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Equation for Computing Ultimate Flexural Strength of Circularly Retrofitted Concrete Columns

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A simplified design equation was proposed in this paper to evaluate ultimate flexural strength of reinforced concrete columns retrofitted by circular steel tube. Ultimate axial load versus moment interactive curve of a circularly retrofitted column section, which is defined by the proposed design equation, consists of two parabolas. The proposed design equation has a very simple mathematical expression and enables structural engineer to easily take into account confinement effect of circular steel tube on ultimate flexural strength of the retrofitted column.

Keywords: *Ultimate flexural strength, Axial load ratio, Circularly retrofitted column, Confinement effect*
終局曲げ耐力，軸力比，円形横補強柱，拘束効果

1. INTRODUCTION

Several recent earthquakes (e.g., Northridge, 1994; Hyogoken-Nanbu, 1995; Taiwan, 1999; India Gujarat, 2001) have caused substantial damages to a great number of reinforced concrete bridge piers and building columns. Severe damages observed in many concrete frame buildings during these earthquakes demonstrated inadequacy of the previous design and emphasized again the urgent need to enhance the earthquake-resisting capacity of older concrete structures and/or their members.

Jacketing a reinforced concrete column with steel tube or plate is one of the effective measures to upgrade earthquake-resistant capacities of the existing column. While several recent researches [1]-[4] have verified effectiveness of the lateral confining steel tube in enhancing flexural and shear strength of concrete column, there has not yet been a rational design equation available to compute the ultimate strength of retrofitted concrete columns. Needless to say, to make rational design, the desirable design equation for ultimate strength should be simple and can take confinement effect of steel tube into consideration.

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Purpose of this paper is to propose a simplified design equation to calculate the ultimate flexural strength of retrofitted concrete columns. The proposed equation has a simple and explicit expression for the axial load versus ultimate flexural strength interactive curve of a retrofitted column section. It will be developed by utilizing a equivalent stress block for the compressed concrete recently proposed by the authors [5], and can take confinement effect by steel tube on the ultimate flexural strength into consideration. Another characteristic of the proposed design equation is that it enables structural engineers to simply compute the flexural strength of retrofitted columns by hand.

Study of this paper will focus on the columns retrofitted by circular steel tube or plate. Development of design equation to compute the flexural strength of squarely retrofitted columns is in progress, and will be reported in near future.

2. EQUIVALENT RECTANGULAR STRESS BLOCK

This section briefly reviews background and outline of a stress block for the compressed concrete in circular column, since it forms basis of the design equation that will be developed in the next section. Also, the calculation procedure and results of the flexural strength obtained using the stress block are presented to provide basic information

about the features of ultimate flexural strength of circularly retrofitted concrete columns.

2.1 Concept of the equivalent stress block

The interaction diagram between the axial load N and the ultimate moment (referred to as N-M interaction diagram hereafter) is usually adopted to compute the flexural strength of concrete columns. Generally, for a given column, its' N-M interaction diagram can be obtained by conducting flexural analysis, i.e., by computing the moment-curvature response of the column section under various levels of axial load.. If a sound stress-strain model for the concrete confined by steel tube is assumed, the flexural analysis can give fairly accurate prediction of the flexural strength, in which confinement effect of steel tube can be taken into account. This general approach, however, is very tedious and needs help of a computer program, since it involves an iterative procedure to find the depth of the neutral axis for the internal forces to balance the external applied load. Due to its complexity, this approach isn't a favorite method for structural engineers.

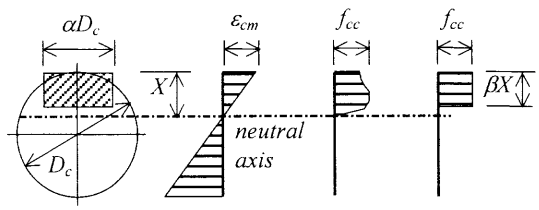
On the other hand, if the stress state of the compressed concrete at the peak moment can be simplified, it is possible to develop a simpler calculation procedure for the N-M interaction diagram. By conducting parametric study of flexural response for circularly retrofitted columns, the authors have proposed an equivalent rectangular stress block to replace the actual stress state of the compressed concrete in a circular column section as the first step toward simplification of flexural strength calculation [5]. Fig.1 shows outline of the stress block for confined concrete.

As shown in Fig. 1, the equivalent stress block has width of αD_c and depth of βX with uniform stress f_{cc}, where D_c is the diameter of core concrete, X is the depth of the neutral axis, and f_{cc} ($=K f_p$) is the compressive strength of confined concrete. Utilizing the stress block, the axial compressive force N_B and the first moment M_B sustained by the concrete at ultimate state can be calculated as follows:

$$N_B = \alpha\beta K f_p D_c X \tag{1}$$

$$M_B = \alpha\beta K f_p D_c X\left(\frac{D_c}{2} - \frac{\beta}{2}X\right) \tag{2}$$

where α and β are the stress block parameters, K is the strength enhancement ratio of confined concrete, f_p is the unconfined concrete strength, D_c is the diameter of concrete core, and X is the depth of the neutral axis. Obviously, if the stress block parameters α and β as well as the strength enhancement ratio K are known, one can complete draw the



(a) concrete section (b) strain profile (c) actual stress (d) stress block

Fig.1 Outline of the stress block at ultimate state

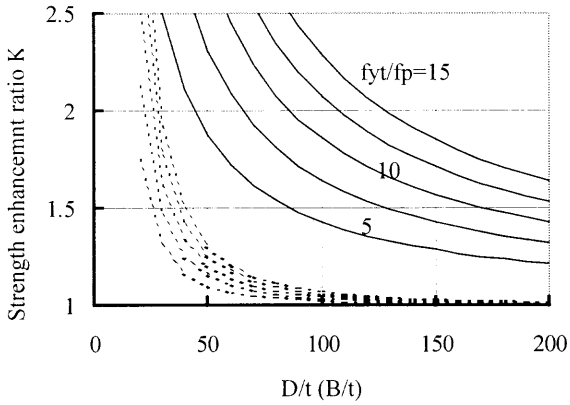


Fig.2 Strength enhancement ratio of confined concrete

ultimate N-M interaction diagram for a given column.

Based on the theoretical results concerning with the flexural behavior of circularly retrofitted columns, the authors have developed expressions to compute the stress block parameters α and β in forms of [5]

$$\alpha\beta = A(K, X_n) - B(K, X_n)\frac{f_p}{42} \tag{3}$$

$$\frac{\beta}{2} = C(K, X_n) - D(K, X_n)\frac{f_p}{42} \tag{4}$$

where

$$A(K, X_n) = \frac{0.723 + 0.061K}{0.112 + X_n} X_n \tag{5}$$

$$B(K, X_n) = \frac{0.048K^{-2}}{0.072K^{-1.5} + X_n} X_n \tag{6}$$

$$C(K, X_n) = (0.476 + 0.051K)(1 - 0.132X_n^2) \tag{7}$$

$$D(K, X_n) = 0.017\left[1 - (0.024 + 0.187K)X_n^2\right] \tag{8}$$

in which X_n is the normalized depth of the neutral axis.

The strength enhancement ratio K of concrete confined by circular steel tube can be obtained by applying famous Richart formula as follow

$$K = \frac{f_{cc}}{f_p} = 1 + 4.1\frac{2}{D/t - 2}\frac{f_{yt}}{f_p} \tag{9}$$

where f_{cc} is the confined concrete strength, f_{yt}, t, and D are the yield strength, thickness, and outside diameter of the steel tube, respectively.

Fig. 2 shows relationships between the strength enhancement ratio and the wall thickness of circular steel tube. For comparison, the same relationships for the concrete confined by square steel tube are also plotted in Fig.2 using dotted lines [6]. One can see from Fig. 2 that confinement effectiveness of the circular steel tube is much higher than that of square steel tube. For a column of 1500 mm in diameter, retrofitting the column with 10 mm thick circular steel tube having f_{yt} = 300 MPa could bring 53% strength gain to the concrete with f_p = 30 MPa as compared with the only 2% strength gain provided by confinement of square steel tube having same thickness.

2.2 Application of the equivalent stress block

To compute the ultimate *N-M* interaction diagram of circularly retrofitted columns using the proposed stress block, they are assumed that 1) the tensile strength of concrete can be neglected, 2)the concrete sustained axial force as well as moment can be replaced by those of the stress block , 3) the longitudinal bar is a rigid-plastic material, and 4) the longitudinal bars can be replaced with an equivalent axial steel tube with same reinforcement area (see Fig. 3).

Based on these assumption, the calculation procedure for ultimate *N-M* interaction diagram doesn't involve iterative calculation, and can be summarized as follows.

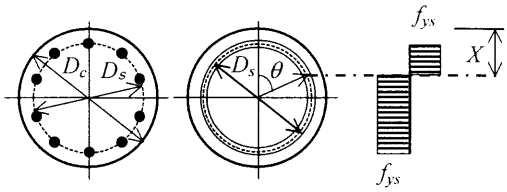
1) Calculate the strength enhancement ratio K of concrete by substituting D/t ratio and yield strength f_{yt} of the steel tube and the unconfined concrete strength f_p, into Eq. (9).
2) Give an initial value (e.g. 0.02π) to θ, the radial angle of the section and calculate the normalized depth of the neutral axis X_n by Eq. (10).

$$X_n = X/D_c = 0.5(1 - D_s \cos\theta / D_c) \tag{10}$$

where D_c is the diameter of confined core concrete, and D_s is the distance between the centroids of the longitudinal bars. (see Fig. 3)

3) Calculate the stress block parameters $\alpha\beta$ and $\beta/2$ from Eq (3) through Eq. (8) and compute the axial force N_B and moment M_B sustained by the concrete by Eqs. (1) and (2).
4) Based on the last two assumptions above mentioned, the axial force N_s and moment M_s sustained by longitudinal bars can be calculated as

$$N_s = \frac{f_{ys} p_g D_c^2 (2\theta - \pi)}{4} \tag{11}$$



(a) longitudinal bars (b) equivalent steel tube (c) stress distribution

Fig. 3 Equivalent axial steel tube of the longitudinal bars

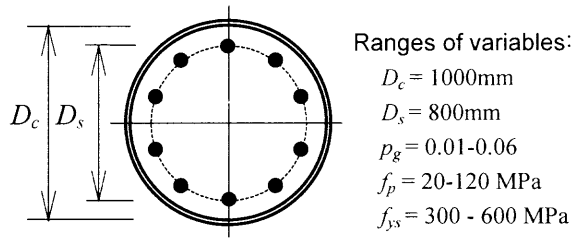


Fig. 4 Details of sample column section

$$M_s = \frac{f_{ys} p_g D_c^2 D_s \sin\theta}{4} \tag{12}$$

where f_{ys} and p_g are the yield strength and the steel ratio of the longitudinal bars, respectively.

5) The normalized axial force n as well as the moment m corresponding to the given radial angle θ are given by

$$n = \frac{N_B + N_s}{A_g f_p} = \frac{4}{\pi}\alpha\beta \bullet K \bullet X_n + \frac{1}{\pi}\frac{f_{ys} p_g}{f_p}(2\theta - \pi) \tag{13}$$

$$\begin{aligned} m &= \frac{M_B + M_s}{A_g D_c f_p} \\ &= \frac{4}{\pi}\alpha\beta \bullet K \bullet X_n\left(\frac{1}{2} - \frac{\beta}{2}X_n\right) + \frac{1}{\pi}\frac{f_{ys} p_g}{f_p}\frac{D_s}{D_c}\sin\theta \end{aligned} \tag{14}$$

6) The normalized ultimate *n-m* interaction diagram can be completely determined by incrementing θ till θ = π and repeating the calculation step 2) through step 5) above.

2.3 Features of the ultimate *n-m* interactive diagram

To investigate feature of the ultimate *n-m* interactive curve for circularly retrofitted concrete columns, parametric study was conducted. Fig. 4 shows details of the sample section as well as varying ranges of the main variables. As obvious in Fig. 4, parametric study covered a wide range of concrete strength and steel amount, which implies the wide applicability of the proposed equivalent stress block in terms of concrete strength and amount of longitudinal bars.

Fig. 5 plots several ultimate *n-m* interactive curves of

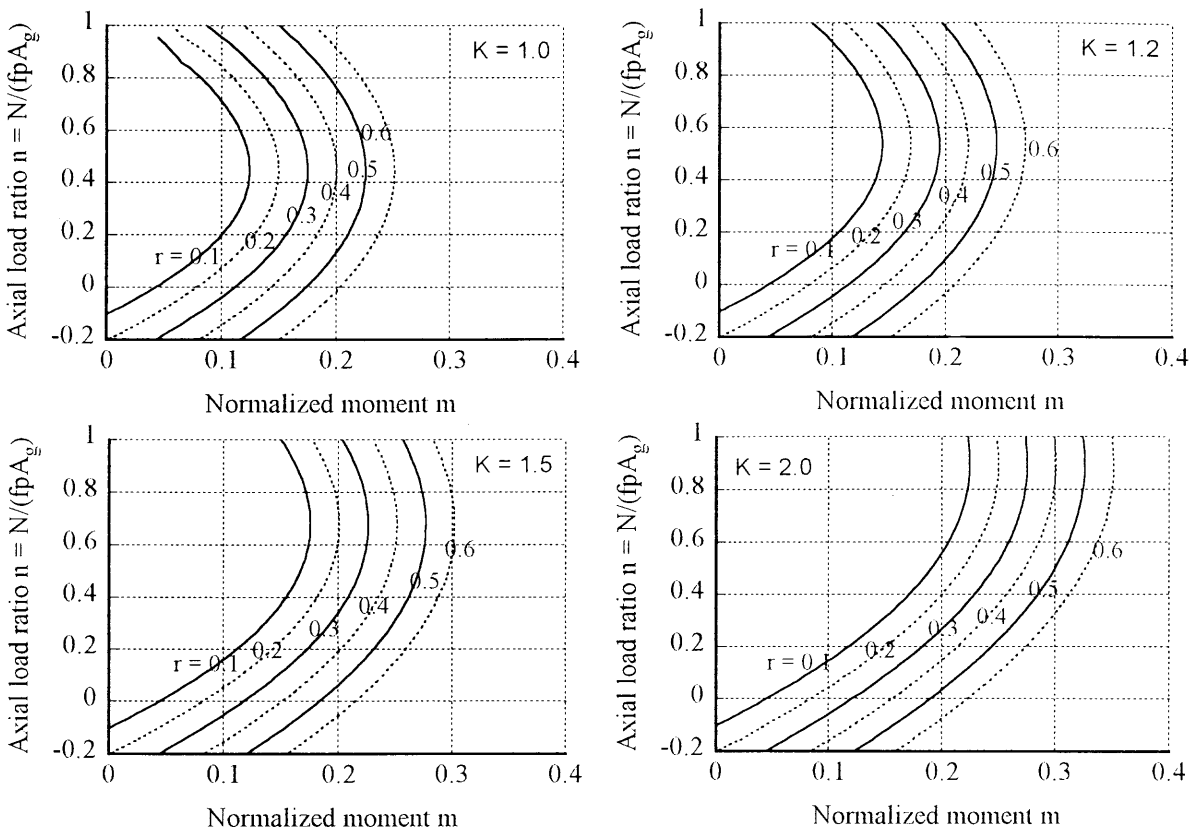


Fig. 5 Calculated n-m interactive curves of the sample column

the sample column section obtained utilizing the calculation procedure described above, while Fig. 6 shows relationships between the normalized ultimate moment m and the neutral axis depth X_n. In Figs. 5 and 6, confinement effect of circular steel tube on the flexural strength is represented by the strength enhancement ratio K of confined concrete as defined by Eq. (9), and the factor r expresses the normalized steel ratio of the longitudinal bars and is given by

$$r = \frac{f_{ys} p_g}{f_p} \tag{15}$$

From Figs. 5 and 6, one can observe the following features in the n-m interactive curves of circularly retrofitted concrete columns:

1) For the circular columns confined by the steel tube with a specific K value, the n-m interactive curves expand outwards as the amount of the longitudinal steel. On the other hand, the normalized moments m, as a function of the axial load ratio n, tend to reach their maximums at a axial load level that is nearly independent of the amount of longitudinal bars.
2) The axial load ratio n_o at the peak point of each n-m interactive curve increases with the K value, while the peak moment m_o at each n-m interactive curve increases as increment of both the K value and the amount of longitudinal bars.
3) Regardless of the confinement degree (the K value) and the amount of longitudinal bars, the ultimate n-m curve reaches its peak moment as the normalized neutral axis depth X_n varies between 0.52 through 0.55 with a mean value of 0.53.

3. PROPOSAL OF A DESGIN EQUATION

The calculation method using the equivalent stress block does really simplify the computation of flexural strength for the circularly retrofitted columns as compared with the method based on the general flexural analysis. However, this method is still complicated as can be seen from Eq(3) through Eq. (8). To provide a simpler and more powerful tool enabling structural engineers to more quickly and reasonably conduct the retrofitting design, based on the features of ultimate n-m interactive diagrams for circularly retrofitted columns, a design equation is developed in this section.

3.1 Development of the design equation

Based on the results of parametric studies described in the previous section, the ultimate n-m interactive curve can be symbolized as the curve ABC shown in Fig. 7.

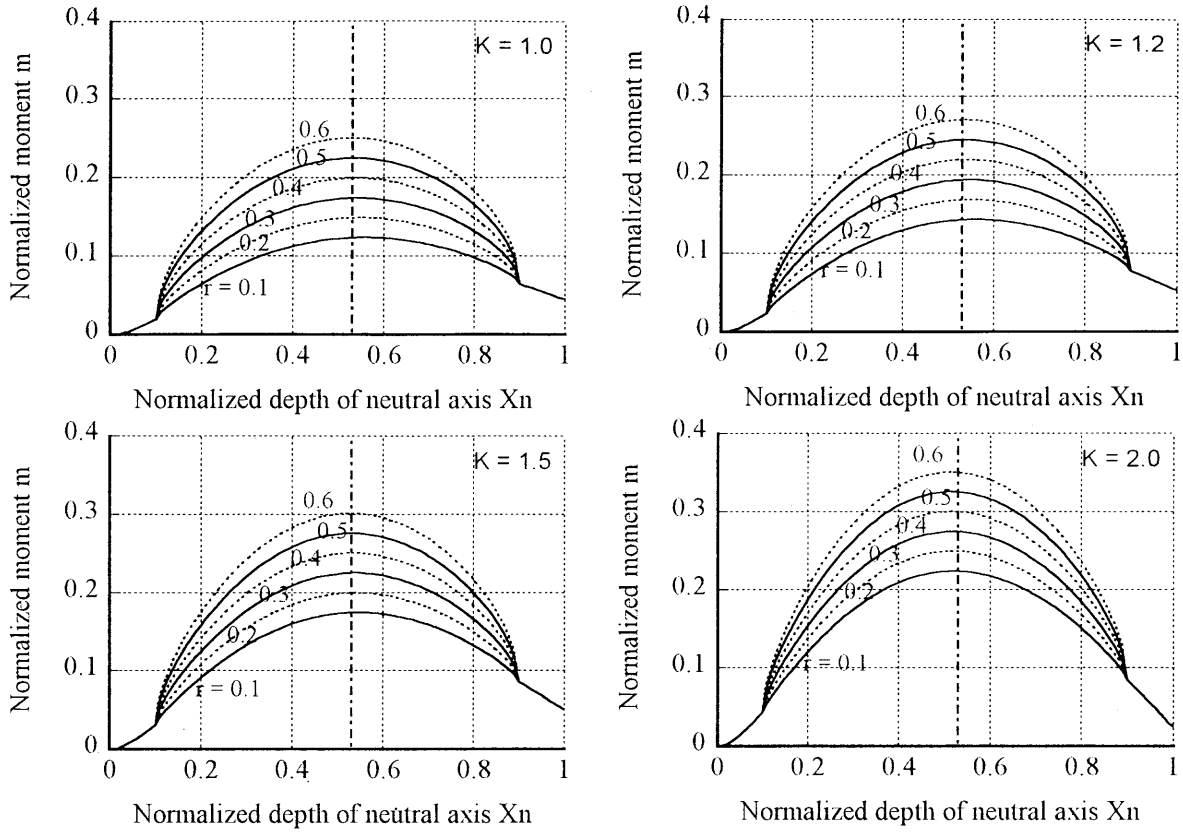


Fig. 6 Calculated relationships between the normalized neutral axis depth X_n and the moment m

Obviously, to represent the features observed in ultimate n-m interactive diagrams, the design equation, which defines the relationship between the normalized moment m and the axial load ratio n, should satisfy the following conditions:

$$
\begin{aligned}
&m\big|_{n=-r} = 0 \\
&m\big|_{n=K+r} = 0 \\
&m\big|_{n=n_0} = m_0 \\
&\left.\frac{\partial m}{\partial n}\right|_{n=n_0} = 0
\end{aligned}
\tag{16}
$$

where m_0 and n_0 are the peak moment of each n-m curve and the corresponding axial load ratio, respectively.

The four conditions given by Eq. (16) can determine a single cubic curve. However, a cubic curve involves four constants, which might results in new complexity.

By carefully analyzing the ABC curve shown in Fig. 7, one can see that dividing the n-m curve into two parts, the ascending portion AB and the descending part BC, makes it possible to develop much simpler expressions for the n-m curve. In fact, the AB curve can be taken as the ascending half of a parabola ABA', while the BC curve as the descending half of a parabola C'BC.

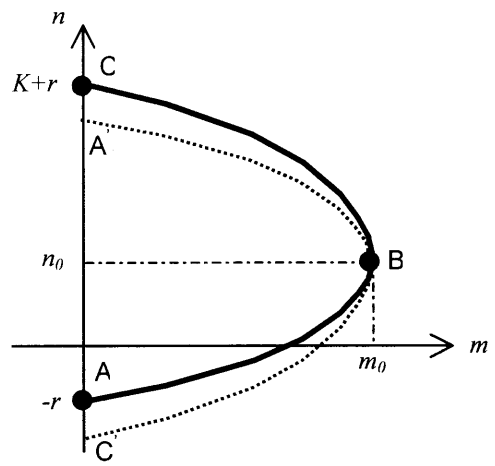


Fig. 7 Idealization of the n-m interactive curve

Considering that the both parabolas have the line defined by $n=n_0$ as their common symmetric axis, the mathematical expression for the ultimate n-m curve can be simply written as follow

$$
m = \begin{cases} m_0\left[1-\left(\dfrac{n-n_0}{n_0+r}\right)^2\right], & n \le n_0 \\ m_0\left[1-\left(\dfrac{n-n_0}{n_0-K-r}\right)^2\right], & n > n_0 \end{cases}
\tag{17}
$$

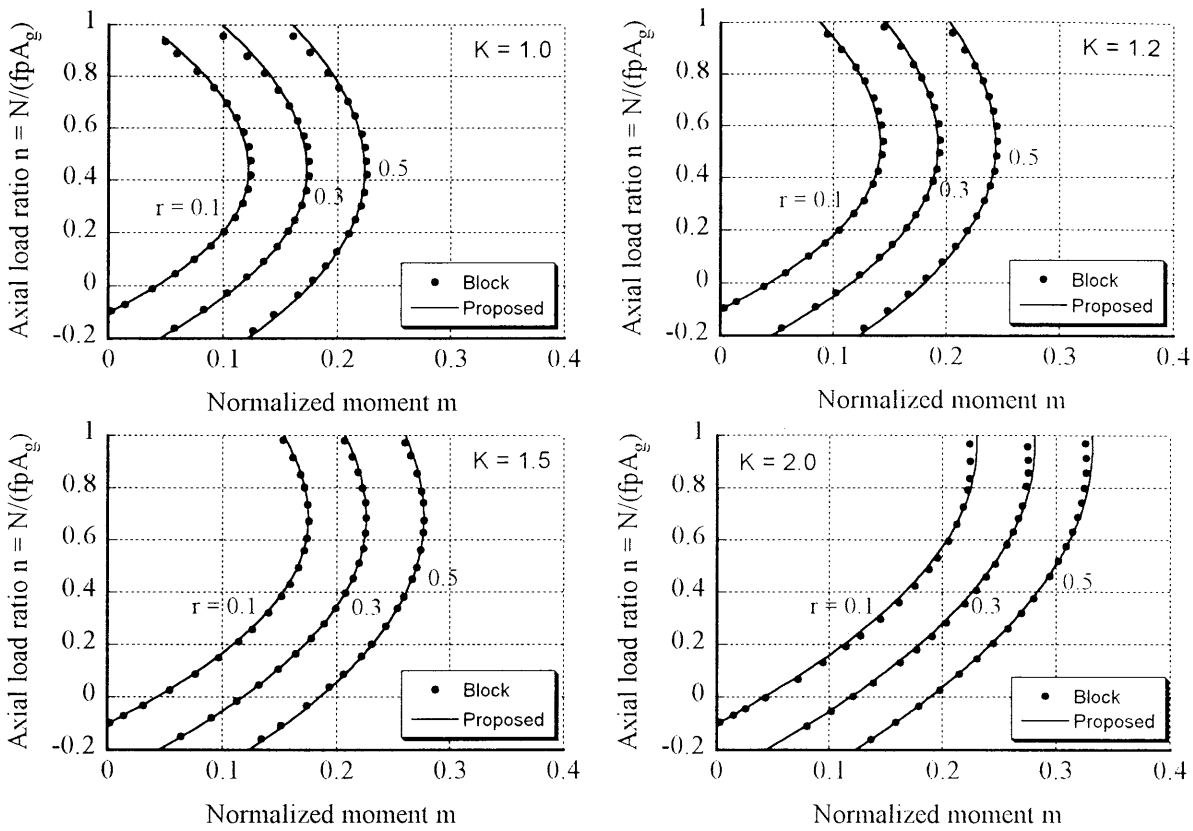


Fig. 8 Verification of validity of the proposed design equation

The two parabolas defined by Eq. (17) are continuous at the peak point of *n-m* curve. Apparently, the proposed design equation is a two-parameter model. Only if the peak moment m_0 and the corresponding axial load ratio n_0 are given, one can completely determine the ultimate *n-m* curve.

3.2 Determination of m_0 and n_0

As described in the previous section, each *n-m* curve reaches its peak as the normalized depth of neutral axis is between 0.52 and 0.55 with a mean value of 0.53 regardless of the values of both K and r, which are defined by Eq.(9) and Eq.(15), respectively. Hence, substituting X_n=0.53 into Eqs. (3) and (4), and then Eqs. (13) and (14), one can obtain close approximations for m_0 and n_0. Noting that as X_n=0.53,

$$\theta \approx \frac{\pi}{2} \Rightarrow \sin\theta \approx 1.0$$

exists, and conducting regression analysis on the approximations of m_0 and n_0 one can derive simpler expressions to compute m_0 and n_0 as follows:

$$n_0 = \frac{1}{\pi}\left(0.1K^2 + 1.3K - 2.2K^{-1} f_p \times 10^{-3}\right) \quad (18)$$

$$m_0 = \frac{1}{\pi}\left[0.31K + (0.61K - 0.85) f_p \times 10^{-3} + r\frac{D_s}{D_c}\right] \quad (19)$$

It can be seen from Eq. (17) through Eq. (19) that the proposed design equation enables structural engineers to calculate the ultimate flexural strength by hand.

3.3 Validity of the proposed design equation

To verify validity of the proposed design equation, the ultimate *n-m* curves obtained using the design equation are compared with the curves calculated using the equivalent stress block in Fig. 8. Legends "Proposed" and "Block" in Fig. 8 represent these two kinds of curves, respectively.

One can observe from Fig. 8 that the *n-m* curves computed using the proposed design equation agree very well with the curves obtained using the equivalent stress block. Since the calculation method using the stress block has been proved to be able to give a reasonably accurate prediction to the flexural strength of circularly retrofitted columns [5], it is reasonable to conclude that the proposed equation can also predict the flexural strength of circular columns well.

4. TRIAL DESIGN OF RETROFITTED COLUMNS

Rational design of a retrofitted column means choice of proper thickness of the steel tube to ensure the retrofitted column adequate ultimate strength. Generally, there are two approaches to the ultimate flexural design of the retrofitted columns as listed below:

1) *Verification design*: This approach is useful in evaluating the ultimate moment of the retrofitted columns after the thickness and the material property of the confining steel tube were firstly designed.

2) *Strength capacity design*: This approach is to determine necessary thickness of the steel tube to enhance the ultimate strength of an older column to a specific level.

To better understand how to use the proposed design equation for circularly retrofitted column, a trial design is presented below by conducting the verification design.

Example

A reinforced concrete column has a diameter of 1000 mm and a shear span ratio $M/(VD_c)$ of 3.0, hence a distance of 3000 mm from the critical section to the point of contraflexure. The longitudinal steel is twenty D25 deformed bars (25.5 mm diameter) placed uniformly along the perimeter with 80 mm of cover to the steel centroid. The concrete has a compressive strength of f_p =30 MPa, while the longitudinal steel has a yield strength of f_{ys} = 300 MPa. The axial load is 3770 kN. If the column is retrofitted with circular steel tube having a yield strength of f_{yt} = 300 MPa and thickness of 10mm, calculate the flexural strength and the corresponding lateral force of the column after retrofitted

Solution

1) Column properties

$$D_c = 1000\,\text{mm},\quad D_s = 1000 - 2\times 80 = 840\,\text{mm}$$

$$A_g = \frac{\pi}{4}D_c^{\,2} = \frac{3.14}{4}\times 1000^2 = 7.85\times 10^5\,\text{mm}^2$$

$$p_g = \frac{24\times 507}{7.85\times 10^5} = 0.0155$$

$$r = \frac{p_g f_{ys}}{f_p} = \frac{0.0155\times 300}{30} = 0.155$$

$$n = \frac{N}{A_g f_p} = \frac{3770\times 1000}{7.85\times 10^5\times 30} = 0.16$$

2) Material properties and strength enhancement ratio K

$$D = D_c + 2t = 1000 + 2\times 10 = 1020\,\text{mm}$$

$$\frac{D}{t} = \frac{1020}{10} = 102$$

$$f_p = 30\text{MPa}$$

$$f_{yt} = 300\,\text{MPa}$$

$$K = 1 + 4.1\frac{2}{D/t-2}\frac{f_{yt}}{f_p} = 1 + \frac{8.2}{102-2}\times\frac{300}{30} = 1.82$$

3) Calculation of m_0 and n_0

From Eqs. (18) and (19)

$$n_0 = \frac{1}{\pi}\left(0.1K^2 + 1.3K - 2.2K^{-1}f_p\times 10^{-3}\right)$$

$$= \frac{1}{3.14}\left(0.1\times 1.82^2 + 1.3\times 1.82 - \frac{2.2}{1.82}\times 30\times 10^{-3}\right)$$

$$= 0.847$$

$$m_0 = \frac{1}{\pi}\left[0.31K + (0.61K - 0.85)f_p\times 10^{-3} + r\frac{D_s}{D_c}\right]$$

$$= \frac{1}{3.14}\left[0.31\times 1.82 + (0.61\times 1.82 - 0.85)\times 30\times 10^{-3} + 0.155\times\frac{840}{1000}\right]$$

$$= 0.228$$

4) Computation of the flexural strength

$$\because\ n = 0.16 < n_0 = 0.847$$

from Eq. (17), one can obtain

$$\therefore m = m_0\left[1 - \left(\frac{n-n_0}{n_0+r}\right)^2\right]$$

$$= 0.228\times\left[1 - \left(\frac{0.160-0.847}{0.847+0.155}\right)^2\right]$$

$$= 0.124$$

then the flexural strength for the retrofitted column can be calculated as follow

$$M_u = mA_g D_c f_p = \frac{0.124\times 7.85\times 10^5\times 1000\times 30}{1000\times 1000}$$

$$= 2920\,\text{kN - m}$$

5) Calculation of the corresponding lateral force

$$\because \frac{M}{VD_c} = 3.0$$

$$\therefore V_u = \frac{M}{3.0D_c} = \frac{2920\times 1000}{3.0\times 1000} \approx 970\text{kN}$$

Answer

M_u = 2920 kN-m, V_u = 970 kN

5. CONCLUSIONS

A simple design equation has been proposed in this paper to calculate the ultimate flexural strength of concrete columns retrofitted by circular steel tubes. The Proposed design equation has a very simple expression and can take into account confinement effect of steel tubes on the flexural strength. Because of its simplicity and comprehensiveness, the proposed design equation can provide a powerful tool enabling structural engineers to conduct rational retrofitting design for the existing concrete columns.

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