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Shimojo, Masataka

Shao, Tao

Nakano, Yutaka
University Farm, Faculty of Agriculture, Kyushu University

Kakihara, Hidetoshi
Laboratory of Regulation in Metabolism and Behavior, Graduate School of Bioresource and Bioenvironmental Sciences, Kyushu University

他

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Introducing Viewpoints of Mechanics into Basic Growth Analysis (XIV) Growth Dynamics and Related Problems based on Mathematical Properties of Modified Differential Equation for Growth –

Masataka SHIMOJO*, Tao SHAO, Yutaka NAKANO[1], Hidetoshi KAKIHARA[2] and Manabu TOBISA[3]

Laboratory of Regulation in Metabolism and Behavior, Division of Animal and Marine Bioresource Sciences, Department of Bioresource Sciences, Faculty of Agriculture, Kyushu University, Fukuoka 812–8581, Japan and Department of Grassland and Forage Science, College of Animal Sciences and Technology, Nanjing Agricultural University, Weigang 1, Nanjing 210095, the People's Republic of China
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This study was conducted to investigate growth dynamics and related problems by analyzing mathematical properties of whole solutions to modified differential equation for growth. This modified differential equation was obtained by relating basic growth function with its first and second derivatives. The results obtained were as follows. (1) Sixteen functions were whole solutions to the modified differential equation for growth. Each of weight, relative growth rate (RGR), time and exponential function with base *e* was given positive and negative signs, namely symmetries in them with respect to ± signs. Functions in which time was replaced with space were also taken up. (2) The conservation of positive weight, by avoiding negative weight, required the existence of the past and that of the environmental space. (3) There were time reversal and space inversion theoretically in functions. But actually there was a breakdown of those symmetries, because moving forward in time is the principle of information transmission and there is an effect of gravity determining the upper and lower sides. (4) The hypothesis that RGR in functions was a mixture of positive and negative RGRs was discussed in relation to metabolic turnover, compensatory growth and homeostasis. (5) The form of the modified differential equation for growth suggested analogies to laws of motion. It was suggested from the present study that growth dynamics and related problems resulted from conservation, symmetry and analogy in whole solutions to modified differential equation for growth.

INTRODUCTION

Forage production is indispensable to ruminant production from the viewpoint that ruminant animals convert plant fibers into milk and meat for human consumption (Minson, 1990). Basic growth analysis has been considered a simple method to investigate the plant growth (Blackman, 1919; Watson, 1952; Radford, 1967; Hunt, 1990). There are reports of introducing viewpoints of mechanics into basic growth analysis (Shimojo, 2007; Shimojo *et al.*, 2006, 2007a, 2007b, 2008, 2009a, 2009b, 2009c, 2009d, 2009e), where modified differential equation for growth is taken up to investigate some aspects of growth phenomena. This problem, however, requires reexamination with more detailed explanation and self–criticism based on mathematics.

The present study was designed to investigate growth dynamics and related problems by analyzing mathematical properties of whole solutions to modified differential equation for growth.

[1] University Farm, Faculty of Agriculture, Kyushu University, Kasuya, Fukuoka 811–2307

[2] Laboratory of Regulation in Metabolism and Behavior, Graduate School of Bioresource and Bioenvironmental Sciences, Kyushu University

[3] Faculty of Agriculture, Miyazaki University, Miyazaki 889–2192, Japan

* Corresponding Author (E–mail: mshimojo@agr.kyushu–u.ac.jp)

MATHEMATICAL PROPERTIES OF MODIFIED DIFFERENTIAL EQUATION FOR GROWTH

Modified differential equation for growth

Differential equation for relative growth rate (RGR) of a forage plant is given by

$$\mathrm{RGR} = (1/W)\cdot(dW/dt) = r_t, \quad (1)$$

where W = weight of a forage plant, t = time, r_t = RGR. Solving differential equation (1) leads to basic growth function (2),

$$W = W_0 \cdot \exp(r_t \cdot t), \quad (2)$$

where $W_0 = W$ at $t = 0$.
Absolute growth rate (AGR) and growth acceleration (GA) are given by

$$\mathrm{AGR} = dW/dt = r_t \cdot W_0 \cdot \exp(r_t \cdot t), \quad (3)$$

$$\mathrm{GA} = d^2W/dt^2 = r_t^2 \cdot W_0 \cdot \exp(r_t \cdot t). \quad (4)$$

Relating basic growth function (2), its first derivative (3) and second derivative (4) gives

$$\frac{dW/dt}{W} = \frac{d^2W/dt^2}{dW/dt} = r_t, \quad (5)$$

therefore,

$$(dW/dt)^2 = W \cdot (d^2W/dt^2). \quad (6)$$

Differential equation for growth (6) takes a modified form, when compared with differential equation (1) of the original form.

Extended solutions to modified differential equation for growth

The normal solution to modified differential equation (6) is given by

$$dW/dt = \sqrt{W \cdot (d^2W/dt^2)}, \quad (7)$$

$$\rightarrow W = W_0 \cdot \exp(r_t \cdot t). \quad (2)$$

When based on mathematical rules, extracting the square root allows both positive and negative dW/dt to appear. This is the first extension that is given by

$$dW/dt = \pm\sqrt{W \cdot (d^2W/dt^2)}, \quad (8)$$

$$\rightarrow W = W_0 \cdot \exp((\pm r_t) \cdot t). \quad (9)$$

In addition, positive and negative dW/dt seem to be given by other terms if given ± signs. This is a logical leap, but might be expected to give some results to growth dynamics. Thus,

$$\begin{aligned} dW/dt &= \pm\sqrt{W \cdot (d^2W/dt^2)} \\ &= \pm\sqrt{(W_0 \cdot \exp(r_t \cdot t)) \cdot (r_t^2 \cdot W_0 \cdot \exp(r_t \cdot t))} \\ &= \pm\sqrt{(W_0)^2 \cdot (r_t)^2 \cdot (\exp(r_t \cdot t))^2} \\ &= (\pm W_0) \cdot (\pm r_t) \cdot (\pm \exp((\pm r_t) \cdot t)), \quad (10) \end{aligned}$$

therefore,

$$W = (\pm W_0) \cdot (\pm \exp((\pm r_t) \cdot t)). \quad (11)$$

Every term except t in function (11) is given ± signs, and function (11) includes function (9).

Positive weight conservation related to time and space

Positive weight conservation related to time

The following two functions are chosen from function (11) to investigate positive weight conservation related to time.

$$W = W_0 \cdot \exp(r_t \cdot t), \quad (2)$$

$$W = (-W_0) \cdot \exp(r_t \cdot t). \quad (12)$$

Function (2) expresses the weight increase in a forage plant. Function (12) expresses a corresponding reduction of weight, where the negative sign shows that substances are deprived from environment to be absorbed by the plant for its growth. The sum of functions (2) and (12) gives zero, the weight conservation that makes us think of energy conservation.

However, the actual weight of environmental substances is given by the positive value. The negative weight problem in function (12) is avoided by the following function (Shimojo *et al.*, 2009c),

$$W = W_0 \cdot \exp((-r_t) \cdot (-t)). \quad (13)$$

The negative r_t in function (13) has already appeared in functions (9) and (11). However, going back to the past, which is given by negative t, appears for the first time in function (13). This shows that the existence of the past is required to ensure the positive weight of environmental substances that are used for the plant growth. The problem of time reversal in function (13) is taken up in the section of symmetries in time and in space.

Positive weight conservation related to space

Replacing time (t) with space (x, y, z) in function (13) gives the following weight–space relationships,

$$W = W_0 \cdot \exp((-r_x) \cdot (-x)), \quad (14)$$

$$W = W_0 \cdot \exp((-r_y) \cdot (-y)), \quad (15)$$

$$W = W_0 \cdot \exp((-r_z) \cdot (-z)). \quad (16)$$

These three functions show that the existence of the environmental space surrounding the plant is required to ensure the positive weight of substances that are used for the plant growth. The problem of space inversion in functions (14), (15) and (16) is taken up in the section of symmetries in time and in space.

Symmetries in time and in space

Inserting function (13) into function (11) and applying this operation to space give

$$W = (\pm W_0) \cdot (\pm \exp((\pm r_t) \cdot (\pm t))), \quad (17)$$

$$W = (\pm W_0) \cdot (\pm \exp((\pm r_x) \cdot (\pm x))), \quad (18)$$

$$W = (\pm W_0) \cdot (\pm \exp((\pm r_y) \cdot (\pm y))), \quad (19)$$

$$W = (\pm W_0) \cdot (\pm \exp((\pm r_z) \cdot (\pm z))). \quad (20)$$

These four functions show theoretical symmetries both in time and in space with respect to ± signs.

However, symmetries actually break in both cases. We discuss time reversal and space inversion based on the following two coordinates,

$$(t, x, y, z), \quad (21) \qquad (-t, -x, -y, -z), \quad (22)$$

where normal time and space for (21), time reversal and space inversion for (22).

The symmetry in time actually breaks, because time passes from the past to the future. This is probably due to the fact that the information about things surrounding us spatially moves forward in time to reach us. What we see now is a phenomenon that took place in the past. Therefore, function (13),

$$W = W_0 \cdot \exp((-r_t) \cdot (-t)), \quad (13)$$

is rewritten as follows so as to ensure the passage of time from the past to the future,

$$W = W_0 \cdot \exp(r_t \cdot (t + \tau))$$

$$= \exp(r_t \cdot \tau) \cdot W_0 \cdot \exp(r_t \cdot t), \quad (23)$$

where $\tau > 0,\ -\tau \leq t \leq 0$.
Comparing function (23) with function (2),

$$W = W_0 \cdot \exp(r_t \cdot t), \quad (2)$$

shows that moving along the time axis does not change the basic form of function. This is related to the energy conservation that is one of the principles in nature.

The space inversion gives upside down, front–back reversal and left–right reversal. These conserve symmetries in space, if there is not the effect of gravity. However, the existence of gravity determines the upper and lower sides, leading to a breakdown of symmetry in space.

Mathematical expression of converting environmental substances into forage plant

Functions (24) and (2) are chosen from function (17) to express the conversion of environmental substances into plant,

$$W = (-W_0) \cdot (-\exp(r_t \cdot t)), \quad (24)$$

$$W = W_0 \cdot \exp(r_t \cdot t). \quad (2)$$

Function (24) shows mathematically that substances deprived from the environment are converted into forage plant, as shown by the equality with function (2) that expresses the plant growth.

Hypothetic mixture of positive RGR and negative RGR

We take up functions (2) and (25), which are chosen from function (17), in order to discuss what a mixture of $\pm r_t$ might be expected to give,

$$W = W_0 \cdot \exp(r_t \cdot t). \quad (2)$$

$$W = W_0 \cdot \exp((-r_t) \cdot t). \quad (25)$$

Function (2) expresses the increase in weight, and function (25) expresses the decrease in weight. We set up a hypothesis, though it looks strange, that r_t in both functions is a mixture of positive r_t and negative r_t. This operation is given by function (26), from which, function (2) and function (25) will be derived as special cases. Thus,

$$W = W_0 \cdot \exp((\alpha \cdot r_t + \beta \cdot (-r_t)) \cdot t)$$

$$= W_0 \cdot \exp((\alpha - \beta) \cdot r_t \cdot t), \quad (26)$$

where $\alpha > 0,\ \beta > 0$.

Values of $(\alpha - \beta)$ implying metabolic turnover

In function (26), substituting 1 for $(\alpha - \beta)$ gives function (2), and substituting −1 for $(\alpha - \beta)$ gives function (25). There is a degradation of metabolites and degraded metabolites are partly lost from the plant, followed by compensation by the synthesis. This is metabolic turnover, and applying this to function (26) leads to that α and β imply coefficients of synthesis and degradation for loss, respectively. The change in weight is, therefore, given by the difference between them. There is a weight gain when synthesis exceeds degradation for loss, $(\alpha - \beta) = 1$. In the opposite case, $(\alpha - \beta) = -1$, there is a weight loss. When $(\alpha - \beta) = 0$, this gives a dynamic equilibrium. Therefore, in basic growth function, $(\alpha - \beta)$ takes values of (−1, 0, 1). This suggests, boldly writing at the risk of making mistakes, that function (26) is associated with metabolic turnover.

Extended values of $(\alpha - \beta)$ implying compensatory growth and homeostasis

We hypothesize in function (26) that $(\alpha - \beta)$ is extended to take the following values,

$$(\alpha - \beta) < 1, \quad \text{therefore,} \quad (\alpha - \beta) \cdot r_t < r_t, \quad (27)$$

$$1 < (\alpha - \beta), \quad \text{therefore,} \quad r_t < (\alpha - \beta) \cdot r_t. \quad (28)$$

When there occurs a shift from (2) to (27), this may be caused by reduced activities of photosynthesis and nutrients uptake by the plant,

$$(\alpha - \beta) \cdot r_t = r_t, \quad (2) \quad \rightarrow \quad (\alpha - \beta) \cdot r_t < r_t. \quad (27)$$

If there is a shift from (27) to (28), this may be a compensatory growth that is caused by increased growth activities higher than (2),

$$(\alpha - \beta) \cdot r_t < r_t, \quad (27) \quad \rightarrow \quad r_t < (\alpha - \beta) \cdot r_t. \quad (28)$$

When there occurs a shift from (29) to (30), this may be caused by homeostasis that tries to maintain a stable condition of the plant,

$$(\alpha - \beta) \cdot r_t < 0, \quad (29) \quad \rightarrow \quad (\alpha - \beta) \cdot r_t = 0. \quad (30)$$

Suggested analogies between growth dynamics and laws of motion

Suggested analogy to Newton's equation of motion

Modified differential equation for growth (6) looks like Newton's equation of motion (31),

$$dp/dt = m \cdot (d^2r/dt^2), \quad (31)$$

where p = momentum, t = time, m = mass of an object, r= position, dp/dt = force, d^2r/dt^2 = acceleration of motion,

$$(dW/dt)^2 = W \cdot (d^2W/dt^2), \quad (6)$$

where W = weight of a forage plant, t = time, dW/dt =

AGR (absolute growth rate), d^2W/dt^2 = GA (growth acceleration).

There seems to be an analogy in appearance between function (31) for motion and function (6) for growth, a term–to–term correspondence that is suggested between the two functions. Since dp/dt is a force of motion, $(dW/dt)^2$ may be a force of growth.

Suggested analogy to Newton's law of universal gravitation

We choose functions (32) and (33) from function (10),

$$dW/dt = W_0 \cdot r_t \cdot \exp(r_t \cdot t), \tag{32}$$

$$dW/dt = (-W_0) \cdot r_t \cdot \exp(r_t \cdot t). \tag{33}$$

The product of functions (32) and (33) gives

$$(dW/dt)^2 = (W_0 \cdot r_t \cdot \exp(r_t \cdot t)) \cdot ((-W_0) \cdot r_t \cdot \exp(r_t \cdot t)) = (-r_t^2) \cdot (W_0 \cdot \exp(r_t \cdot t)) \cdot (W_0 \cdot \exp(r_t \cdot t)). \tag{34}$$

Function (34) may be interpreted as follows. (a) There are two terms of $W_0 \cdot \exp(r_t \cdot t)$; if the first one is animal body weight, then the second one corresponds to the feed weight that is expressed in terms of animal body weight. (b) The value of $(r_t)^2$ is constant, because r_t is calculated as a constant value so as to give functions (32) and (33). (c) $(dW/dt)^2$ is regarded as a force, as shown in the subsection of suggested analogy to Newton's equation of motion.

Newton's law of universal gravitation (F) is given by

$$F = (-G \cdot M \cdot m)/d^2, \tag{35}$$

where G = gravitational constant, M and m = mass of the two objects, d = distance between the two objects.

Comparing equation (34) with equation (35) suggests that there is a virtual force of attracting that might operate between the animal and the feed without the effect of distance. This interpretation is, boldly writing at the risk of making mistakes, based on an analogy that is suggested between function (34) and the numerator of function (35). When both the animal (A) and the feed (F) are transported from some other places, inserting distances that they are transported (d_A and d_F) into function (34) leads to

$$(dW/dt)^2 = (-r_t^2) \cdot (W_0 \cdot \exp(r_t \cdot t) \cdot d_A) \cdot (W_0 \cdot \exp(r_t \cdot t) \cdot d_F)/(d_A \cdot d_F). \tag{36}$$

Function (36) suggests three things. (i) The higher the weight and relative growth rate of the animal are, the stronger the virtual attracting force operating between the animal and the feed is. (ii) The content in round brackets in the right–hand side shows something like food–mileage that is given by the product of weight and distance. (iii) Distances can be reduced between the numerator and the denominator in the right–hand side, being capable of inserting an arbitrary distance that allows both self–sufficiency and dependence on imports to occur in animal agriculture.

Conclusions

It is suggested from the present study that growth dynamics and related problems result from conservation, symmetry and analogy in whole solutions to modified differential equation for growth.

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