id} \\ g(x) = \partial_x F_{id}(x) = 20a_{id}(x - x_{id})^3 + 12b_{id}(x - x_{id})^3 + 3c_{id}(x - x_{id})^2 + g_{id}(x - x_{id}) + g_{id} \\ \chi(x) = \partial_x^2 F_{id}(x) = 20a_{id}(x - x_{id})^3 + 12b_{id}(x - x_{id})^3 + 3c_{id}(x - x_{id})^2 + g_{id}(x - x_{id}) + g_{id} \\ \chi(x) = \partial_x^2 F_{id}(x) = 20a_{id}(x - x_{id})^3 + 12b_{id}(x - x_{id})^3 + 3c_{id}(x - x_{id}) + g_{id} \\ \chi(x) = \partial_x^2 F_{id}(x) = 20a_{id}(x - x_{id})^3 + 12b_{id}(x - x_{id}) + g_{id}(x - x_{id}) + g_{id} \\ \chi(x) = \partial_x^2 F_{id}(x) = 20a_{id}(x - x_{id})^3 + 12b_{id}(x - x_{id}) + g_{id}(x - x_{id}) + g_{id}(x - x_{id}) + g_{id}(x - x_{id}) + g_{id}(x - x_{id})^2 + g_{id}(x - x_{id}) + g_{id}(x - x_{id}$$

Code of CIP-5th method



CIP-5th method is very simple and we can easily develop CIP-5th code based on CIP-3rd method code.

Check performance of the CIP-5th method by comparing the CIP-3rd and CIP-5th methods

Case-1: Advection of step shape

(b) Results of Φ distribution (t=0.5)

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(a) test parameters



(c) Results of Φ distribution(t=2.0)

(c') Zoomed part



CIP-5th method \rightarrow Width of step is thinner. \rightarrow Phase accuracy is better.

Case-2: Advection of sin wave

(a) test parameters

- Physical condition
 - 1D convection of sin wave
 - Speed c=1
 - Space length : L=2
 - Time length : **T=0.6**, **3.0**
- Numerical analysis condition
 - Num. of meshse: 100
 → dx=0.02
 - Width of time step
 → dt=0.015→CFL=0.75
 - \rightarrow one wave/5grids
 - Initial condition
 - \rightarrow Sin wave



CIP-5th → Damping of amplitude is very smaller .
 → Phase accuracy is better.

Let's study the impact of CIP-5th on the parareal iteration!

Parareal_CIP (3rd, 5th)



δt: time step width of fine solver: fit
δT: time step width of coarse solver: δ

fixed by the CFL condition: $\Delta x/v < \delta t$ $\delta T >> \delta t$

Flow of Parareal_CIP is same for CIP-3rd and CIP-5th methods.

Numerical test: Parameters

Test problem	 C = 1.0 and Space [0,3] × Time: [0, 2.0] (a) advection of step shape (b) advection of step like wave
Space and time descritaization	 dx=0.01, 200meshes (10grids/wave) →L=dx × 200=2 δt =0.001(CFL=0.1) Boundary condition : continuous
PinT condition	 Number of time slices: 20 Rfc = 25 (δT= 0.025)

Initial condition



"Step" and "Smooth curves" are used as initial condition.

→ When smoothness of curve increases, the number of high wave number waves decrease in curves.

Residual during the parareal iteration: Influence of CIP5th and reducing the number of high wave number waves



 CIP method and reduce of the number of high wave number waves improves the convergence.
 ★ CIP 5th has not so much effectiveness than CIP3rd.
 → Reason why not yet clear ?



but case by case.

CIP-5th looks not so much effective than CIP-3rd, really ? Then, check the profile of variable along iteration •••



Profile show that CIP-5th is effective even for first sate of the iteration!

Summary and Future Work

Summary:

- Increasing the accuracy of phase calculation by the CIP method is effective for improvement of parereal convergence.
 - High wave number modes cause the difficulty of pararel convergence via decreasing accuracy of phase calculation.



Parareal algorithm should be evaluated using step convection problem to dig up issues.



 Phase of high wave number modes tends to become worse by the numerical damping even if we use the CIP method.

Then, the improvement of the CIP3rd, 5th is need.

Future work



Methods that are tried in this study

nain method (1) CIP scheme: advecting the phase information by the gradient of value Φ.
 (A) Next, We try to combination of STRS scheme and CIP method.
 (B) Control of the residual rebound



See you next in Roscoff Marine Station, Frace at May, PinT 2018 meeting.