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Pei, Yan
Graduate School of Design, Kyushu University

Takagi, Hideyuki
Faculty of Design, Kyushu University

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Comparative Study on Fitness Landscape Approximation with Fourier Transform

Yan Pei

Graduate School of Design, Kyushu University
Fukuoka, Japan
peiyan@kyudai.jp

Hideyuki Takagi

Faculty of Design, Kyushu University
Fukuoka, Japan
<http://www.design.kyushu-u.ac.jp/~takagi>

Abstract—We propose to apply n dimensional discrete Fourier transform (DFT) to a fitness landscape, search an elite individual using obtained principal frequency component and accelerate evolutionary computation (EC) search. A comparative evaluation with our previous works is conducted using eight benchmark functions. The evaluation shows that our proposed approach can obtain the accurate fitness landscape than that with 1 dimensional DFT, and EC acceleration performance can be improved significantly. However, it needs more computational time in the process of conducting n dimensional DFT than that in 1 dimension. We also investigate the computational complexity of the two approaches and some related issues.

Keywords—evolutionary computation; Fourier transform; fitness landscape; approximation; acceleration

I. INTRODUCTION

Evolutionary computation (EC) acceleration is a promising study topic in EC community and several related acceleration studies were proposed [3]. They devise population initialization, fitness evaluation and selection, population reproduction, EC algorithm adaptation and additional local search in EC algorithm [7].

In EC iterative search process, learning the problem's structure, predicting the promising search region and conducting a local search have been realized as promising approaches to accelerating EC. Reference [6] approximates fitness landscape in original search space to obtain elite to enhance EC search, and Reference [2] extends Reference [6]'s works, which conducts this kinds of approximation in each projected one dimensional search space and synthesizes each dimensional elite into the next iterative search. From Reference [4], it concludes that projecting one dimensional search space for approximating fitness landscape approach can save the computational cost into 1/2-1/3 of original approach.

Fourier transform is a useful analysis tool to obtain the frequency information of a search space. By the obtained frequency, amplitude and phase information, the fitness landscape is easily approximated by a trigonometric function with principal frequency component. From the approximated fitness landscape, we can predict the promising search region and conduct a local search to accelerate EC search.

The objective of this paper is to investigate EC acceleration performance with approximated fitness landscape by a trigonometric function with the obtained principal frequency component by n -D DFT and to conduct a comparative evaluation with the approaches in References [2], [5], and [6]. From this empirical study, the computational complexity of the two approaches and some related issues are investigated and discussed.

We proposed to use DFT for approximating fitness landscape and accelerating EC search [5]. There are five steps for implementing this objective. They include re-sampling in the original search space, conducting 1 dimensional Fourier transform, filtering principal frequency component, conducting 1 dimensional inverse Fourier transform and obtaining elite into next generation search.

The main difference between this paper and our previous work [5] is to conduct n -D DFT to obtain the principal frequency component information, and we expect that elite obtained by n -D DFT is closer to the global optimum than that obtained by 1-D DFT. N -D DFT can obtain the accurate frequency information than that obtain by 1-D DFT, but needs more computational time. We need to balance the used computational time and the obtained solution quality for a concrete problem or real word application.

The remainder of this paper is structured as follows. An overview of Fourier transform of 1-D and n -D is given in Section II. Fundamentals of the principal frequency component obtained by n -D DFT and acceleration method design are summarized in Section III. In Section IV, we use differential evolution as an evaluation tool to analysis the proposed algorithms' performance with eight benchmark functions. In Section V, we discuss the proposed algorithms in detail and an outlook on open topics and potential opportunities are given. Finally, we present the further research direction and make a conclusion of the whole paper (Section VI).

II. DISCRETE FOURIER TRANSFORM

A. 1 Dimensional DFT

The principal of the fast discrete Fourier transform (1-D FFT) is to cut the long series into the shorter ones and to use the periodic and symmetric characteristics to reduce the calculation times of DFT [1]. There are two kinds of FFT; one is decimation in time algorithm (DIT) and the other is decimation in frequency algorithm (DIF).

$$X(k) = x(2r)W_N^{2rk} \pm W_N^k x(2r+1)W_N^{(2r+1)k} \quad (1)$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} [x(n) + (-1)^{(k)} x(n + \frac{N}{2})] W_N^n k \quad (2)$$

Equations (1) and (2) show the fundamental principal of the DIT and DIF, respectively. In Equation (1), algebraic $+$ and $-$ are taken when $k \in [0, \frac{N}{2} - 1]$ and when $k \in [\frac{N}{2}, N - 1]$, respectively. They can reduce the computational complex of DFT from $O(N^2)$ to $O(N \log N)$. In this paper, we use DIF as the main analysis tool to transfer original EC individual series into frequency space for obtaining frequency information of original search space (see Figure 1).

Table I

BENCHMARK FUNCTIONS USED IN EC EXPERIMENTAL EVALUATIONS, WHERE RANGE, D , AND C REFER TO THE RANGES OF THE PARAMETER VALUES, THE DIMENSION OF EACH FUNCTION, AND ITS CHARACTERISTICS, RESPECTIVELY. M, U, N, AND S REFER TO MULTIMODAL, UNIMODAL, NON-SEPARABLE AND SEPARABLE, RESPECTIVELY. n AND m MEAN SAMPLING NUMBER IN 1^{st} DIMENSION AND IN 2^{nd} DIMENSION.

No.	Name	Test function	Range	D	C	n	m	Base frequency #	Sampling #
F1	Sphere	$f(x) = \sum_{i=1}^n x_i^2$	[-5.12,5.12]	1vs.2	US	256	256	256	8
F2	Rosenbrock	$f(x) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2$	[-2.048,2.048]	1vs.2	UN	256	256	256	256
F3	DeJong-Step	$f(x) = \sum_{i=1}^n \lfloor x_i \rfloor$	[-5.12,5.12]	1vs.2	US	1024	1024	1024	8
F4	Quantic & Noise	$f(x) = \sum_{i=1}^n ix_i^4 + Gauss(0, 1)$	[-1.28,1.28]	1vs.2	US	256	256	256	256
F5	Shekel's Foxholes	$f(x) = [0.02 + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^2 (x_i - a_{ij})^6}]^{-1}$	[-65.536,65.536]	1vs.2	MS	256	256	256	8
F6	Rastrigin	$f(x) = (10n) + \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i))$	[-5.12,5.12]	1vs.2	MS	1024	1024	1024	8
F7	Schweffel 2.26	$f(x) = \sum_{i=1}^n (-x_i \sin(\sqrt{ x_i }))$	[-512,512]	1vs.2	MS	1024	1024	1024	8
F8	Griewank	$f(x) = 1 + \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos(\frac{x_i}{\sqrt{i}})$	[-512,512]	1vs.2	MN	256	256	256	256

B. n Dimensional DFT

N fast Fourier transform (n -D FFT) is considered as conducting one time 1-D FFT with M_i points when the other dimensional value is certain, and this kind of 1-D FFT conducts $\prod_{k=1, k \neq i}^n M_k$ times. M_i is the sampling point in the i -th dimension.

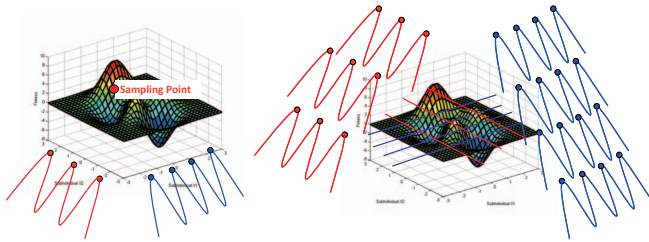
For a brief explanation, the 2-D FFT and inverse 2-D FFT are Equations (3) and (4) (see Figure 1).

$$X(k, l) = [\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n) W_M^{km} W_N^{ln}] R_{M,N}(k, l) \quad (3)$$

$$x(m, n) = \frac{1}{N} [\sum_{k=0}^{M-1} \sum_{l=0}^{N-1} X(k, l) W_M^{-km} W_N^{-ln}] R_{M,N}(m, n) \quad (4)$$

$$R_{M,N}(m, n) = \begin{cases} 1 & 0 \leq m \leq M-1, 0 \leq n \leq N-1, \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

M and N are the range of original 2 dimensional range, m and n are the respective sampling data in each dimension, l and k are the FFT base frequency number in each 1 dimensional FFT, respectively. The computational complex of 2 dimensional FFT is $O((m+n)N \log N)$, i.e. $(m+n)$ times of 1 dimensional FFT.

Figure 1. 1-D and n -D DFT.

III. APPROXIMATING FITNESS LANDSCAPE BY FOURIER TRANSFORM TO ACCELERATE EVOLUTIONARY SEARCH

A. Frequency Information Obtained by n -D DFT

We use n -D DFT to obtain the frequency characteristics of the original search space (Equation (3)). The sampling

method, 1-D-GLB [5], is used in our evaluation. The number of concrete sampling points, i.e. n and m in Equation (3), sampling data number in each one time DFT and base frequency number setting are shown in Table I.

B. Fitness Landscape Approximation using Frequency Information

Our original proposal is to use n -D DFT to obtain frequency characteristics of a fitness landscape, and to resolve the complexity of EC fitness landscape using simple principal frequency component from frequency landscape characteristic. After obtaining the frequency information of a fitness landscape, we use it to approximate the fitness landscape to accelerate EC convergence.

Accordance with the Stone-Weierstrass theorem, that is trigonometric can universal approximate any continuous function with the periodical of 2π , so we use the Equation (6) as the regression model to approximate original EC search space.

$$EC(x) = \sum_{i=0}^N \cos(2\pi\omega_i X + B_i) \quad (6)$$

Although the original fitness landscape is not always with the periodical of 2π , we can universally approximate a local landscape with any numbers of base function on a compact set to arbitrary accuracy in local. However, in practical point of view, too many base functions mean higher computational complexity, we just use one base function to approximate the original fitness landscape in this paper.

C. Evolution Control Approaches

After obtaining a concrete regression model with a principal frequency component, we obtain the local or global landscape characteristics of EC search space in 1-D dimension and n -D dimension. From this landscape characteristics, the more EC search information can be obtained than that little number of individuals supporting.

To accelerate EC, we analyze an EC fitness landscape with this regression model and direct EC search or make new elite into the next generation. Peak point must be locate around the points of $k(1/\omega) + b$ and $k(1/2\omega) + b$ for the maximum problem and the minimum problem, respectively. So if we can find out the respect individuals with this characteristic in original search space and put it into the next generation evolution, EC can be accelerated.

IV. EXPERIMENTAL EVALUATIONS

A. Evaluation Design

We use the DeJong five standard functions (F1 - F5), Ras-trigin function (F6), Schwefel function (F7) and Griewank function (F8) as benchmark functions to compare the acceleration performance of 1-D DFT and 2-D DFT. Their dimensions and search ranges of all parameters are described in Table I. All these function optimization tasks are posed as minimization problems with the optimal solution being the point with the lowest value. Their landscapes have a variety of characteristics. They include both continuous and discontinuous, convex and non-convex, unimodal and multi-modal, variable sparable and non-sparable. For comparing the performance with the previous acceleration methods, we use the EC acceleration approaches of References [2], [5] and [6] to test those benchmark functions and make a comparative evaluation.

B. Optimization Method and Sign Tests

Differential evolutionary (DE/best/1/bin) is used as an optimization method to evaluate the proposed approaches. We test each benchmark function up to 100 generations with 50 trial runs. Figure 2 shows the average convergence curves of the best fitness values of 50 trial runs for all eight benchmark functions, and Figures 3 and 4 show their sign test results at each generation. Abbreviations used in Figures 2, 3 and 4 are given in Table II.

Table II
ABBREVIATIONS USED IN FIGURES 2, 3 AND 4.

abbreviations	DE which fitness landscape is regressed by
DE-N	no any method (standard DE)
DE-F1D, DE-F2D	1-D or 2-D FFT approach
DE-LR	a two-degree Lagrange interpolation
DE-LS	a line power function least squares approximation
DE-TB	a two-degree power function least squares approximation in original search space with the best sampling [2], [4]

C. Experiment Results

Accordance with those test results, we can conclude that:

- (1) Our proposed n -D DFT approximation approach can accelerate all eight benchmark functions, except F3, F4, and F5.
- (2) Our proposed n -D DFT approximation approach has the better acceleration performance than that of 1-D DFT approximation approach, except F3, F4, and F5.
- (3) N -D DFT approximation approach can accelerate F7's search, but 1-D DFT approximation approach cannot.

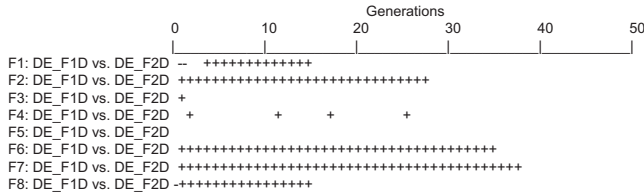


Figure 3. Sign test results for 50 trial runs of (DE-F1D vs. DE-F2D) per generation. The + and - marks mean that A is significantly better and poorer than B, respectively, for "A vs. B" ($p < 0.05$).

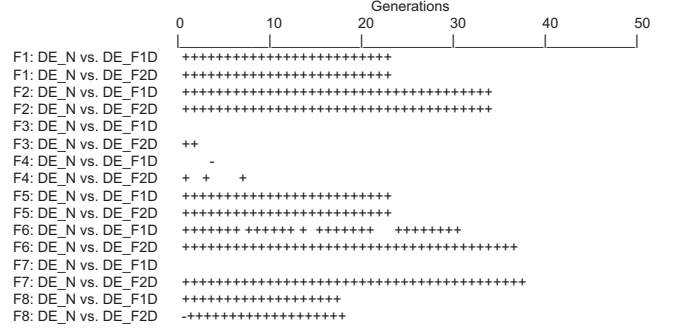


Figure 4. Sign test results for 50 trial runs of (DE-N vs. DE-F1D), (DE-N vs. DE-F2D) per generation. The + and - marks' meaning as in Figure 3.

V. DISCUSSION

A. Approach Performance

Our proposed n -D DFT approach can obtain the better acceleration performance than 1-D DFT approach from the evaluation results, except F3, F4, and F5, which the global optima locates in bound. N -D DFT approach can obtain more frequency information and transfer to relative full-scale fitness landscape thanks to conducting 1- n DFT in each dimensional several time. However, the 1-D DFT approach just conduct 1-D DFT with certain point one time in each dimension. This is the reason why the acceleration performance by n -D DFT approach better than 1-D DFT approach.

B. Fourier Transform

Although n -D DFT approximation approach can obtain relatively enough frequency information than that obtained by 1-D DFT approximation approach, its performance depends on the location of sampling points for n -D DFT conducts. If each location is around the global optimum, n -D DFT approximation approach can obtain the better acceleration performance, but when they are far from the global optima, its performance may not be great. The acceleration performance obtained by n -D DFT approximation approach also depends on base frequency setting and sampling data number.

C. Computational Complexity

Theoretically, the computational complexities of n -D FFT and 1-D FFT are $(\prod m_i)N \log N$ and $nN \log N$, respectively. It means that n -D FFT computational complexity is $(\prod m_i)/n$ times of that of 1-D FFT; n -D FFT approximation approach is costly approach. For practical applications, if the 1-D FFT approximation approach has the relatively acceptable acceleration performance, which similar to that of n -D FFT approximation approach, we should consider to use 1-D FFT approximation approach, of course, it depends on benchmark tasks and real world applications.

VI. CONCLUSION AND FUTURE WORK

In this paper, we introduced n -D DFT to approximate fitness landscape to accelerate EC search and compared with that of by 1-D DFT. It has the better acceleration performance. However, this approach needs more computational cost than that of 1-D DFT. From the practical points of view,

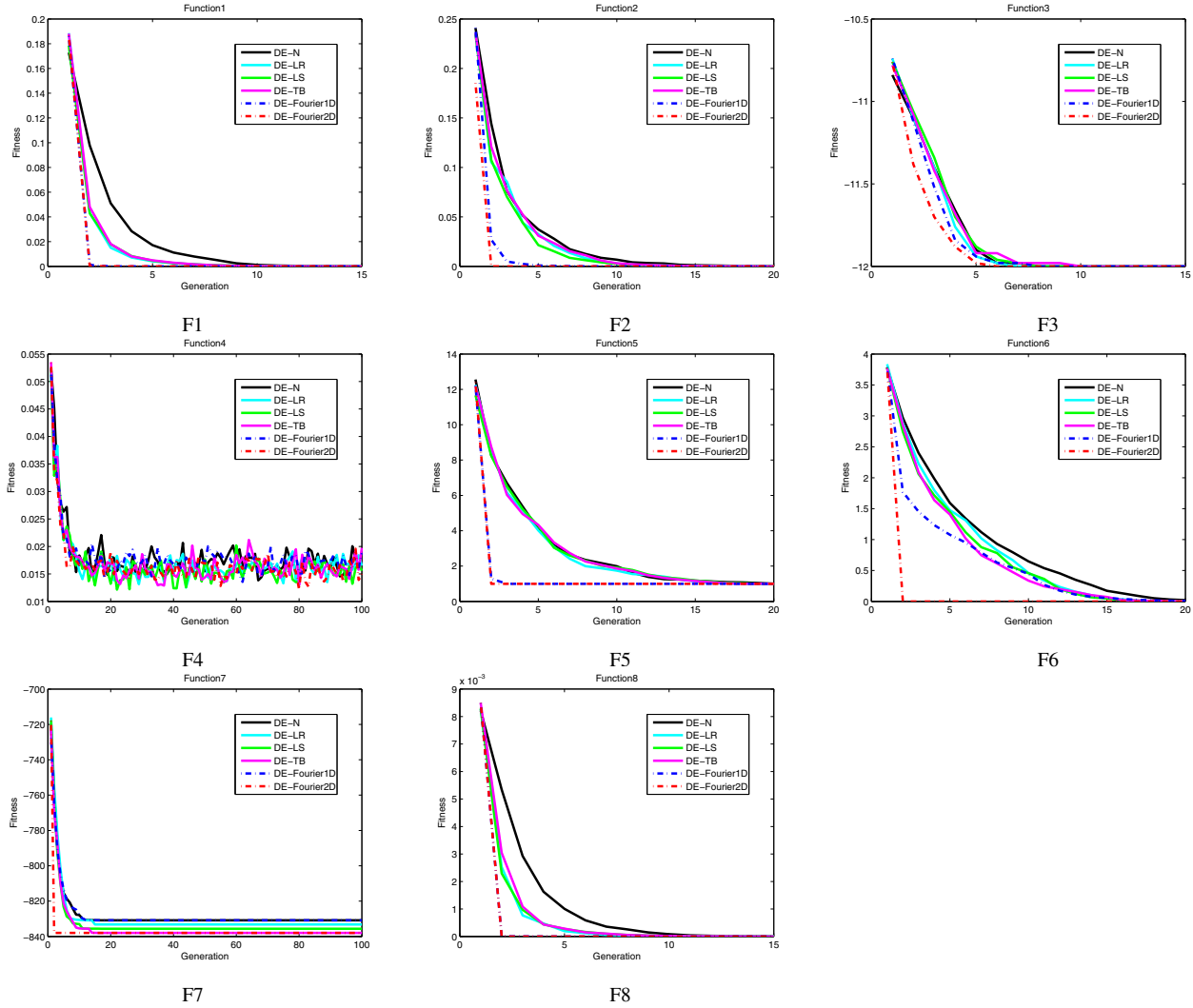


Figure 2. Average convergence curves of 50 trial runs for F1-F8.

we should consider the balance between the computational cost and solution quality when applying this approach to the real world applications.

The locations of sampling points, base frequency setting and sampling data number are the crucial factors for obtaining the better acceleration performance by n -D DFT. There are the best configuration between those factors and approximation or EC acceleration performance for a specific landscape or a real world applications. We will study this topic in the future.

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