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# Introducing Viewpoints of Mechanics into Basic Growth Analysis – (XV) Relationships between Richards Growth Function and Basic Growth Function –

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This study was designed to investigate relationships between Richards growth function and basic growth function by comparing them using growth dynamics. There are inflection points in Richards growth function, but basic growth function expresses exponential curves only. To compare these two functions that are greatly different, the first and second derivatives were incorporated into each of them. The results obtained were as follows. Richards growth function was related to basic growth function when combined with derivatives and $m = 1$, though $m = 1$ was prohibited in Richards growth function itself. The occurrence of this relationship was a mathematical result of growth dynamics based on differential. It was suggested from the present study that Richards growth function was related to basic growth function when combined with the first and second derivatives.

## INTRODUCTION

The basic growth analysis is a simple method to analyze weight changes of the plant (Blackman, 1919; Watson, 1952; Radford, 1967; Hunt, 1990) and the animal (Brody, 1945). There is, however, a limitation in the application to overall growth curve. The problem of this analysis is that basic growth function gives only an exponential increase, and does not give inflection points that appear in the curve of actual growth. It is well known that there are various improved growth functions that predict actual growth of plants and animals. Logistic function, which is one of them, was related with basic growth function using a concept of growth dynamics (Shimojo *et al.*, 2009). Osumi and Ishikawa (1983) reported that Richards growth function (Richards, 1959) had strong points in describing, analyzing and predicting actual growth, probably a superior function to others in the flexibility to fit growth curves of various types.

The present study was designed to investigate relationships between Richards growth function and basic growth function by comparing them using growth dynamics.

## RICHARDS GROWTH FUNCTION AND BASIC GROWTH FUNCTION AS RELATED USING GROWTH DYNAMICS

### Dynamics of Richards growth function

Richards growth function (Richards, 1959) is given by

$$W_R = A \cdot (1-b \cdot \exp(-k \cdot t))^{1/(1-m)}, \quad (1)$$

where $A$, $b$, $k$ and $m$ are constants, $m \neq 1$, $t$ = time. Introducing viewpoints of mechanics into Richards growth function (1) gives

$$dW_R/dt = (A/(1-m)) \cdot (1-b \cdot \exp(-k \cdot t))^{m/(1-m)} \cdot (b \cdot k \cdot \exp(-k \cdot t)), \quad (2)$$

$$d^2W_R/dt^2 = (A/(1-m)^2) \cdot (1-b \cdot \exp(-k \cdot t))^{(2m-1)/(1-m)} \cdot (b \cdot k^2 \cdot \exp(-k \cdot t)) \cdot (m-1+b \cdot \exp(-k \cdot t)), \quad (3)$$

where $dW_R/dt$ = growth rate, $d^2W_R/dt^2$ = growth acceleration, $m \neq 1$.

Combining functions (1), (2) and (3) leads to

$$(dW_R/dt)^2 = W_R \cdot (d^2W_R/dt^2) \cdot ((b \cdot \exp(-k \cdot t))/(m-1+b \cdot \exp(-k \cdot t))). \quad (4)$$

### Dynamics of basic growth function

Basic growth function is given by

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$$W_B = W_0 \cdot \exp(r \cdot t), \quad (5)$$

where $W$ = weight, $t$ = time, $W_0 = W$ at $t = 0$, $r$ = relative growth rate.

Introducing viewpoints of mechanics into basic growth function (5) gives

$$dW_B/dt = r \cdot W_0 \cdot \exp(r \cdot t), \quad (6)$$

$$d^2W_B/dt^2 = r^2 \cdot W_0 \cdot \exp(r \cdot t), \quad (7)$$

where $dW_B/dt$ = growth rate, $d^2W_B/dt^2$ = growth acceleration.

Combining functions (5), (6) and (7) leads to

$$(dW_B/dt)^2 = W_B \cdot (d^2W_B/dt^2). \quad (8)$$

**Relating Richards growth function with basic growth function using growth dynamics**

It is possible to substitute 1 for $m$ in function (4), though $m \neq 1$ is required in Richards growth function (1) and its first derivative (2) and second derivative (3). Thus,

$$(dW_R/dt)^2$$

$$= W_R \cdot (d^2W_R/dt^2) \cdot ((b \cdot \exp(-k \cdot t))/$$

$$(1-1+b \cdot \exp(-k \cdot t)))$$

$$= W_R \cdot (d^2W_R/dt^2). \quad (9)$$

By $m = 1$, equation (4) results in equation (9). Equation (9) takes the same form as that of equation (8),

$$(dW_R/dt)^2 = W_R \cdot (d^2W_R/dt^2), \quad (9)$$

$$(dW_B/dt)^2 = W_B \cdot (d^2W_B/dt^2). \quad (8)$$

This result suggests a kind of collapse of Richards growth function into basic growth function. However, it was reported that Richards growth function tends to Gompertz function when $m$ tends to 1 (Richards, 1959). This contradiction remains to be investigated.

**Conclusions**

It is suggested from the present study that Richards growth function is related to basic growth function when combined with the first and second derivatives.

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