Proposal of F-F-Objective Optimization for Many Objectives and its Evaluation with a 0/1 Knapsack Problem

Inoue, Makoto
Architect Office Optima Design

Takagi, Hideyuki
Faculty of Design, Kyushu University

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Proposal of F-F-Objective Optimization for Many Objectives and its Evaluation with a 0/1 Knapsack Problem

Makoto Inoue
Architect Office Optima Design
Fukuoka, Japan
http://inoue.s201.xrea.com/

Hideyuki Takagi
Faculty of Design, Kyushu University
Fukuoka, Japan
http://www.design.kyushu-u.ac.jp/~takagi/

***Abstract*—We propose Fewer-Fixed-Objective Optimization (F-F-Objective Optimization), a method for improving the capabilities of evolutionary many-objective optimization. The method is evaluated by applying it to a multi-objective 0/1 knapsack problem. Searching performance in many-objective optimization becomes drastically worse as the number of objectives is increased. To address this problem, the proposed method ranks individuals in subsets of s objectives selected from the total m objectives, where s is a fixed number in [1, m]. The final rank of each individual is determined as the aggregation of its ${}_mC_s$ ranks.**

We begin by introducing the F-F-Objective Optimization concept and illustrating its application to a numerical 5-objective optimization problem. Next, we further investigate the proposed method using an 8-objective 0/1 knapsack problem as an example of a typical many-objective optimization problem. Here we apply multi-objective genetic algorithms (GA) with the proposed method for all values of s from 1 to 8. When $s = 1$, the method is equivalent to the average ranking method or weight-based GA with equal weights, and it is equivalent to conventional evolutionary multi-objective optimization when $s = m$. The method's performance is evaluated using such metrics as hypervolume and the C Metric. Finally, we discuss the proposed method with regards to its convergence characteristics and the diversity of its Pareto solutions.

***Keywords*-evolutionary many-objective optimization, multi-objective 0/1 knapsack problem, all combinations of fewer-fixed-objective optimization.**

I. INTRODUCTION

The optimization performance of Evolutionary Multi-objective Optimization (EMO) degrades with an increasing number of objectives. This class of problem is termed many-objective optimization. Ishibuchi et al. pointed out that the performance degradation was due to the rapid increase in the ratio of non-dominated solutions in whole individuals that occurs after some number of generations [14] when using the Pareto ranking method [9]. With Pareto ranking, an individual is a non-dominated solution if it dominates the other solutions for at least one objective, and it is this property which appears to be responsible for the phenomenon.

To address this problem, we propose an "all combinations of fewer-fixed-objective optimization" (F-F-Objective Optimization) method be used in many-objective optimization [12], [13]. The F-F-Objective Optimization handles a fixed number of objectives, s, which is fewer than the total number of objectives, m ($1 \leq s \leq m$). From here on, we will interchangeably refer to s as the F-F-Objectives. In other words, the proposed method ranks individuals in an s-dimensional objective space rather than in the original m-dimensions. However, unlike ordinal approaches where the number of objectives are reduced, our method ranks each individual in its ${}_mC_s$ combinations of objectives and determines a final rank by aggregating the obtained ${}_mC_s$ ranks to fully use all non-dominant information for all m objectives.

The objective of this paper is to evaluate the characteristics of convergence and diversity of our proposed method by applying a multi-objective genetic algorithm (MOGA) [5] using the proposed method to a multi-objective 0/1 knapsack problem as in[20] and analyze its performance.

Sato et al. proposed a method wherein the combination of fewer objectives used for ranking at every certain generations [17]. Other approaches for improving this problem with many-objective optimization include increasing selection pressure [1], [8], reducing the searching space inside the objective space [6], [7], and introducing evaluation indices during searching [19], [20]. It has been reported that an average ranking method [2], which does not use the Pareto approach to determine winners among non-dominated solutions, does not yield diverse solutions [3], [11], [16].

II. ALL COMBINATIONS OF FEWER-FIXED-OBJECTIVE OPTIMIZATION

A. The Development of the F-F-Objective Optimization Concept

It is easier to determine dominant/non-dominant relationships among individuals when the number of objectives is reduced for Pareto ranking. The Pareto ranking method chooses a non-dominated solution if the individual is superior to the other individuals for at least one objective (see Figure 1(a)) and all other dominant–non-dominant relations between the individual and other individuals on other objectives are ignored. In order that these ignored relationships be made use of, our proposed method sets s against all m objectives ($1 \leq s \leq m$), aggregating the ranks

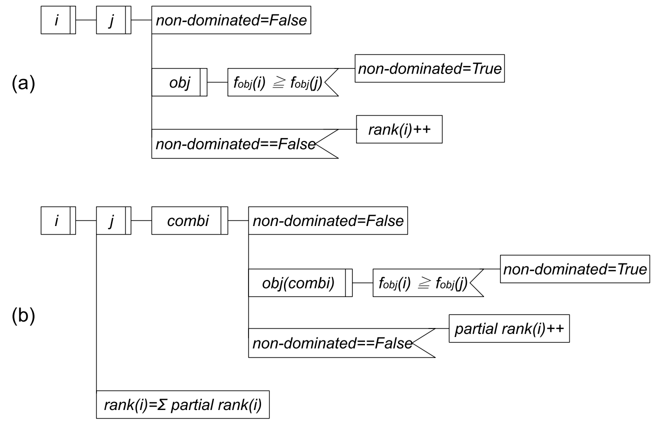

Figure 1. Problem Analyses Diagrams (PAD) [10] for (a) the conventional Pareto ranking method and (b) that of our proposed method, where i is the index of the individual, j is the index of another individual to be compared to the i-th individual ($i \neq j$), obj is the objective number, $f_{obj}()$ is the fitness function, and $combi$ is the combination number.

obtained in the ${}_mC_s$ different objective spaces, and uses this aggregated value as the final rank for the individual (see Figure 1(b)). If $s = 1$, this thus corresponds to the average ranking method in weight-based genetic algorithms (WBGA) [5] and if $s = m$, it corresponds to conventional EMO based on Pareto ranking.

B. A Numerical Example Illustrating the Proposed Method

Our proposed method can be explained using a solution example from [14], i.e. (A, B, C) = (2,2,2,2,2), (1,1,1,1,3), and (3,3,3,3,1) (see Figure 2) for a five-objective task, $m = 5$. When $s = 3$, there are ${}_5C_3$ (=10) combinations of 3 objectives from a total of 5 objectives, and their average ranks are shown in Table I. Similarly, average ranks for other choices of s ($1 \leq s \leq m$) are shown in Table II. The table shows that the smaller s becomes, the bigger the differences between the ranks of the solutions become. Ranking with fewer objectives can thus be seen to reduce the ratio of non-dominated solutions. Note that $s = 1$ and $s = 5$ correspond respectively to the average ranking method and ordinary Pareto ranking method.

Although our proposed method uses a reduced number of objectives, s, to find Pareto solutions, it considers all combinations of the s objectives. The final obtained ranks therefore include information from all m objectives. This implies that the proposed method preserves the characteristics of a Pareto ranking approach despite the reduced number of objectives, s, and that we can thus expect a diversity of solutions.

III. EXPERIMENTAL CONDITIONS AND EVALUATION INDEXES

A. Experimental Conditions

We evaluate our proposed method using a multi-objective 0/1 knapsack problem [20]. The problem is formulated as follows: 8 objectives (i.e. 8 knapsacks) and 100 items; item

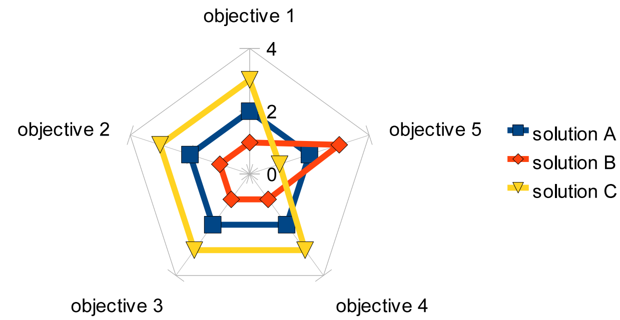

Figure 2. Visualization of the solution example of (A, B, C) = (2,2,2,2,2), (1,1,1,1,3), and (3,3,3,3,1).

Table I
PARETO RANKS OF SOLUTIONS (A, B, C), AND AVERAGE RANKS FOR ALL COMBINATIONS OF THREE OBJECTIVES SELECTED FROM FIVE, I.E. m = 5 AND s = 3.

Obj.	1 2 3	1 2 4	1 2 5	1 3 4	1 3 5	1 4 5	2 3 4	2 3 5	2 4 5	3 4 5	average rank
A	2	2	1	2	1	1	2	1	1	1	1.4
B	1	1	1	1	1	1	1	1	1	1	1.0
C	3	3	1	3	1	1	3	1	1	1	1.8

weights are on the interval [10,100]; the upper limit for total item weight for each knapsack is 0.5 of permissible weight of each knapsack, and individuals that do not satisfy this condition are considered lethal.

Our proposed F-F-Objective Optimization is embedded into MOGA [5] and used to calculate Pareto ranks. The ranks are used without scaling.

We evaluate the performance of the proposed method with eight F-F-Objectives ($s = 1, 2, ..., 8$) where the GA conditions are as follows. There are 200 individuals, a maximum of 500 generations, 10 trial runs, roulette wheel selection, 100% crossover rate with uniform crossover, 1% mutation rate, 0.4924 niche radius σ_{share} [5] as used in [4], and a value of 1 for the power parameter α_{share} in the share function [5]. $s = 1$ and $s = 8$ correspond respectively to 8 single-objective optimizations with equal weights, and ordinary MOGA. An elite strategy is not used; parent individuals are all replaced with their offspring.

The experiments are conducted on an Intel Core2 Duo P8700 at 2.53GHz with 2GB RAM running 32 bit Windows

Table II
THE NUMBER OF COMBINATIONS OF s OBJECTIVES FROM FIVE AND THE AVERAGE RANKS OF THE SOLUTIONS (A, B, C).

the number of fewer-fixed-objectives	s=1	s=2	s=3	s=4	s=5
the number of combinations (${}_5C_s$)	5	10	10	5	1
A	2.0	1.6	1.4	1.2	1.0
B	1.4	1.0	1.0	1.0	1.0
C	2.6	2.2	1.8	1.4	1.0

7. The numerical analysis software used is INRIA Scilab 5.2.1.

B. Evaluation Indexes

Hypervolume and the C Metric, two performance indexes for convergence and diversity, were adopted as the primary measures of the experimental results. In addition, CPU time was used to measure computational cost and maximum spread to measure the diversity of solutions. Detail descriptions of these indexes are as follows.

Hypervolume [15] is the volume of the area occupied by the Pareto-Optimal Solutions (POS) in the multi-objective space. The bigger its value, the better both convergence and diversity of the POS. We use the weighted hypervolume indicator [21] to calculate its value. Since the calculation for all POS would be extremely time consuming, the calculation is only performed for the 200 POS that have the lowest niche count [5].

C Metric [15] indicates the ratio of members of POS set B dominated by members of POS set A and is given by Eq. (1).

$$C(A,B) = \frac{|\{b \in B; \exists a \in A : a \preceq b\}|}{|B|} \quad (1)$$

CPU time is the total calculation time to execute 10 trial runs for the number of F-F-Objectives, s.

Maximum spread (MS) [18] given by Eq.(2) indicates how wide the POS coverage is in the multi-objective space and is used as an index of POS diversity. The bigger MS is, the wider the coverage of the POS. We set F_i^{max}=5500 and F_i^{mim}=10 in the Eq.(2) taking into account of the number of items, 100, for our knapsack problem, the interval of item weights, [10,100], and the upper limit of total item weight of each knapsack, 0.5. POS diversity is respectively least or greatest when MS is close to 0 or 1.

$$MS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{min(f_i^{max}, F_i^{max}) - max(f_i^{min}, F_i^{min})}{F_i^{max} - F_i^{min}}\right)^2} \quad (2)$$

IV. EXPERIMENTAL RESULTS

Figure 3 shows boxplots of the hypervolume for 10 trial runs where $s = 1, 2, ..., 8$. Hypervolume is smallest when s = 8, and it becomes bigger as s is reduced to $7, 6, ...,$. It does not change when s becomes less than or equal to 5.

Figure 4 shows boxplots of all combinations of C Metric for $s = 1, 2, ..., 8$, where A and B in $C(A, B)$ mean the number of F-F-Objectives, s. Generally speaking, POS with smaller s dominate other POS, and the dominated ratio becomes bigger as the difference between A and B increases.

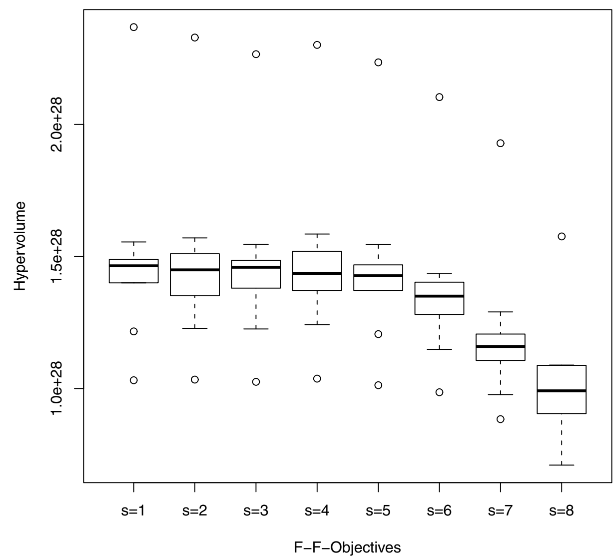

Figure 3. Boxplot of Hypervolume for 10 trial runs for F-F-Objectives, $s = 1, 2, ..., 8$.

Table III
THE RELATIONSHIP BETWEEN F-F-OBJECTIVES AND THE NUMBER OF THEIR COMBINATIONS.

F-F-Objectives	$s = 1$	$s = 2$	$s = 3$	$s = 4$
# of objective combination	${}_8C_1 = 8$	${}_8C_2 = 28$	${}_8C_3 = 56$	${}_8C_4 = 70$
F-F-Objectives	$s = 5$	$s = 6$	$s = 7$	$s = 8$
# of objective combination	${}_8C_5 = 56$	${}_8C_6 = 28$	${}_8C_7 = 8$	${}_8C_8 = 1$

The number of objective combinations, i.e. the number of partial ranks, depends deeply on the number of F-F-Objectives, s. This is illustrated in table III, which shows the number of combinations for each s. With our proposed method, $s = 8$ corresponds to ordinary MOGA and the number of combinations is 1. For $s = 4$, the number of combinations reaches its maximum of 70.

Figure 5 shows the total CPU time in seconds for 10 trial-runs of each F-F-Objective. CPU time is greatest when the number of F-F- Objectives s =4 and least when it is 8.

Figure 6 shows boxplots of maximum spread for 10 trial-runs of each F-F-Objective. In our experimental conditions with eight objectives optimization, maximum spread is the biggest when $s = 8$ and POS covers the widest area in multi-objective space. Maximum spread decreases as s is reduced and becomes stable when s is less than or equal to 5. The range between max. and min. of the boxplot is relatively small when s is eight, but it does not change drastically when s is less than equal six.

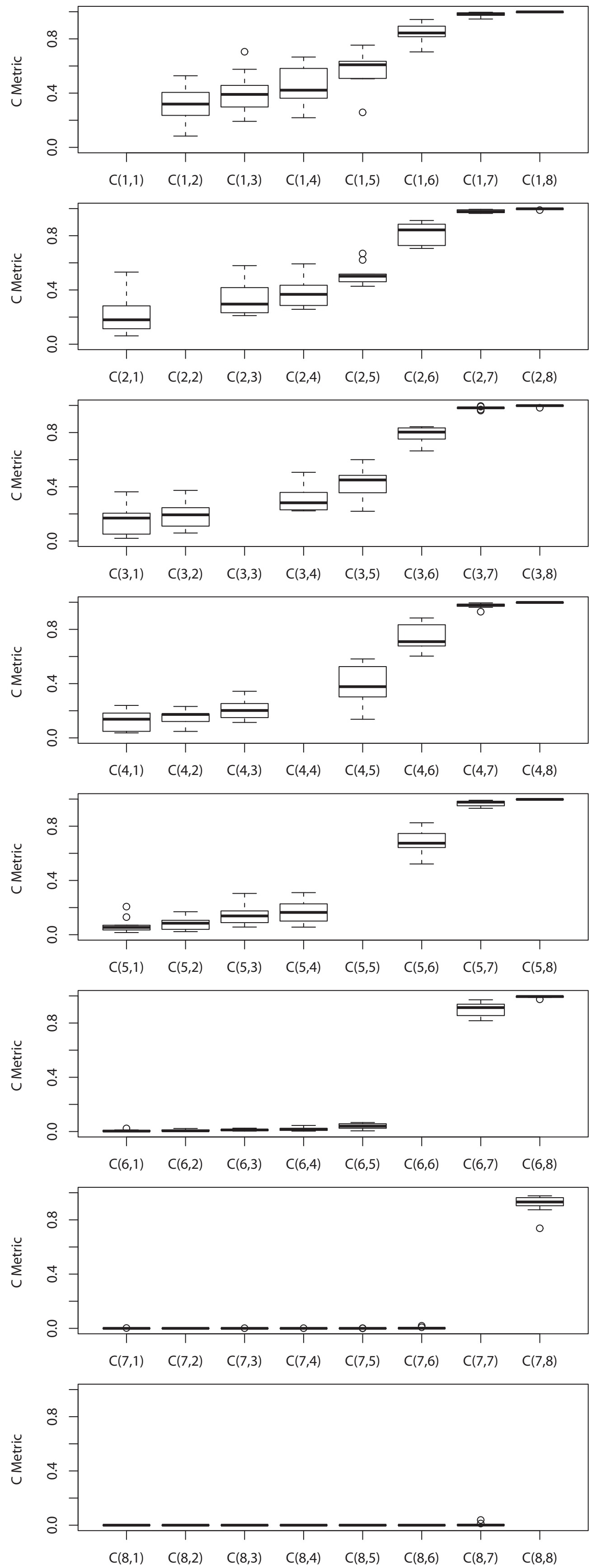

Figure 4. Boxplots of C Metric for all combination of F-F-Objectives, s, after 10 trial-runs. A and B of $C(A, B)$ mean the number of F-F-Objectives, s.

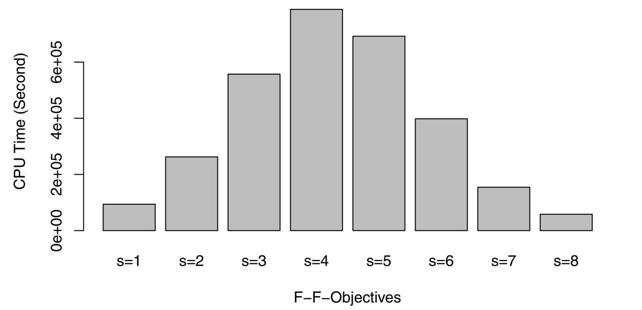

Figure 5. Total CPU time in seconds for 10 trial-runs of each F-F-Objectives, s.

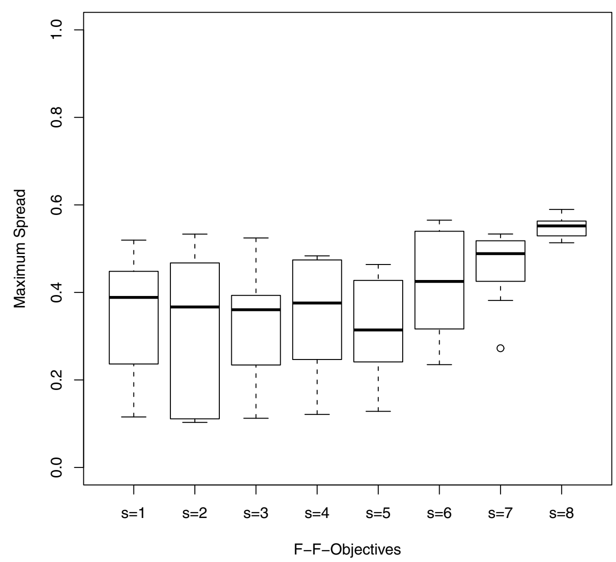

Figure 6. Boxplots of maximum spread for 10 trial-runs of each F-F-Objectives, s.

V. DISCUSSION

As shown in Figure 3, hypervolume increases as s is decreased from 8 to 5. Significant differences among s = 8, 7, 6, and 5 were indicated by the sign test ($p < 0.01$). The same significance was found between s = 5 and 3 ($p < 0.05$), but there were no significant differences for $s \leq 3$. We can say that hypervolume changed significantly when $3 \leq s \leq 8$ in our experiment. As the hypervolume is an evaluation index that expresses the performances of both convergence and diversity, we can say that our proposed F-F-Objective Optimization is effective, at least from the point of view of hypervolume.

As shown in Figure 4, the C Metric suggests that there is tendency with F-F-Objectives for the wider POS to dominate other individuals, and the bigger the difference between two F-F-Objectives the greater the extent to which the POS dominates. Table IV shows the results of sign tests which confirm this tendency. When the difference between A and B is around 2 for $1 \leq s \leq 3$, there is no significant difference between C(A, B) and C(B, A), but it can be observed for $5 \leq s \leq 8$ ($p < 0.01$).

Table IV
SIGN TEST RESULTS BETWEEN $C(A, B)$ AND $C(B, A)$.B, b, –, w, AND W MEAN, RESPECTIVELY, THAT $C(A, B)$ IS SIGNIFICANTLY BETTER THAN $C(B, A)$ ($p < 0.01$), SIGNIFICANTLY BETTER ($p < 0.05$), EVEN WITH $C(B, A)$, SIGNIFICANTLY WORSE THAN $C(B, A)$ ($p < 0.05$), AND SIGNIFICANTLY WORSE ($p < 0.01$).

A \B	1	2	3	4	5	6	7	8
1		–	–	b	B	B	B	B
2	–		–	B	B	B	B	B
3	–	–		–	b	B	B	B
4	w	W	–		b	B	B	B
5	W	W	w	w		B	B	B
6	W	W	W	W	W		B	B
7	W	W	W	W	W	W		B
8	W	W	W	W	W	W	W	

The only part of our proposed method that increases CPU time, due to the many combinations of F-F Objectives, is the Pareto ranking. CPU time increases in proportion to the number of combinations of F-F-Objectives as shown in Table III. The CPU time shown in Figure 5 appear to have a similar relationship, and the s's with the maximum/minimum CPU time in Figure 5 are also those with the maximum/minimum combinations of F-F-Objectives.

The biggest value of the maximum spread was found for $s = 8$, which corresponds to the case where the number of F-F-Objectives was equal to the total number of objectives. The sign test indicates that the maximum spread for $s = 7$ is significantly smaller than for when $s = 8$ ($p < 0.01$), and that it is likewise significantly smaller for $s = 5$ than it is for $s = 7$ ($p < 0.05$). No significance was found for maximum spread when $s < 5$. It is generally said that the diversity of solutions is narrow when maximum spread is small. Thus, with our method, the diversity of our solutions is narrower when s is small.

VI. CONCLUSIONS

We have confirmed the superiority of our proposed method, the F-F-Objective Optimization, to MOGA with regards to hypervolume and C Metric. Evaluated for maximum spread, the proposed method was inferior to MOGA but performed better than the average ranking method and WBGA with equal weights.

The proposed method has characteristics of both ordinal EMO ranking methods using a Pareto approach and the average ranking method or WBGA with equal weights. We have confirmed that characteristics of convergence and diversity are controlled by changing the number of F-F-Objectives, s.

In addition, the proposed method takes into account all objectives for each searching generation. It does not need to analyze which objectives can be reduced or require the user to make decisions to reduce them. As a consequence, computational cost and user load are lessened and no interactive system is required for the decision making. The ranking approach of the proposed method can be used not only for MOGA, as was done in our experiments in this paper, but may also be used in any Pareto ranking-based EMO and even with non-Pareto approaches.

In future research, we will extend this approach by evaluating it with other sizes of objectives and with many other benchmark tasks.

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