Large deviations and nite time ruin probabilities for generalized renewal risk models

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Large deviations and finite time ruin probabilities for generalized renewal risk models

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Abstract. In this paper, we extend the standard renewal risk model to the case where the premium income process is a counting process and the claim sizes and the inter-arrival times are two sequences of negatively associated random variables. For this risk model, the paper investigates the large deviations for the claim surplus process and gives the Lundberg type limiting results on the finite time ruin probabilities.

Keywords. Generalized renewal risk model, finite time ruin probability, large deviation

1. Introduction and main results

In risk theory, the standard renewal risk model has the following structure:

(i) The claim sizes, $X_n, n \geq 1$, form a sequence of independent, identically distributed(i.i.d.) and nonnegative random variables(r.v.s) with a finite mean;

(ii) The inter-arrival times, $T_n, n \geq 1$, form another sequence of i.i.d. and nonnegative r.v.s, which are independent of the r.v.s, $X_n,\ n \geq 1$, and have a finite mean;

(iii) The number of claims in the interval $[0, t]$ is denoted by

$$N(t) = \sup\{n \geq 1 : \sum_{k=1}^{n} T_k \leq t\}, t \geq 0,$$

where, by convention, $\sup \emptyset = 0$. Then the risk reserve process $\{R(t); t \geq 0\}$ is

$$R(t) = u + ct - \sum_{n=1}^{N(t)} X_n, t \geq 0,$$

where $c > 0$ is the constant premium income rate, and $u > 0$ is the initial surplus of the insurance company. The net profit condition is $c\mathbb{E}T_1 > \mathbb{E}X_1$.

As pointed out by Hu [4], from a realistic point of view, the premium income process should depend on the number of customers who buy the insurance portfolios. Clearly, this number is a r.v. in a given time interval $[0, t], t \geq 0$. In this paper, we will model this process by a counting process, not necessarily be a linear function. In the standard renewal risk model, the claim sizes, $X_n, n \geq 1$ and the inter-arrival times, $T_n, n \geq 1$ are two sequences of independent r.v.s. In this paper, we will consider the negatively dependent claim sizes and the negatively dependent inter-arrival times. We will assume that $\{X_n, n \geq 1\}$ and $\{T_n, n \geq 1\}$ are two negatively associated sequences. By definition, a sequence $\{Z_n, n \geq 1\}$ is said to be negatively associated(NA) if, for any $m \geq 2$ and any disjoint nonempty subsets A and B of $\{1, 2, \cdots, m\}$

$$cov(f(Z_i : i \in A), g(Z_j : j \in B)) \leq 0,$$

where f and g are any coordinatewise nondecreasing functions such that the moment involved exists. For details, one can refer to Joad-Dev and Proschan [5].

In this paper, we will consider the following risk model:

(i) The claim sizes, $X_n, n \geq 1$ are NA and nonnegative r.v.s with a common distribution F and a finite mean;

(ii) The inter-arrival times, $T_n, n \geq 1$, form another sequence of NA identically distributed and nonnegative r.v.s, which are independent of the r.v.s, $X_n,\ n \geq 1$ and have a finite mean;

(iii) The number of claims in the interval $[0, t]$ is denoted by

$$N(t) = \sup\{n \geq 1 : \sum_{k=1}^{n} T_k \leq t\}, t \geq 0.$$

Let $\lambda(t) = \mathbb{E}(N(t)), t \geq 0$;

(iv) The number of customers who buy the insurance portfolios in the time interval $[0, t]$ is denoted by $M(t), t \geq 0$, which is independent of $\{X_n, n \geq 1\}$ and $\{T_n, n \geq 1\}$, and has a finite mean $\delta(t) = \mathbb{E}(M(t)), t \geq 0$.

We will call such a risk model as above a generalize dependent renewal risk model(GDRRM). When $X_n, n \geq 1$ and $T_n, n \geq 1$ are independent r.v.s, and $\{N(t); t \geq 0\}$ and $\{M(t); t \geq 0\}$ are two Poisson processes, this is the generalized compound Poisson risk model, which was introduced by Hu [4]. For the GDRRM, the risk reserve process $\{R(t); t \geq 0\}$ is given by

$$R(t) = u + c_1 M(t) - \sum_{n=1}^{N(t)} X_n, t \geq 0,$$

while the claim surplus process $\{S(t); t \geq 0\}$ is

$$S(t) = \sum_{n=1}^{N(t)} X_n - c_1 M(t) := Y(t) - c_1 M(t), t \geq 0,$$

where $c_1 > 0$ is the premium of a single insurance portfolio(i.e. the price of the insurance portfolio), and $u > 0$ is the initial surplus of the insurance company. The net profit condition is $c_1\delta(t) > \lambda(t)\mathbb{E}(X_1), t \geq 0$. The time of ruin is

$$\tau(u) = \inf\{t \geq 0 : R(t) < 0\} = \inf\{t \geq 0 : S(t) > u\}.$$

Hereafter, all limit relationships are for $t \to \infty$ unless otherwise stated, and, for two positive functions $a(t)$ and $b(t)$, we write $a(t) \sim b(t)$ if $\lim a(t)/b(t) = 1$; write $a(t) = O(1)b(t)$ if $\limsup a(t)/b(t) < \infty$ and write $a(t) = o(1)b(t)$ if $\lim a(t)/b(t) = 0$. For a proper distribution V on $(-\infty, \infty)$, let $\overline{V}(x) = 1 - V(x), x \in (-\infty, \infty)$ be its tail.

We shall restrict ourselves to the case of heavy-tailed claim distributions. So we first introduce some heavy-tailed distribution classes. We say that a distribution V on $[0, \infty)$ belongs to the extended regular variation class, if there are some $0 < \alpha \leq \beta < \infty$ such that

$$s^{-\beta} \leq \liminf \frac{\overline{V}(st)}{\overline{V}(t)} \leq \limsup \frac{\overline{V}(st)}{\overline{V}(t)} \leq s^{-\alpha} \quad \text{for all } s \geq 1,$$

denoted by $V \in ERV(-\alpha, -\beta)$. If $\alpha = \beta$, we say that V belongs to the regular variation class and write $V \in \mathcal{R}_{-\alpha}$.

A larger class is the so-called dominated variation class. By definition, a distribution V on $[0, \infty)$ belongs to dominated variation class, denoted by $V \in \mathcal{D}$, if

$$\liminf \frac{\overline{V}(ty)}{\overline{V}(t)} > 0, \quad \text{for all } y > 1.$$

A subclass of $\mathcal{D}$ is the consistent variation class. By definition, a distribution V on $[0, \infty)$ belongs to the consistent variation class, denoted by $V \in \mathcal{C}$, if

$$\lim_{y \downarrow 1} \liminf_{t \to \infty} \frac{\overline{V}(ty)}{\overline{V}(t)} = 1.$$

It is well known that the following inclusions are proper

$$\mathcal{R}_{-\alpha} \subset ERV(-\alpha, -\beta) \subset \mathcal{C} \subset \mathcal{D}.$$

One can refer to Embrechts et al. [3] and Cline and Samorodnitsky [2], etc.

For the generalized compound Poisson risk model, under the condition $F \in ERV(-\alpha, -\beta), 1 < \alpha \leq \beta < \infty$, Hu [4] has investigated the probabilities of large deviations of $S(t)$ and the Lundberg type limiting results of the finite time ruin probabilities. In this paper, these problems will be discussed for the GDRRM with a claim distribution $F \in \mathcal{D}$. Throughout this paper, we assume that the counting processes $\{N(t); t \geq 0\}$ and $\{M(t); t \geq 0\}$ satisfy the following condition:

Condition 1.1 $\quad \dfrac{M(t)}{\delta(t)} \xrightarrow{\mathbb{P}} 1$ and

$$p = \sup_{t \geq t_0} \frac{\delta(t)}{\lambda(t)} \in (0, \infty) \quad \text{for some } t_0 > 0.$$

Note that, when $\{N(t); t \geq 0\}$ and $\{M(t); t \geq 0\}$ are two renewal counting processes generated by the i.i.d. r.v.s, using the strong laws of large numbers for renewal counting processes and the elementary renewal theorem, we know that Condition 1.1 is satisfied. Recently, Yang and Wang [9] have obtained the elementary renewal theorem for the NA r.v.s (see Lemma 2.1 below), so Condition 1.1 is also satisfied for the renewal counting processes generated by the NA r.v.s.

Before giving the main results, we introduce some notation. Let V be a distribution concentrated on $[0, \infty)$. For any $y > 1$, we set

$$\overline{V}_*(y) = \liminf \frac{\overline{V}(ty)}{\overline{V}(t)},$$

and then define

$$L_V = \lim_{y \downarrow 1} \overline{V}_*(y), \quad J_V^+ = \inf\{-\frac{\log \overline{V}_*(y)}{\log y} : y > 1\}.$$

It is well known that $V \in \mathcal{D} \Longleftrightarrow L_V > 0 \Longleftrightarrow J_V^+ < \infty$(see the proof of Lemma 3.5 of Tang and Tsitsiashvili [7] and Bingham et al. [1]). For the claim sizes, $X_n, n \geq 1$, let $A \subset \{1, 2, \cdots\}$, $\sigma(X_i : i \in A)$ is a σ-field generated by $X_i, i \in A$ and define

$$\varphi(1) = \sup_{n \geq 2} \sup_{k \geq 1} \sup_{C \in \sigma(X_i : 1 \leq i \leq n, i \neq k)\, D \in \sigma(X_k), P(D) > 0} |P(C|D) - P(C)|.$$

Obviously, $0 \leq \varphi(1) \leq 1$ and when $X_n : n \geq 1$ are independent, $\varphi(1) = 0$. This notation was introduced by Wang et al. [8].

Now we present the main results.

Theorem 1. *For the GDRRM, suppose that Condition 1.1 is satisfied and $F \in \mathcal{D}$. Then, for any fixed $\gamma > 0$,*

$$(1) \quad 1 \leq \liminf_{t \to \infty} \inf_{x \geq \gamma\lambda(t)} \frac{\mathbb{P}(Y(t) > x)}{\lambda(t)\overline{F}(x)} \leq \limsup_{t \to \infty} \sup_{x \geq \gamma\lambda(t)} \frac{\mathbb{P}(Y(t) > x)}{\lambda(t)\overline{F}(x)} \leq L_F^{-1};$$

$$(2) \quad \max\{(1 - \varphi(1))L_F, \overline{F}_*(1 + \mathbb{E}(X_1)\gamma^{-1})\} \leq \liminf_{t \to \infty} \inf_{x \geq \gamma\lambda(t)} \frac{\mathbb{P}(Y(t) - \mathbb{E}(Y(t)) > x)}{\lambda(t)\overline{F}(x)} \leq \limsup_{t \to \infty} \sup_{x \geq \gamma\lambda(t)} \frac{\mathbb{P}(Y(t) - \mathbb{E}(Y(t)) > x)}{\lambda(t)\overline{F}(x)} \leq L_F^{-1};$$

$$(3) \quad \overline{F}_*(1 + c_1 p\gamma^{-1}) \leq \liminf_{t \to \infty} \inf_{x \geq \gamma\lambda(t)} \frac{\mathbb{P}(S(t) > x)}{\lambda(t)\overline{F}(x)} \leq \limsup_{t \to \infty} \sup_{x \geq \gamma\lambda(t)} \frac{\mathbb{P}(S(t) > x)}{\lambda(t)\overline{F}(x)} \leq L_F^{-1}$$

and for any fixed $\gamma > c_1 p$,

$$(4) \qquad \max\{(1-\varphi(1))L_F, \overline{F}_*(1+\mathbb{E}(X_1)\gamma^{-1})\}L_F \le \liminf_{t\to\infty} \inf_{x\ge\gamma\lambda(t)} \frac{\mathbb{P}(S(t)-\mathbb{E}(S(t))>x)}{\lambda(t)\overline{F}(x)} \le \limsup_{t\to\infty} \sup_{x\ge\gamma\lambda(t)} \frac{\mathbb{P}(S(t)-\mathbb{E}(S(t))>x)}{\lambda(t)\overline{F}(x)} \le L_F^{-2}.$$

In particular, if $F \in \mathcal{C}$ *and* $X_n, n \ge 1$ *are independent, then for any fixed* $\gamma > c_1 p$,

$$(5) \qquad \lim_{t\to\infty} \sup_{x\ge\gamma\lambda(t)} \left| \frac{\mathbb{P}(S(t)-\mathbb{E}(S(t))>x)}{\lambda(t)\overline{F}(x)} - 1 \right| = 0.$$

Remark 1. If $\{N(t); t \ge 0\}$ and $\{M(t); t \ge 0\}$ are two Poisson processes then Condition 1.1 is satisfied. So from (5), Theorem 2.1 of Hu [4] can be obtained.

Before giving the Lundberg type limiting results on the time of ruin $\tau(u)$, we introduce two parameters, namely,

$$\overline{\alpha} = \liminf \frac{-\log \overline{F}(t)}{\log t} \quad \text{and} \quad \underline{\alpha} = \limsup \frac{-\log \overline{F}(t)}{\log t}.$$

Clearly, $\overline{\alpha} \le \underline{\alpha}$. A further useful fact is that

$$\overline{\alpha} = \sup\{\alpha \ge 0 : \mathbb{E}(X_1^\alpha) < \infty\},$$

(see Rolski et al. [6]). Since $\mathbb{E}X_1 < \infty$, then $\overline{\alpha} \ge 1$. If $F \in \mathcal{D}$ then there exists an $M > 0$ such that $t^M \overline{F}(t) \to \infty$. Hence by the contradiction, it is easy to show that $\underline{\alpha} < \infty$. In particular, if $F \in ERV(-\alpha, -\beta)$, then $\overline{\alpha} = \alpha$ and $\underline{\alpha} = \beta$. If $F \in \mathcal{R}_{-\alpha}$ then $\overline{\alpha} = \underline{\alpha} = \alpha$.

Theorem 2. *For the GDRRM, suppose that Condition 1.1 is satisfied and* $F \in \mathcal{D}$. *Then, for any fixed* $0 < x \le 1$ *and* $y > 0$,

$$(6) \qquad \liminf_{u\to\infty} \frac{\log \mathbb{P}(\tau(u) \le yu^x)}{\log u} \ge x - \underline{\alpha}$$

and

$$(7) \qquad \limsup_{u\to\infty} \frac{\log \mathbb{P}(\tau(u) \le yu^x)}{\log u} \le x - \overline{\alpha}.$$

In particular, if $F \in \mathcal{R}_{-\alpha}$ *for some* $\alpha > 1$ *then for any fixed* $0 < x \le 1$ *and* $y > 0$,

$$(8) \qquad \lim_{u\to\infty} \frac{\log \mathbb{P}(\tau(u) \le yu^x)}{\log u} = x - \alpha.$$

In the next section, the main results will be proved.

2. Proofs of main results

We first give a lemma.

Lemma 1. *Suppose that* $\{Z_n, n \ge 1\}$ *is a sequence of NA identically distributed and nonnegative r.v.s with a finite mean.* $N_1(t) = \sup\{n \ge 1 : \sum_{k=1}^n Z_k \le t\}, t \ge 0$. *Then*

$$(9) \qquad \frac{N_1(t)}{t} \to \frac{1}{\mathbb{E}(Z_1)} \quad a.s.; \qquad \frac{\mathbb{E}(N_1(t))}{t} \to \frac{1}{\mathbb{E}(Z_1)}$$

and for any $\theta > 0$ *and some* $\varepsilon > 0$,

$$(10) \qquad \sum_{k>(1+\theta)\mathbb{E}(N_1(t))} (1+\varepsilon)^k \mathbb{P}(N_1(t) \ge k) = o(1).$$

Proof. The relation (9) is Theorem 6.1 of Yang and Wang [9]. We now prove (10). For any $h > 0$, let $f(h) = -\log \mathbb{E}(e^{-hZ_1})$, then $f(h) > 0$ and

$$\lim_{h\to 0} \frac{f(h)}{h} = \lim_{h\to 0} \frac{\mathbb{E}(e^{-hZ_1}Z_1)}{\mathbb{E}(e^{-hZ_1})} = \mathbb{E}(Z_1).$$

Thus, for any fixed $\theta > 0$, there exists an $h_0 = h_0(\theta)$ such that

$$(11) \qquad (1+\frac{\theta}{2})\frac{f(h_0)}{\mathbb{E}(Z_1)} > (1+\frac{\theta}{4})h_0.$$

Take $s = \log(1+\varepsilon)$, then letting ε be sufficiently small such that

$$(12) \qquad 0 < s < \theta(\theta+4)^{-1} f(h_0).$$

Thus, by Property P_2 of Joad-Dev and Proschan [5], (11), (9) and (12), we have that when t is sufficiently large

$$\begin{aligned}
&\sum_{k>(1+\theta)\mathbb{E}(N_1(t))} (1+\varepsilon)^k \mathbb{P}(N_1(t) \ge k) \\
&= \sum_{k>(1+\theta)\mathbb{E}(N_1(t))} (1+\varepsilon)^k \mathbb{P}(\sum_{i=1}^k Z_i \le t) \\
&\le \sum_{k>(1+\theta)\mathbb{E}(N_1(t))} e^{sk+h_0 t} (\mathbb{E}(e^{-h_0 Z_1}))^k \\
&\le e^{h_0 t} \int_{(1+\theta)\mathbb{E}(N_1(t))}^{\infty} e^{sy} (\mathbb{E}(e^{-h_0 Z_1}))^y \mathrm{d}y \\
&= e^{h_0 t} (f(h_0))^{-1} \int_{(1+\theta)\mathbb{E}(N_1(t))f(h_0)}^{\infty} \exp\left\{\left(\frac{s}{f(h_0)} - 1\right) y\right\} \mathrm{d}y \\
&= (f(h_0)-s)^{-1} e^{h_0 t} \exp\left\{\left(\frac{s}{f(h_0)} - 1\right)(1+\theta)\mathbb{E}(N_1(t)) f(h_0)\right\} \\
&\le (f(h_0)-s)^{-1} e^{h_0 t} \exp\left\{\left(\frac{s}{f(h_0)} - 1\right)\left(1+\frac{\theta}{2}\right)\frac{t}{\mathbb{E}(Z_1)} f(h_0)\right\} \\
&\le (f(h_0)-s)^{-1} \exp\left\{\left(\frac{s}{f(h_0)}\left(1+\frac{\theta}{4}\right) - \frac{\theta}{4}\right) h_0 t\right\} \\
&= o(1).
\end{aligned}$$

□

Proof of Theorem 1. For the proofs of (1) and (2), using Theorem 2.2 of Wang et al. [8], we only need to show that $\{N(t); t \ge 0\}$ satisfies the following condition: for some $\alpha > J_F^+$ and any $\theta > 0$,

$$(13) \qquad \mathbb{E}\left((N(t))^\alpha 1(N(t) > (1+\theta)\lambda(t))\right) = O(1)\lambda(t),$$

where $1(A)$ denotes the indicator function of an event A.

By Lemma 1, we know that for any $\theta > 0$ and some $\varepsilon > 0$,

$$\sum_{k>(1+\theta)\lambda(t)} (1+\varepsilon)^k \mathbb{P}(N(t) \geq k) = o(1).$$

It follows from (9) that $N(t) \to \infty, a.s.$ and $\lambda(t) \to \infty$. Thus, for any fixed $\alpha > 0$ and $\theta > 0$, when t is sufficiently large,

$$\begin{aligned}&\mathbb{E}\left((N(t))^\alpha 1(N(t) > (1+\theta)\lambda(t))\right)\\ &\leq \mathbb{E}\left((1+\varepsilon)^{N(t)} 1(N(t) > (1+\theta)\lambda(t))\right)\\ &\leq \sum_{k>(1+\theta)\lambda(t)} (1+\varepsilon)^k \mathbb{P}(N(t) \geq k) = o(1).\end{aligned}$$

So (13) is satisfied.

For the proof of (3), on the one hand, by (1), for any fixed $\gamma > 0$,

$$\begin{aligned}(14)\qquad &\limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(S(t) > x)}{\lambda(t)\overline{F}(x)}\\ &= \limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t) > x + c_1 M(t))}{\lambda(t)\overline{F}(x)}\\ &\leq \limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t) > x)}{\lambda(t)\overline{F}(x)} \leq L_F^{-1}.\end{aligned}$$

On the other hand, for any $0 < \theta < 1$,

$$\begin{aligned}(15)\qquad &\mathbb{P}(S(t) > x)\\ &= \mathbb{P}(Y(t) > x + c_1 M(t))\\ &= \sum_{n=0}^{\infty} \mathbb{P}(Y(t) > x + c_1 n)\mathbb{P}(M(t) = n)\\ &\geq \sum_{n\leq(1+\theta)\delta(t)} \mathbb{P}(Y(t) > x + c_1 n)\mathbb{P}(M(t) = n).\end{aligned}$$

Using (1) and Condition 1.1, for any fixed $\gamma > 0$,

$$\begin{aligned}&\liminf_{t\to\infty} \inf_{x\geq\gamma\lambda(t)} \sum_{n\leq(1+\theta)\delta(t)} \frac{\mathbb{P}(Y(t) > x + c_1 n)}{\lambda(t)\overline{F}(x)} \times \mathbb{P}(M(t) = n)\\ &\geq \liminf_{t\to\infty} \inf_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t) > x + c_1(1+\theta)\delta(t))}{\lambda(t)\overline{F}(x)} \times \mathbb{P}(M(t) \leq (1+\theta)\delta(t))\\ &\geq \liminf_{t\to\infty} \inf_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t) > x + c_1(1+\theta)\delta(t))}{\lambda(t)\overline{F}(x + c_1(1+\theta)\delta(t))} \times \frac{\overline{F}(x(1 + (1+\theta)c_1 p\gamma^{-1}))}{\overline{F}(x)}\\ &\geq \liminf_{x\to\infty} \frac{\overline{F}(x(1 + (1+\theta)c_1 p\gamma^{-1}))}{\overline{F}(x)}\\ &= \overline{F}_*(1 + (1+\theta)c_1 p\gamma^{-1}).\end{aligned}$$

Then letting $\theta \downarrow 0$, by (15) we have that for any fixed $\gamma > 0$,

$$\liminf_{t\to\infty} \inf_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(S(t) > x)}{\lambda(t)\overline{F}(x)} \geq \overline{F}_*(1 + c_1 p\gamma^{-1}),$$

which combined with (14) yields that (3) holds.

Now we prove (4). For any $0 < \theta < 1$,

$$\begin{aligned}(16)\quad &\mathbb{P}(S(t) - \mathbb{E}(S(t)) > x)\\ &= \sum_{n=0}^{\infty} \mathbb{P}(Y(t) - \mathbb{E}(Y(t)) > x - c_1\delta(t) + c_1 n) \times \mathbb{P}(M(t) = n)\\ &= \left(\sum_{n<(1-\theta)\delta(t)} + \sum_{(1-\theta)\delta(t)\leq n\leq(1+\theta)\delta(t)} + \sum_{n>(1+\theta)\delta(t)}\right)\\ &\quad \mathbb{P}(Y(t) - \mathbb{E}(Y(t)) > x - c_1\delta(t) + c_1 n)\mathbb{P}(M(t) = n)\\ &\equiv J_1 + J_2 + J_3.\end{aligned}$$

For J_1, by (2), Condition 1.1 and $F \in \mathcal{D}$, for any fixed $\gamma > c_1 p$,

$$\begin{aligned}(17)\quad &\limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{J_1}{\lambda(t)\overline{F}(x)}\\ &\leq \limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t) - \mathbb{E}(Y(t)) > x - c_1\delta(t))}{\lambda(t)\overline{F}(x - c_1\delta(t))} \times \frac{\overline{F}(x(1 - c_1 p\gamma^{-1}))}{\overline{F}(x)}\mathbb{P}(M(t) < (1-\theta)\delta(t))\\ &\leq L_F^{-1} \limsup_{t\to\infty} \frac{\overline{F}(x(1 - c_1 p\gamma^{-1}))}{\overline{F}(x)} \times \lim_{t\to\infty} \mathbb{P}(M(t) < (1-\theta)\delta(t))\\ &= 0.\end{aligned}$$

For J_3, by (2) and Condition 1.1, for any fixed $\gamma > 0$,

$$\begin{aligned}(18)\quad &\limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{J_3}{\lambda(t)\overline{F}(x)}\\ &\leq \limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t) - \mathbb{E}(Y(t)) > x)}{\lambda(t)\overline{F}(x)} \times \mathbb{P}(M(t) > (1+\theta)\delta(t))\\ &\leq L_F^{-1} \lim_{t\to\infty} \mathbb{P}(M(t) > (1+\theta)\delta(t)) = 0.\end{aligned}$$

For J_2, since $0 < \theta < 1 < (c_1 p)^{-1}\gamma$ for any fixed $\gamma > c_1 p$, by using (2) and Condition 1.1, we have

$$\begin{aligned}(19)\quad &\limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{J_2}{\lambda(t)\overline{F}(x)}\\ &\leq \limsup_{t\to\infty} \sup_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t) - \mathbb{E}(Y(t)) > x - c_1\theta\delta(t))}{\lambda(t)\overline{F}(x - c_1\theta\delta(t))} \times \limsup_{x\to\infty} \frac{\overline{F}(x(1 - c_1\theta p\gamma^{-1})}{\overline{F}(x)}\\ &\leq L_F^{-1} \frac{1}{\overline{F}_*\left((1 - c_1\theta p\gamma^{-1})^{-1}\right)}\end{aligned}$$

and

$$(20)\quad \liminf_{t\to\infty} \inf_{x\geq\gamma\lambda(t)} \frac{J_2}{\lambda(t)\overline{F}(x)} \geq \liminf_{t\to\infty} \inf_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(Y(t)-\mathbb{E}(Y(t))>x+c_1\theta\delta(t))}{\lambda(t)\overline{F}(x+c_1\theta\delta(t))} \times \liminf_{x\to\infty} \frac{\overline{F}(x(1+c_1\theta p\gamma^{-1})}{\overline{F}(x)}$$
$$\geq \max\{(1-\varphi(1))L_F, \overline{F}_*(1+\mathbb{E}(X_1)\gamma^{-1})\} \times \overline{F}_*\left(1+c_1\theta p\gamma^{-1}\right).$$

Thus, by (16) - (20), letting $\theta\downarrow 0$, we have that (4) holds.

For the proof of (5), recall that if $F\in\mathcal{C}$ then $L_F=1$, and if $X_n, n\geq 1$ are independent then $\varphi(1)=0$. Hence (5) can be immediately obtained from (4). □

Proof of Theorem 2 For any fixed $0<x\leq 1$ and $y>0$, take $0<a<y^{-1}\mathbb{E}(T_1)$. Then for any fixed $a<\gamma<y^{-1}\mathbb{E}(T_1)$, by (3) and (9)

$$\liminf_{u\to\infty} \frac{\mathbb{P}(S(yu^x)>u))}{\lambda(yu^x)\overline{F}(u)} \geq \liminf_{t\to\infty} \inf_{x\geq\gamma\lambda(t)} \frac{\mathbb{P}(S(t)>x))}{\lambda(t)\overline{F}(x)} \geq \overline{F}_*(1+c_1pa^{-1}).$$

Hence, by (9), for any $0<\varepsilon<1$, when u is large enough,

$$\begin{aligned}&\mathbb{P}(\tau(u)\leq yu^x)\\ &\geq \mathbb{P}(S(yu^x)>u))\\ &\geq (1-\varepsilon)\overline{F}_*(1+c_1pa^{-1})\lambda(yu^x)\overline{F}(u)\\ &\geq (1-2\varepsilon)\overline{F}_*(1+c_1pa^{-1})(\mathbb{E}T_1)^{-1}yu^x\overline{F}(u).\end{aligned}$$

Thus

$$\liminf_{u\to\infty} \frac{\log\mathbb{P}(\tau(u)\leq yu^x)}{\log u} \geq x+\liminf_{u\to\infty}\frac{\log\overline{F}(u)}{\log u} = x-\underline{\alpha}.$$

Similarly, for any fixed $0<x\leq 1$ and $y>0$, by (1) and (9), for any $0<\varepsilon<1$, when u is large enough

$$\begin{aligned}\mathbb{P}(\tau(u)\leq yu^x) &\leq \mathbb{P}(Y(yu^x)>u))\\ &\leq (1+\varepsilon)L_F^{-1}\lambda(yu^x)\overline{F}(u)\\ &\leq (1+2\varepsilon)L_F^{-1}(\mathbb{E}T_1)^{-1}yu^x\overline{F}(u).\end{aligned}$$

Thus

$$\limsup_{u\to\infty} \frac{\log\mathbb{P}(\tau(u)\leq yu^x)}{\log u} \leq x+\limsup_{u\to\infty}\frac{\log\overline{F}(u)}{\log u} = x-\overline{\alpha}.$$

□

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