Nonlinear regression modeling via Compressed Sensing

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Abstract

We consider the problem of constructing nonlinear regression models in case that the dimension of data is less than the number of basis functions. We propose a procedure where the smooth curve are effectively estimated along with the technique of regularization method. We give a simple idea for applying the Restricted Isometry Property to curve fitting. Simulation results and real data analysis demonstrate that our methodology performs well in various situations.

Key Words and Phrases: Basis expansion, Compressed sensing, Nonlinear regression, Regularization, Restricted isometry property.

1 Introduction

Nonlinear regression models have been used to analyze data with complex structure in various situations. As a useful technique for analysing diverse phenomena, a nonlinear regression model based on basis expansions is widely used (Konishi and Kitagawa, 2008). The essential idea behind basis expansions is to express a regression function as a linear combination of known functions, called basis functions (Bishop, 2006; Hastie et al., 2009). In constructing the nonlinear regression model, various functions are used to represent a regression function. For example, B-splines (Eilers and Marx, 1996; de Boor, 2001; Imoto and Konishi, 2003) and radial basis functions (Hastie et al., 2009) involving Gaussian basis functions are used.

Suppose that we have $n$ independent observations $\{(y_i, x_i); i = 1, 2, \cdots , n\}$, where $y_i$ are random response variables and $x_i$ are explanatory variables. We consider a regression problem in the following sense.

\[ y_i = \beta^T \phi(x_i) + z_i, \quad i = 1, \cdots , n, \]  

(1)
where $\beta = (\beta_1, \cdots, \beta_m)^T$ is a unknown coefficient vector, $\phi(x) = (\phi_1(x), \cdots, \phi_m(x))^T$ is a vector of basis functions and $z$ is an unknown noise term. Our goal is to reconstruct coefficient vector $\beta \in \mathbb{R}^m$. Here we are interested in $n < m$ case and in whether or not it is possible to reconstruct $\beta$ with accuracy. In this case, there is an ill-posed problem. Although in recent years we have seen the development of various types of nonlinear model, in $n < m$ case we can not have good estimations. As follows, we propose one approach for the problem. In this note, our approach based on the compressed sensing which is relied on the restricted isometry property (Candès, 2006; Donoho, 2006).

It is crucial issue to determine the tuning parameters, including regularization parameter and variance parameter associated with Gaussian basis functions. We choose these parameters using cross-validation.

This paper is organized as follows. Section 2 describes the framework of basis expansions and our proposed basis function models. In Section 3 we present an estimation for nonlinear regression models. In Section 4 we investigate the performance of our nonlinear regression modeling techniques through simulations and real data analysis. Some concluding remarks are presented in Section 5.

2 Nonlinear regression model with basis functions

Now, we expand a coefficient parameter $\beta_j (j = 1, \cdots, m)$ in equation (1) as follows:

$$\beta_j = w^T \psi(t_j), \quad j = 1, \cdots, m,$$

(2)

where $w = (w_1, \cdots, w_m)^T$ is a unknown coefficient vector, $t_j (j = 1, \cdots, m)$ are temporary data such as equally-spaced points in the range of $x$ and $\psi(t) = (\psi_1(t), \cdots, \psi_m(t))^T$ is a vector of basis functions.

As a result, regression model (1) is expressed as follows:

$$y = \Phi \Psi w + z$$

(3)

where $y = (y_1, \cdots, y_n)^T$, $z = (z_1, \cdots, z_n)^T$, $\Phi = (\phi(x_1), \cdots, \phi(x_n))^T$ and $\Psi = (\psi(t_1), \cdots, \psi(t_m))^T$. Here, we employ $B$-splines functions of degree 3 with equidistant knots (Imoto and Konishi, 2003; Konishi and Kitagawa, 2008) as $\phi(\cdot)$ and the Gaussian basis
functions whose centers are temporary data points \( t_j \) as \( \psi(\cdot) \), that is,

\[
\psi_j(t) = \exp \left\{ -\frac{(t - t_j)^2}{h} \right\}, \quad j = 1, \cdots, m, \tag{4}
\]

where \( h \) is a width parameter.

## 3 Estimation

We propose estimating \( w \) as the solution to the convex optimization problem

\[
\min_{\tilde{w} \in \mathbb{R}^m} \| \tilde{w} \|_1 \quad \text{subject to} \quad \| y - A\tilde{w} \|_2 \leq \varepsilon \tag{5}
\]

where \( A = \Phi \Psi \) and \( \varepsilon \) is an upper bound on the noisy contribution, that is \( \| z \|_2 \leq \varepsilon \).

If a new coefficient vector \( w \) is sparse or nearly sparse and the matrix \( A \) obeys a condition known as the restricted isometry property (RIP) introduced below, the optimization problem solution \( w^* \) recover \( w \) exactly. It means that a coefficient vector \( \beta \) is reconstructed exactly. That is,

\[
\beta^* = \Psi w^* \tag{6}
\]

therefore,

\[
y^* = \Phi \beta^* \tag{7}
\]

In order to realize that \( w \) is sparse and \( A \) obeys RIP, we use large size design matrix and a good set of basis function matrices \( \Phi \) and \( \Psi \), though it depends on data set. We introduce a concept of mathematical backgrounds.

**Definition 1 (RIP)**

For each integer \( s \in \mathbb{N} \), define the isometry constant \( 0 < \delta_s < 1 \) of a matrix \( A \) as the smallest number such that the inequalities

\[
(1 - \delta_s) \| w \|_2^2 \leq \| A w \|_2^2 \leq (1 + \delta_s) \| w \|_2^2 \tag{8}
\]

hold for all \( s \)-sparse vector \( w \). A vector is said to be \( s \)-sparse if it has at most \( s \) nonzero entries. (Candès, 2008)
**Theorem 1** (Noisy Recovery) \[3\].

Assume that \( \delta_2 < \sqrt{2} - 1 \) and \( \| z \|_2 \leq \varepsilon \). Then the solution \( w^* \) obeys

\[
\| w^* - w \|_2 \leq C_0 s^{-\frac{1}{2}} \| w - w_s \|_1 + C_1 \varepsilon
\]  

(9)

where \( C_0, C_1 \) are explicitly given constants. (Candès, 2008)

This theorem assesses how good is the solution \( w^* \). That is, it shows how much the solution \( w^* \) reconstruct the true coefficient vector \( w \) with accuracy. Next, we show simulation results and real data analysis demonstrate that our methodology performs well in various situations.

**4 Numerical examples**

**4.1 Simulation study**

In this section we report some simulation studies done to compare the performance of our proposed method with that of the elastic net (Zou and Hastie, 2005), the adaptive lasso (Zou, 2006), and the ridge (Horel and Kennard, 1970). In all examples, we used nonlinear regression model based on \( B \)-spline functions of degree 3 basis functions. In \( m < n \) case, the number of basis functions \( m \) and regularization parameters \( \lambda, \alpha \) (elastic net) were selected by using the criterion Cross-validation (CV) (Stone, 1974), and we computed the adaptive weights for the adaptive lasso using OLS coefficients. In \( n < m \) case, we extended to the coefficient vector by Gaussian basis functions and regularization parameters \( \lambda, \alpha \) and a variance of Gaussian basis functions \( h \) were selected by CV, and we computed the adaptive weights for the adaptive lasso using Ridge coefficients. As follows, we considered functions as true regression model.

\[
(a) \quad u(x) = \sin(5\pi x) \sin(10\pi x) \\
(b) \quad u(x) = \sin(2\pi x) \exp(-5x) \\
(c) \quad u(x) = \sin(2\pi x^3) \\
(d) \quad u(x) = 4 \cos(8\pi x) \exp(-5x) + \cos(2\pi x)
\]

Then we repeatedly generated random samples \( \{(y_i, x_i); i = 1, 2, \cdots, 100\} \) using the true regression model \( y_i = u(x_i) + z_i \). The design points are equally-spaced in \([0, 1]\) and the
noises $z_i$ are independently, normally distributed with mean 0 and standard deviation $\sigma$, where $\sigma$ is taken as $\sigma = 0.2R_u$, and $R_u$ is the range of $u(x)$ over $x \in [0, 1]$. In $n < m$ case, we put $m = 200$. In this setting, we performed 100 repetitions, then we compare the performance of the averages of the mean square errors (AMSE) defined by
\[
\text{MSE} = \frac{\sum_{n}^{100} (u(x_n) - \hat{y}_n)^2}{100}
\]
and the SD indicates standard deviations for the AMSE to assess the goodness of curve fitting. Table 1 summarizes the simulation results in $n < m$ case. Table 2 summarizes the simulation results in $m < n$ case.

4.2 Real data analysis

We report the analysis of the motorcycle impact data (Härdle, 1990; Silverman, 1985). The motorcycle impact data are a series of measurements of head acceleration in units of gravity and times in millisecond after impact. Although we used the same setting in above simulations, the number of basis functions $m$ is 2000. Figure 1 shows the motorcycle impact data and estimated curve using our proposed procedure.
Table 1: Comparison of results for simulations in $n < m$ case

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<th>Adaptive lasso</th>
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Table 2: Comparison of results for simulations in $m < n$ case

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5 Concluding remarks

We have proposed a procedure where the smooth curve are effectively estimated along with the technique of regularization method in case that the dimension of data is less than the number of basis functions. We have given a simple idea for applying the restricted isometry property to curve fitting. The simulation results and real data analysis reported here demonstrate the effectiveness of the proposed modeling strategy in terms of prediction accuracy.

The two assumptions that \( \mathbf{w} \) is sparse or nearly sparse and \( \mathbf{A} \) obeys RIP are important to our methodology. However, it is hard to know whether or not \( \mathbf{w} \) is sufficiently sparse and \( \mathbf{A} \) obeys RIP because in real data analysis \( \mathbf{\beta} \) is unknown. It remarks that in case of curve fitting \( \Phi \) represents a wavelet basis functions or Gaussian basis functions whose variance is small and a number of basis functions is populous. Therefore, \( \mathbf{w} \) will be sufficiently sparse, and with respect to RIP, we can almost show that \( \mathbf{A} \) is nearly orthogonal matrix.

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