# A Nonlinear Analysis on the Euro Exchange Rate Using MF-DFA

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# A Nonlinear Analysis on the Euro Exchange Rate Using MF-DFA

Meifen Chu

Abstract

The objective of this study is to examine the nonlinear characteristics of the Euro exchange rate. Many nonlinear approaches have been applied in the finance field. Multifractal detrended fluctuation analysis (MF-DFA) is one of the popular methodologies employed to detect the nonlinear properties of a time series. The MF-DFA analysis is executed by removing the trend of a time series and abstracting its multifractality. This paper examines the features of the entire Euro exchange rate series and the sub-periods during the Global Financial Crisis and European Sovereign Debt Crisis. The overall pattern of the Euro exchange rate is determined. Further, the degrees of complexity during the two crisis periods are detected.

Keywords: Euro Exchange Rate, Nonlinear Time Series Analysis, Fractal, Multifractal Detrended Fluctuation Analysis

## 1. Introduction

Extensive previous studies have proved that most financial time series follow nonlinear processes (Mandelbrot, B., 1982; Campbell, J. Y., et al., 1997). Many researchers have also detected that foreign exchange markets bear nonlinearity rather than linearity (Kantz, H. and Schreiber, T., 1997; Chu, M., 2007). Many researchers have employed the ARCH (Autoregressive Conditional Heteroscedastic) model which was proposed by Engle (Engle, R. F., 1982) to conduct estimation. Among the nonlinear properties, some statistical properties, such as long-term memory in time series, fat-tailed distributions, and multifractal characteristics are frequently observed and become much more important. The multifractal concept has been widely applied and has been recognized as one of the key features in complex systems. This paper focuses on multifractal analysis and its application.

Fractal means self-similarity, namely symmetry at different degrees. Fractal phenomena are encountered everywhere, not only in physiological data such as heart records, texture in images of the natural terrain, variations of traffic flow, phase transition, electrolysis, and chemical oscillation, but also in economic

records, such as the distributions of income, the fluctuations of foreign exchange rates and stock indices, and so on. These phenomena can hardly be described by common mathematical methods. Benoit B. Mandelbrot first developed the nonlinear theory to analyze the phenomena in the 1970s and called them "fractals" (Mandelbrot, B., 1982). In recent years, using the fractal theory to analyze the foreign exchange market has become one of the forefront research fields in finance and computer science. In this paper, the trended fluctuation based multifractal theory is employed to analyze the characteristics of the Euro exchange rate. The Euro has evolved into the world's 2nd strongest currency since it was adopted as the sole official currency of European Union in 1999. Even though it is a young currency, it has experienced several crises during its 17 years. Analyzing its fluctuation and characteristics has become important.

The rest of this paper is organized as follows. Section 2 introduces the multifractal analysis based on detrended fluctuation analysis. Section 3 includes a brief review of the Euro exchange rate. In section 4, empirical analysis is applied to the Euro exchange rate. The conclusion is given in section 5 and directions for future research are specified in the closing section.

## 2. Multifractal Detrended Fluctuation Analysis

When one mentions nonlinearity, one may be reminded the chaos which is the typical nonlinearity of time series. There are also other nonlinear properties, like self-similarity which is called a fractal feature or multifractal characteristic. If the fractal dimension is constant, it is called mono-fractal; if the fractal dimension is not constant but follows a distribution, it is referred to as multifractal (MF). Multifractality can be detected by using wavelet transform-based analysis and the detrended fluctuation approach. Both are popularly applied to find the fractal spectra. Similar results have been found using the two approaches (Kantelhardt, J. W., et al., 2002, 2003). However, detrended fluctuation based multifractal analysis is convenient to compute and more reliable (Kantelhardt, J. W., et al., 2002, 2003). Recently, numerous researchers have adopted the multifractal detrended fluctuation analysis (MF-DFA) to characterize time series patterns. This paper uses this simple and efficient approach to conduct a multifractal analysis.

Detrended fluctuation analysis (DFA) was first proposed by Peng (1994) to examine the fractal properties of a time series by removing the time series trend. This method was quickly extended from physics to medical and engineering science. In 2002, Kantelhardt, J. W., et al. (2002) extended it to MF-DFA, which is now popularly applied not only in physics, medical and engineering science but also in economics. The following are the steps to calculate the fractal properties.

**Step 1.**

Let $x(i)$ represent a time series ($i=1,2,\cdots$). Generate a time series $y(i)$ from the next formula.

$$y(i) = \sum_{k=1}^{i} \left( \mathrm{x}(\mathrm{k}) - \bar{x} \right) \tag{1}$$

while $x(k)$ is the $k^{th}$ point of the time series and $\bar{x}$ is their average.

**Step 2.**

Create $N_s$ boxes of small length $s$ to cover all the $y(i)$. The trend of a given box $v(1 \le v \le N_s)$ is calculated by using the least-squares method in a q-order poly nominal. $y_v(i)$ is the optimized solution, which represents the best-fit curve. Then, remove the trend by subtracting $y_v(i)$ from $y(i)$, $i=1,2,\cdots,N_s$. The fluctuations are calculated as follows:

$$F_2(s,v) = \frac{1}{s} \sum_{i=1}^{s} \left( \left| y((v-1)s+i) - y_v(i) \right| \right)^2 \tag{2}$$

**Step 3.**

Compute the q-order moment by averaging the appropriate function of $F_q(s)$ so that the following scaling relation with box size $s$ can be gained.

$$F_2(s,v) = \left\{ \frac{1}{N_s} \sum_{v=1}^{N_s} F_2(s,v)^{q/2} \right\}^{1/q} \sim s^{h(q)} \tag{3}$$

Here, $h(q)$ is the exponent depending on $q$. The multifractal scaling exponent $\tau(q)$ is depending on $h(q)$ with the following relationship.

$$\tau(q) = qh(q) - D_f \tag{4}$$

**Step 4.**

The multifractal spectrum is calculated by a Legendre transform of $\tau(q)$ as defined by

$$f(\alpha) \equiv \alpha q - \tau(q), \;\; \alpha \equiv \frac{d\tau(q)}{dq} \tag{5}$$

where $f(\alpha)$ is defined as dimension of the time series. If the value of $\alpha$ is constant, the time series is monofractal, otherwise, it is multifractal.

In section 4, the application of MF-DFA is conducted using real foreign exchange rate data and the singularity of the series is then captured.

## 3. A brief review of the Euro exchange rate

The European Union (EU) began to be competitive with world largest economy (the USA), once its sole currency (the Euro) was introduced in January 1999 as the official currency of the Eurozone. At the beginning of the establishment of the single currency, only 15 countries joined the EU, with a population of 3.76 hundred million. However, the EU has expanded dramatically since then, and now has 28 member-countries with a population of 5.08 hundred million. 19 of the 28 member-countries and a few non-EU countries have adopted the Euro as their only official currency. There have been many discussions of the advantages and disadvantages of the introduction of the Euro. However, there is no doubt that the EU economy has grown to be one of the most important economies in the world. This can be seen in Figure 1, which charts the GDP share of major world economies. Specifically, the EU surpassed the USA to have the top GDP share between 2003 and 2014. The Euro's trading volume has substantially increased, and its share as a settlement currency was 29.5% of all, just after the US dollar (Figure 2.). Policy makers and researchers have paid much more attention to the Euro's movement because its fluctuations definitely affect the EU, the US, and other world economies.

Through to the present time, a large volume of research has been conducted on the Euro exchange rates. R. Shams (2005) made a deep study of the volatile movement of the Euro between 1999 and 2004. By comparing the fundamentals, he confirmed that the beliefs about the European economy has become more important, which reinforce the Euro exchange rate. De Zwart, et al. (2007) concluded in their research that both fundamentals and technical factors are important in predicting the Euro exchange rate. However, they found that its short-term movement was related to the technical factors. Further, De Grauwe and Grimaldi

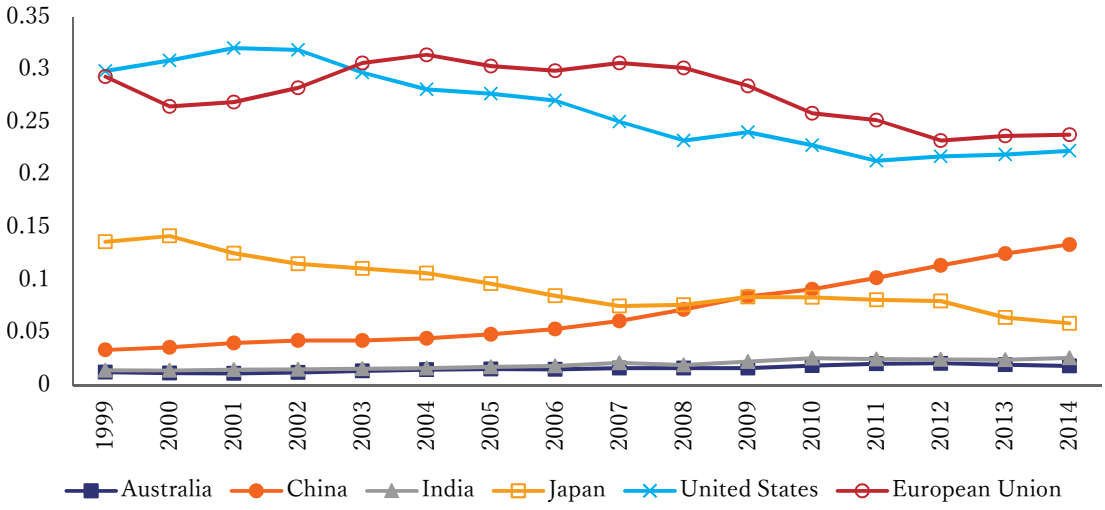


Figure 1. GDP Share in the World

Source: World Bank Database

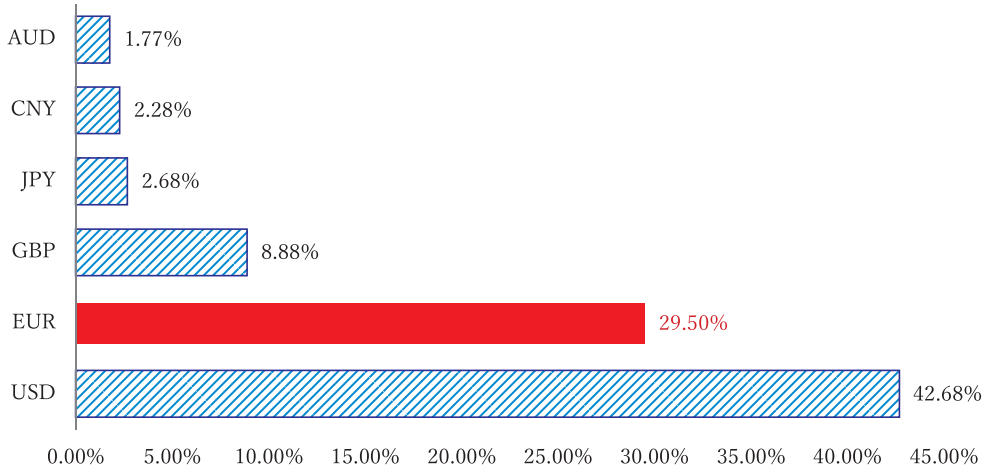


Figure 2. Most used Payment Currencies in the World

Source: European Central Bank Database

(2002) proposed that the fluctuation of Euro exchange rate has no correlation with fundaments. De Grauwe and Vanteenkiste (2001) also mentioned that the exchange rate is nonlinear. Moreover, in countries with low inflation, the exchange rate movements were not influenced by the fundamentals. Most earlier studies focused on whether the fundamentals determined the exchange rate. P. De Grauwe (2000) analyzed the influencing factors for both the EU and USA, but he only focused on the fundamentals. Nowadays, financial crises happen frequently and various unexpected factors affect exchange rate movements. The studies on risk factors are still insufficient. Furthermore, nonlinear analysis of the Euro exchange rate is extremely rare. This paper aims to remove the above limitations and employ a nonlinear approach to detect the characteristics of the Euro exchange rate.

## 4. Empirical analysis

### 4.1 Data Set

In this paper, the daily closing data for the EURO vs US Dollar (EURO/USD), EURO vs Japanese Yen (EURO/YEN), EURO vs British Pound Sterling (EURO/POUND), and EURO vs South African Rand (EURO/RAND) are used. The time period is from 1999.01.04 to 2016.12.31 (year, month, day). Data were extracted from the database provided by European Central Bank. As mentioned earlier, during the evolution of Euro, some major financial events happened, which may have influenced the structure of the Euro exchange rate system. This paper focuses on two of the most important events. The first is the Global Financial Crisis, which happened in 2008. The other event is the European Sovereign Debt Crisis, which started in late 2009. These two events are supposed to have impacted the Euro exchange rate the most heavily. In the following,

a multifractal analysis is used to detect whether there are any significant differences during the periods of the Global Financial Crisis and the European Sovereign Debt Crisis. In this paper, the two important periods each cover 4 years respectively. (One is from 2005 to 2009, and the other one is from 2010 to 2014).

### 4.2 Time Series Description

Figure 3 shows the time series of the exchange rates for the EURO/USD, EURO/YEN, EURO/POUND, and EURO/RAND. Taken as a whole, it is clear that the EURO/RAND exchange rate moves completely differently from the other rates from 1999 to 2008.

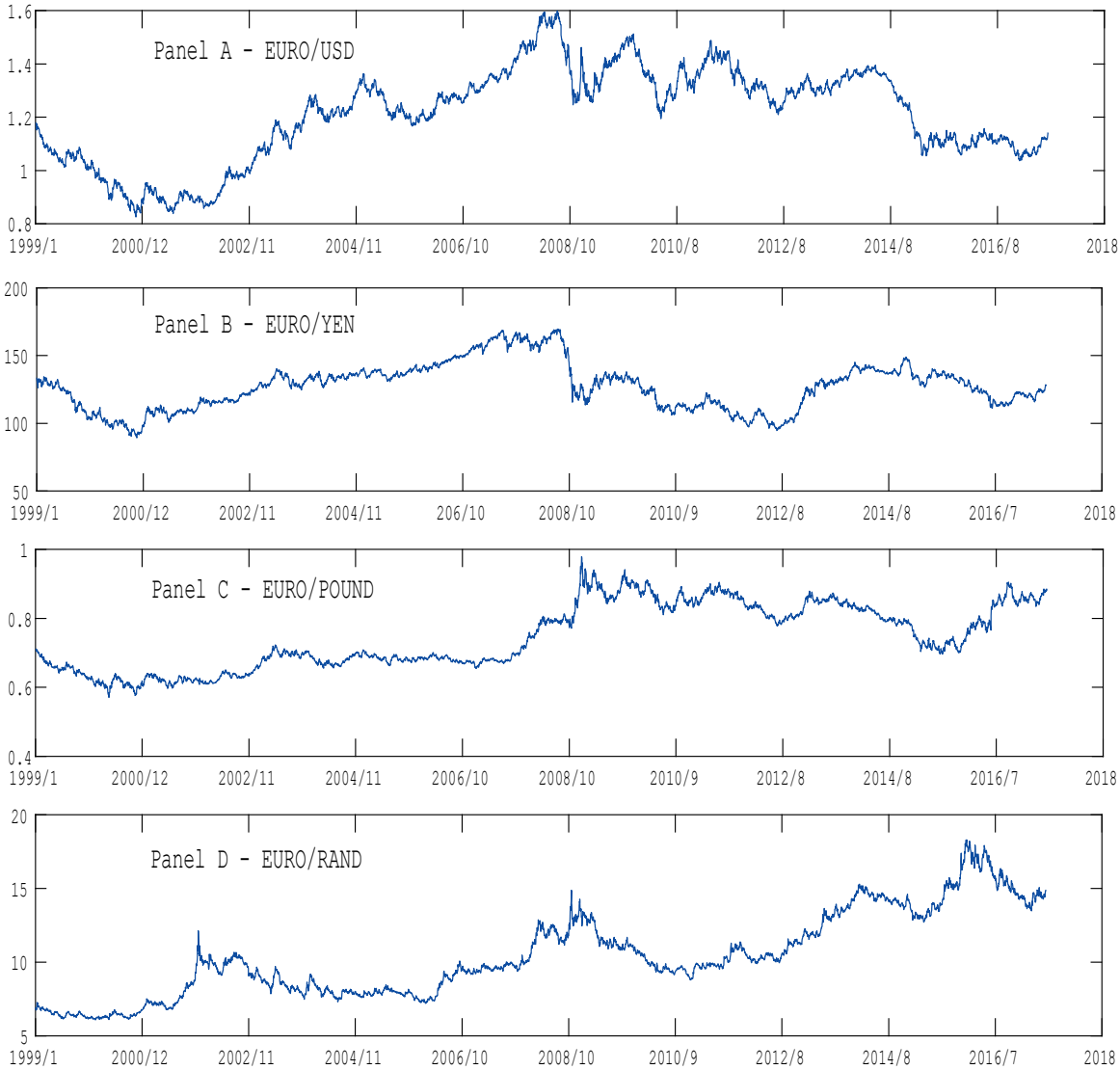


Figure 3. Time series plots of Four Exchange Rates

Source: European Central Bank Data Base

For the EURO/USD, EURO/YEN, and EURO/POUND, their movements are similar from 1999 to 2008. After an initial decrease starting with the birth of the EURO and lasting for almost one year, the EURO rates against USD, YEN, and POUND went up and became stronger. However, these persistent upward movements did not last long due to the global financial crisis that happened in 2007 in the USA. The EURO rates versus the three currencies sharply and unexpected declined in 2008. Thus, the three-currency series displayed diverse moving patterns and fluctuated much more severely since 2008. As a whole, the four series became unstable after the global financial crisis. Further, in late 2013, high waves appeared in all four series. The EURO/USD and EURO/POUND dropped greatly. In the contrast, the EURO/YEN and EURO/RAND rates moved upwards. These large fluctuations were caused by the European Sovereign Debt Crisis. This means the crisis damaged the EURO dramatically as well.

The EURO/RAND kept quite stable for almost two years after the adoption of EURO in the EU in January 1999. It began to move up from 2001 and reached its first peak at the beginning of 2002. However, it dropped suddenly from the peak and continued slipping down until 2004. Between 2004 to 2006, it was comparatively stable. However, it soared up again in 2007 and climbed to the 2$^{nd}$ peak in October 2008. It displayed a big bubble, but that bubble burst quickly. The EURO/RAND rate slipped down suddenly and kept declining until the end of 2010. However, it rose again and a 3$^{rd}$ bubble was formed in 2015, and the 3$^{rd}$ peak was at the end of 2015. A large decline was checked in 2016. In brief, the EURO/RAND fluctuated drastically with three huge waves during the Euro's history.

### 4.3 Multifractal analysis results

Now to examine the multifractal features of the four series - EURO/USD (Series 1), EURO/YEN (Series 2), EURO/POUND (Series 3), and EURO/RAND (Series 4). In Figure 4, the fluctuation spectra and the multifractal degrees of these four-currency series are displayed respectively. This demonstrates that all four series have fractal characteristics. It can be suggested that the Euro exchange rate expressed in any of the four currencies move volatilely and unstably.

Based on the multifractal theory, the difference between $\alpha_{max}$ and $\alpha_{min}$ (denoted as $\Delta\alpha$) is the statistic to examine the strength of the multifractality. The wider the difference, the stronger the strength of multifractal. The YEN moves volatilely with a spectrum width close to 0.5. The USD moves comparatively calmly with a similar spectrum at 0.4. Meanwhile, the POUND's movement is only narrow with a spectrum width of 0.38. This shows that the POUND is more stable when compared with the other currencies. Regarding the RAND movement, it fluctuated the most. The spectrum width order is as follows:

$\Delta\alpha(\text{RAND}) = 0.62786715452841 > \Delta\alpha(\text{YEN}) = 0.497603517 > \Delta\alpha(\text{USD}) = 0.415196306 > \Delta\alpha(\text{POUND}) = 0.382280585$.

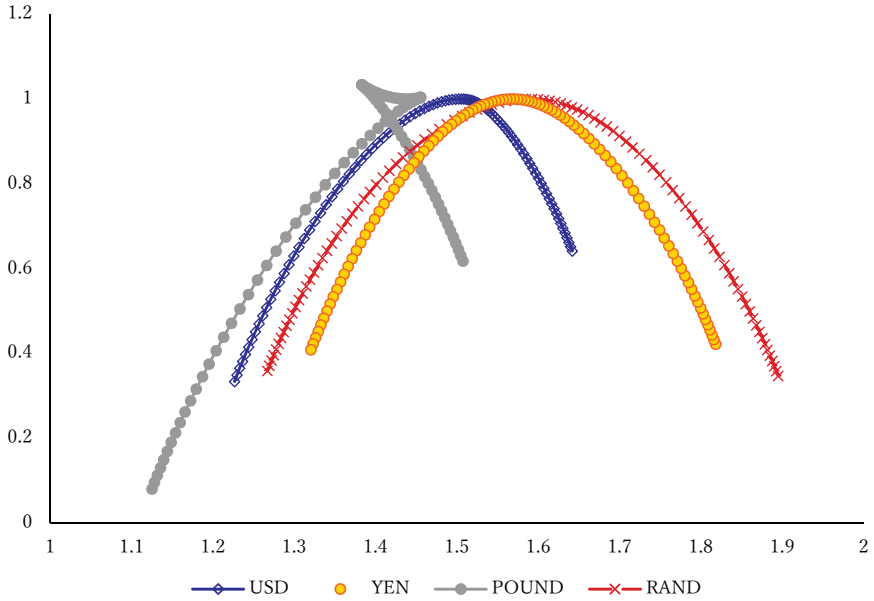


Figure 4. Multifractality of the Four Exchange Rates

Table 1 Correlations between the EURO and Other Currency's Exchange Rates

| | EURO/ AUD | EURO/ CAD | EURO/ RMB | EURO/ POUND | ERUO/ HKD | EURO/ RUPEE | EURO/ YEN | EURO/ KRONE | EURO/ USD | EURO/ RAND |
|---|---|---|---|---|---|---|---|---|---|---|
| EURO/AUD | 1.00000 | 0.38807 | 0.12776 | 0.32513 | 0.14214 | 0.28982 | 0.40974 | -0.33562 | 0.13660 | 0.59861 |
| EURO/CAD | | 1.00000 | 0.87677 | 0.82604 | 0.84975 | 0.92423 | 0.86408 | 0.25513 | 0.85154 | 0.21018 |
| EURO/RMB | | | 1.00000 | 0.88492 | 0.99990 | 0.97419 | 0.88130 | 0.44043 | 0.99997 | 0.10771 |
| EURO/POUND | | | | 1.00000 | 0.87806 | 0.91089 | 0.89174 | 0.36241 | 0.87907 | 0.16926 |
| ERUO/HKD | | | | | 1.00000 | 0.97495 | 0.85960 | 0.45244 | 0.99984 | 0.06254 |
| EURO/RUPEE | | | | | | 1.00000 | 0.94338 | 0.29254 | 0.97417 | 0.29033 |
| EURO/YEN | | | | | | | 1.00000 | 0.21892 | 0.85873 | 0.37719 |
| EURO/KRONE | | | | | | | | 1.00000 | 0.45824 | -0.57606 |
| EURO/USD | | | | | | | | | 1.00000 | 0.05503 |
| EURO/RAND | | | | | | | | | | 1.00000 |

These results clarify that the South African RAND exchange rates move the most largely. The Euro exchange rate expressed in RAND is unstable and is dramatically influenced by South Africa's economy. Whereas the spectrum for the POUND is the narrowest. This means the impact from the Pound on the Euro is the least.

There is the question why the Euro expressed in a developing country's currency (i.e., South African Rand) displays significantly larger fluctuations than the other developed countries. The correlations need to be examined. Table 1 displays the correlations between the EURO and the other currencies.

It can be seen that the USD, YEN, and POUND have greater correlating relationships with the EURO

than other currencies. Especially, the correlation between EURO and USD is the highest. The higher the correlation between EURO and another currency, the greater the other currency's movement may impact the EURO. This makes it clear that EURO movement is largely influenced by the USD. Considering the international trade balance between the EU and the USA is the largest as well compared with those between the EU and other countries. On the other hand, the correlation between the EURO and the RAND is less. This means it is hard for the RAND's movement to influence the EURO exchange rate. The huge fluctuation of EURO/RAND may have resulted from South Africa's unstable economy. Thus, it is reasonable to select the EURO/USD to represent the EURO exchange rate. In the following section, EURO/USD rates are examined.

### 4.4 Comparison of the Global Financial Crisis period and the European Sovereign Debt Crisis period

The multifractal results for the Global Financial Crisis and the European Sovereign Debt Crisis are shown in the following figures. Hereafter, G-period refers to the Global Financial Crisis period while D-period refers to the European Sovereign Debt Crisis period.

Figure 5 displays the original time series for these two periods. It can be observed that fluctuations for the two time-series went in a different way.

Figure 6 presents the $q-\tau(q)$ plots of the two time-series. As explained above, if $\tau(q)$ fits well to a non-linear curve, then the time series is considered to bear multifractality. Contrarily, if the graph of $\tau(q)$ is a straight line, then the time series holds mono-fractality. As seen in Figure 6(b), first, the $\tau(q)$ is approximately expressed by a linear function of $q$, which means multifractality does not exist in the D-period series. While inspecting Figure 6(a), the $q-\tau(q)$ plot is found to deviate from linearity, which is a hallmark for the presence of multifractality.

Figure 7 shows the $\alpha-D(\alpha)$ plots of the series in Global Financial Crisis and the European Sovereign Debt Crisis. Based on multifractal theory, the difference between $\alpha_{\max}$ and $\alpha_{\min}$ (denoted as $\Delta\alpha$) is the statistic to measure the strength of multifractality. The wider the difference, the stronger the strength of multifractal. In Figure 7(a) and Figure 7(b), the width of $\alpha$ for the series of the G-period is larger than that of the D-period. This indicates that the multifractality is stronger in the Global Financial Crisis. Therefore, it can be considered that the Global Financial Crisis did the most damage to the EU economy. Furthermore, the multifractality is weaker after the European Sovereign Debt Crisis.

Besides the results included in the figures, Table 2 presents the calculation results for the values of the singularity width $\Delta(\alpha)$ s for the two series. It can be seen in Table 2 that the $\Delta(\alpha)$ for the period of the Global Financial Crisis is a little wider than that for the period of the European Sovereign Debt Crisis. However, there is no significant difference between these two widths. These results suggest that the two crises impacted the Euro Exchange Rate to almost same level. After the Global Financial Crisis, the EU adopted numerous monetary policies to stabilize the exchange rate, but they seem to have had little effect.

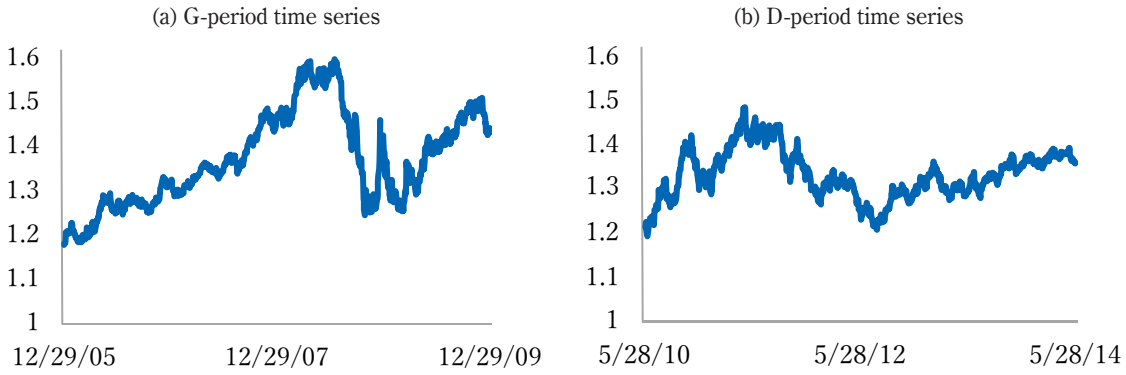


Figure 5.  Plots of the G-period and D-period Series

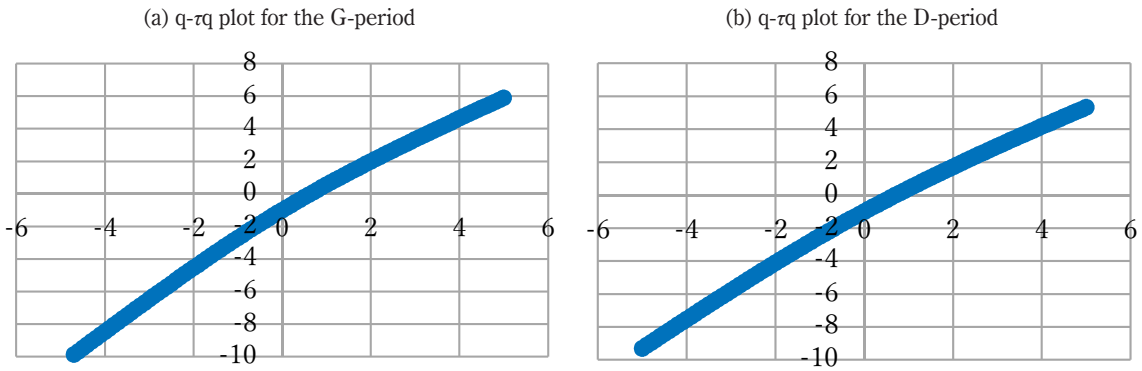


Figure 6.  q-τq plot

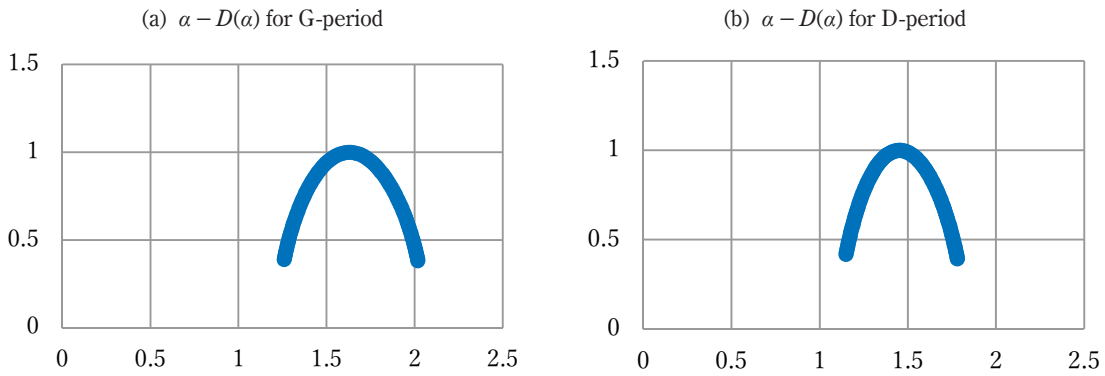


Figure 7  $\alpha - D(\alpha)$

Table 2: Singularity width

| Period / $\alpha$ | G-period | D-period |
|---|---|---|
| $\alpha_{min}$ | 0.381903101 | 0.394189626 |
| $\alpha_{max}$ | 1 | 1 |
| $\Delta\alpha$ | 0.618096899 | 0.605810374 |

## 5. Conclusion

The results of the multifractal analysis show that all these exchange rates (EURO/USD, EURO/YEN, EURO/POUND, EURO/RAND) bear the multifractality. This proves that the Euro exchange rate moves with high fluctuation and display unstable status. It also can be suggested that the Euro is not strong but easily influenced by financial shocks. Another finding can be determined by comparing the characteristics of the periods for the Global Financial Crisis and the European Sovereign Debt Crisis. It is clear that the Global Financial Crisis impacted the Euro exchange rate more. It can be concluded that the Global Financial Crisis caused highly deep and long-term damage to the EU economy. The European Sovereign Debt Crisis is supposed to have resulted from the Global Financial Crisis, but the long recession has not ended yet.

This study's analysis contributed a greater understanding of the impact of two crises on the Euro exchange rate by applying MF-DFA based multifractal theory. MF-DFA has rarely been applied in financial data analysis despite being widely used in physics, medical sciences, and engineering. This study has employed this approach to the Euro exchange rate. Strong multifractality has been detected, which means the series is unstable. Furthermore, through the comparison of the two big crises – the Global Financial Crisis and the European Sovereign Debt Crisis – it can be concluded that the Global Financial Crisis had a deeper and longer-term impact on the European economy.

## 6. Future research

Future research should focus on nonlinearity tests with different approaches. Moreover, determining the structural changes in time series is important when analyzing real financial data. Using the multifractal theory to detect the structural changes in time series is another research topic. Furthermore, future research will examine the effectiveness of EU monetary policies.

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