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Influence of Root Reinforcement of Forest Species on the Slope Stability of Sierra Madre Oriental, Mexico

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Soil erosion and sediment related disasters are one of the most serious environmental problem in Mexico, mainly product of deforestation practices, land use changes and hydro–meteorological events like heavy rainfalls and hurricanes. The application of vegetative measures to restore affected sites should be a priority on mountainous countries like Mexico. The objectives of this study are to estimate the soil reinforcement by roots and to analyze the increment of Factor of safety (Fs) of a studied slope on Sierra Madre Oriental, Mexico from two different scenarios: rainfall and no–rainfall. The estimation of the additional soil reinforcement provided by roots (ΔS) of the four species analyzed ranged from 0.01 to 0.22 kN/m^2 (*Quercus rysopylla*), from 0.006 to 0.08 kN/m^2 (*Pinus pseudostrobus*), from 0.01 to 0.46 kN/m^2 (*Acacia berlandieri*) and from 0.01 to 0.05 kN/m^2 (*Ligustrum lucidum*). The results of the finite element method (FEM) analysis in the different scenarios indicate that the slope is in a stable condition. However, the presence of roots on the slope have a bigger positive effect during rainfall conditions, the Fs increment is especially notable in *Acacia berlendieri* (6.36%) and *Quercus rysopylla* (3.63%). In other hand, in no–rainfall conditions the F$_s$ increment for the same species is *Acacia berlendieri* (4%) and Quercus rysopylla (1.71%). The species that contributed the best to the stability of the studied slope was *Acacia berlendieri* followed by *Quercus rysopylla*, Pinus pseudostrobus and *Ligustrum lucidum*. The F$_s$ improvement due the roots of the species living in Sierra Madre Oriental, Mexico is a very important contribution to understand and to clarify the forest species potential contribution to slope stabilization in the study area.

Key words: Slope stability, Tree roots, Mexico, Sierra Madre

INTRODUCTION

Soil erosion and sediment related disasters are one of the most serious environmental problems in Mexico, where, 24% of the total soil is highly degraded or affected by sediment related disasters as result of deforestation or land use changes. Previous studies have demonstrated that the absence of vegetation increases the erosion in the urbanized forest area. In recent years, meteorological events such as heavy rainfall and hurricanes had magnified the problems in the Sierra Madre Oriental forest slopes. The application of vegetative measures to restore the affected sites could be significant as are ecologically friendly, low in costs and easy in techniques.

The beneficial influence of vegetation on slope stabilization includes modifications of soil moisture, buttressing and arching of roots, surcharge of vegetation biomass above the ground and soil reinforcement by plant roots (Gray and Sotir, 1996). Soil reinforcement is a contributory factor in slope stabilization and is widely investigated and used for site restorations (Waldron, 1977; Wu *et al.*, 1979, Sanchez–Castillo *et al.*, 2014; Zhang *et al.*, 2015).

Roots of forest species increases the shear strength of soil mainly by transferring shear stresses that develop in the soil into tensile resistance, when the shear of the slope makes the fiber deform and causes elongation, which provides enough interface friction and confining stress to lock fiber on place and prevent slippage or pull out. The use of the perpendicular model for the estimation of root reinforcement is a very useful estimation and is supported by laboratory and statistical studied with randomly distributed fibers (Gray and Ohashi, 1983; Maher and Gray, 1990).

This study aims to estimate the soil reinforcement by roots and to analyze the increment of safety factor in forest slope of Chipinque National Park of Sierra Madre Oriental, Mexico.

MATERIALS AND METHODS

Study area

The study was carried out in the slopes of Chipinque National Park in Monterrey, Nuevo Leon, Mexico. Chipinque National Park is located in Cumbres of Monterrey National Park in the northern portion of Sierra Madre Oriental (Fig. 1). This National Park was established in 1939 to protect the native flora and fauna against the spreading urbanization of Monterrey city. The climate in the park changes drastically depending on the elevation and the location; Following Köppen classi-

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fication the following climates are observed in the park. Elevation 600–1,000 m; North tip of the park has a (BWh) desert climate with an average annual temperature of 23°C and an annual precipitation of 217 mm. Elevation 1,000–2,000 m: North tip and the southwest corner of the park has a (BSk) Steppe Climate with an average annual temperature 21°C. Elevation 2,000–3,000 m: Western side of the park has a (Cwb) Oceanic climate with an average annual temperature of 14°C and an annual precipitation of 680 mm. The general vegetation consists of Mixed Pine–Oak forest with species like: *Pinus cembroides*, *Pinus hartwegii*, *Pinus pinceana*, *Pinus pseudostrobus*, *Pinus rudis*, *Pinus teocote*. and Oaks like: *Quercus fusiformis*, *Quercus greggii*, *Quercus hipoxlanta*, *Quercus laeta*, *Quercus rysophylla*. And also submontane shrubs in lower parts with species like: *Acacia berlindieri*, *Acacia rigidula*, *Leucophillum frutescens* and *Cordia boissieri* (Secretariat of Sustainable Development of Nuevo Leon state, 2013).
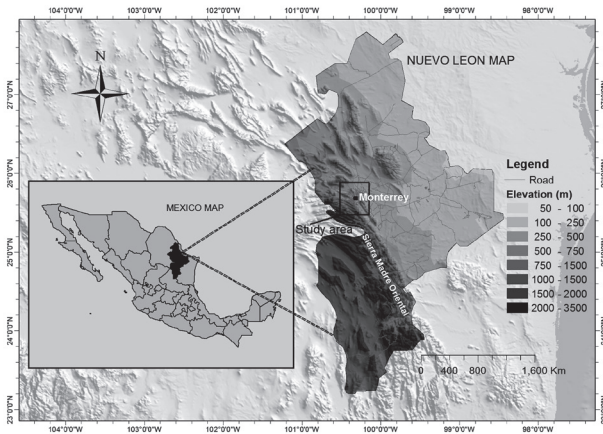

Fig. 1. Study area in Sierra Madre Oriental, Nuevo Leon, Mexico.

Methods

Soil–root reinforcement estimation

Four forest species were selected based on their native characteristics and/or widespread distribution in the slopes of Chipinque National Park in Sierra Madre Oriental, Nuevo Leon: *Quercus rysophylla*, *Pinus pseudostrobus*, *Acacia berlandieri* and *Ligustrum lucidum*. Specimens with root system exposed were selected and root samples were collected, carefully labeled and transported to laboratory for the tensile strength tests. On field, pictures were taken to calculate root area ratio that is defined as the fraction of the soil cross–sectional area occupied by roots per unit area (Gray and Leiser, 1982). The method of root counting by image analysis described by Vogt and Persson (1991) was used to calculate root area ratio, which consists on the mapping and counting of the exposed roots on a trench from images in order to calculate the cross–sectional area and recreate the root system, this method emulates the traditional profile wall method proposed by Bohm (1979).

The tensile strength of an individual root was measured using the universal testing machine Shimazu type SLFL–100 kN following the next procedure: root samples were carefully positioned and were clamped to experimental devices in both extremities. The diameters were measured using a digital caliper Mitutoyo in three different points and the mean diameter was calculated to assign a representative value corresponding to the breaking point of each sample. Root samples were pulled at a constant speed of 10 mm/min. Tensile strength values were measured by the machine load cell and recorded with a data logger. Data was visualized and outputted using the material testing operation software Trapezium X.

The influence of roots on soil reinforcement can be expressed as cohesion term (O'Loughlin, 1974; Wu, 1976; Waldron, 1977) in the Morh–Coulomb failure criteria were the soil–root composite shear strength is calculated as follows:

$$S_r = c' + (\sigma - u)tan\phi' + \Delta S \qquad (1)$$

where c' is the effective cohesion of the soil, σ is the normal stress due the weight of the water and soil of sliding mass, u is the soil pore–water pressure, ϕ' is the effective friction angle of the soil and ΔS is the apparent cohesion provided by the presence of roots.

The theoretical model developed by Wu (1976), Waldron (1977) and Wu *et al.* (1979) was used to estimate the shear strength increase due the presence of roots. This model assumes that roots are flexible, elastic and perpendicular oriented to the slip surface, when the soil layer is moving. The mobilization of tensile resistance of the fibres can be translated into tangential and normal components. Assuming that soil friction angle is not affected (O'Loughlin, 1974) the additional cohesion provided by roots can be calculated as:

$$\Delta S = t_r (sin\beta + cos\beta tan\phi') \qquad (2)$$

where t_r is the average mobilized tensile strength of roots per unit area of the soil and β is the angle of root distortion in the shear zone. Sensitivity analyzes show that the values of $(sin\beta + cos\beta tan\phi')$ can be approximated as 1.2 for $30° < \phi' < 40°$ and $48° < \beta < 72°$ (Wu *et al.*, 1979; Wu, 1995). Thus, equation can be simplified as:

$$\Delta S = 1.2t_r \qquad (3)$$

and t_r can be calculated as:

$$t_r = \sum_{i=0}^{n} T_{ri} (A_{ri}|A) \qquad (4)$$

where T_{ri} is the tensile strength of an individual root (i) and $(A_{ri}|A)$ is the root area ratio or proportion of root cross sectional area to soil cross sectional area A.

– Slope stability analysis

The slope fails because the shear strength of its material is not sufficient to endure the shear stresses to

which is being subjected from its surface. The factor of safety against the slope failure can be calculated as:

$$Fs = \frac{\tau_f}{\tau} \qquad (5)$$

where τ_f is the shear strength of the slope material, which is calculated through Coulomb's criterion as:

$$\tau = c + \sigma_n\, tan\phi \qquad (6)$$

and τ is the shear stress on the slip surface. It can be calculated as:

$$\tau_f = c_f + \sigma_n\, tan\phi_f \qquad (7)$$

where the factors shear strength parameters c_f and ϕ_f are:

$$c_f = \frac{c}{SRF} \qquad (8)$$

$$\phi_f = tan^{-1}\left(\frac{tan\,\phi}{SRF}\right) \qquad (9)$$

where *SRF* is strength reduction factor. This method has been referred to as the 'shear strength reduction method' (SSR). It is a simple approach that involves a systematic search for a stress reduction factor (*SRF*) or factor of safety value that brings a slope failure limits (Hammah *et al.*, 2004). To achieve the correct *SRF*, it is essential to trace the value of Fs that will just cause the slope to fail. In the finite element method, a continuum is divided into "elements" that are separated by imaginary lines or surfaces and are assumed to be interconnected only at a finite number of nodal points situated on the boundaries (Matsui and San, 1992; Dawson *et al.*, 1999; Zienkiewicz and Taylor, 2000; Hammah *et al.*, 2004). In this study slope stability analysis is performed using the finite element method (FEM) with strength reduction technique in 2 dimensions by FEM computer code developed by Geotechnical Laboratory of Gunma University GUSLOPE Version 1.00. The FEM analysis was conducted adding the coefficient of additional cohesion provided by roots (ΔS) to the soil properties of the slope, in

two different scenarios: with rainfall and without rainfall. Rainfall information during hurricane "Emily" in 2005 was used (Fig. 2). The rainfall condition presented during hurricane Emily was used because follows the pattern observed in previous disastrous events: High intensity of rainfall during a very short duration (less than 24 hrs), similar rainfall patterns have been described as triggering factors of sediment related disasters in previous studies (Larsen and Simon, 1993; Crosta and Frattini, 2001; Guzzeti *et al.*, 2008; Saito *et al.*, 2014; Chen, 2016)

RESULTS AND DISCUSSION

Soil reinforcement by roots

The additional soil reinforcement provided by roots (ΔS) for the four species are analyzed ranged from 0.01 to 0.22 kN/m² (*Quercus rysophylla*), from 0.006 to 0.08 kN/m² (*Pinus pseudostrobus*), from 0.01 to 0.46 kN/m² (*Acacia berlandieri*) and from 0.01 to 0.05 kN/m² (*Ligustrum lucidum*). Distribution of minimum, maximum and average values is showed in Fig. 3.
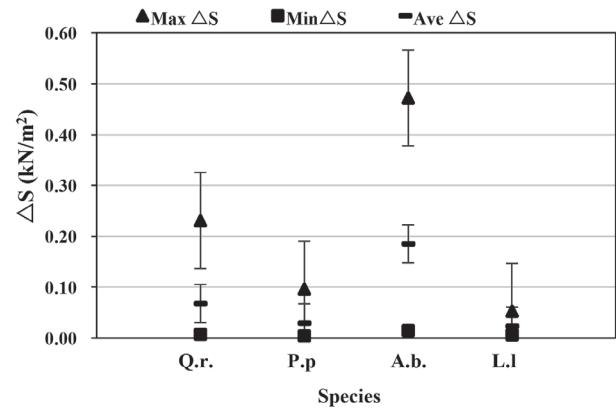
Fig. 3. The additional soil reinforcement provided by roots (ΔS). (Q.r.= *Quercus rysophylla*, P.p.= *Pinus pseudostrobus*, A.b.= *Acacia berlandieri*, L.l.= *Ligustrum lucidum*).
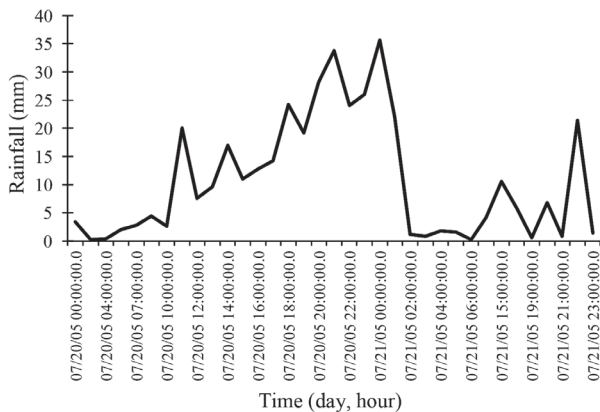
The coefficient of additional soil reinforcement provided by roots depends on quality of root material expressed as root area ratio (RAR) at the shear plane, that is why comparison with the results of other studies is difficult as the results typically reported, which have a big spatial variability (Burroughs and Thomas, 1977; O'Loughlin and Watson, 1979; Ziemer, 1981; Coppin and Richards, 1990; Abernethy and Rutherfurd, 2001). Root area ratio declines sharply with depth, because the roots samples tested in this study correspond to small diameters far from the trunk, due to test machine limitations, values are quite low comparing other studies (Morgan and Rickson, 1995; Abernethy and Rutherfurd, 2001; Tosi, 2007). Besides root area ratio and subsequently root reinforcement is influenced by species genetic, conditions and soil and slope characteristics (Bischetti *et al.*, 2005; Stokes *et al.*, 2002) which limit results to a regional area.

Fig. 2. Rainfall condition during hurricane "Emily" in 2005.

Slope stability analysis

The results of FEM analysis are described as the factor of safety (Fs), when the factor of safety is 1.0 or bigger means the slope is in a stable condition, but when the factor of safety is minor than 1.0 indicates that slope is in an unstable condition. A slope model was delimited (Fig. 4) with soil properties showed in Table 1, the maximum values of coefficient of additional cohesion provided by roots (ΔS) were added as: *Quercus rysophylla*: 0.22 kN/m²; *Pinus* pseudostrobus: 0.08 kN/m²; *Acacia berlandieri*; 0.46 kN/m²; and *Ligustrum lucidum*: 0.05 kN/m².

The factor of safety in the two scenarios without the presence of roots resulted as Fs = 1.1 during rainfall conditions and Fs = 1.75 without rainfall conditions. The slope under normal conditions is stable in both scenarios. However, the objective of this study is to find out the effect of roots of forest species on the stability of slopes, for this purpose the coefficient of additional cohesion

provided by roots (ΔS) was added to the soil properties of the slope model. The increases of Fs without rainfall was as follows: from Fs=1.75 to Fs=1.78 (*Quercus rysophylla*); Fs=1.76 (*Pinus pseudostrobus*); Fs=1.82 (*Acacia berlandieri*); Fs=1.75 (*Ligustrum lucidum*). During rainfall conditions from *Fs*=1.1 to *Fs*=1.14 (*Quercus rysophylla*); Fs=1.12 (*Pinus pseudostrobus*): Fs=1.17 (*Acacia berlandieri*); Fs=1.11. (*Ligustrum lucidum*) (Table 2). Clearly, the Fs of the studied slopes is incremented due the presence of roots in some percentage, in agreement with previous studies (Abernethy and Rutherfurd, 2001; Simon and Collinson 2002; Pollen and Simon, 2005; Tosi, 2007; Docker and Hubble 2008).

CONCLUSIONS

The results of additional soil reinforcement provided by roots (ΔS) for the four species analyzed ranged from
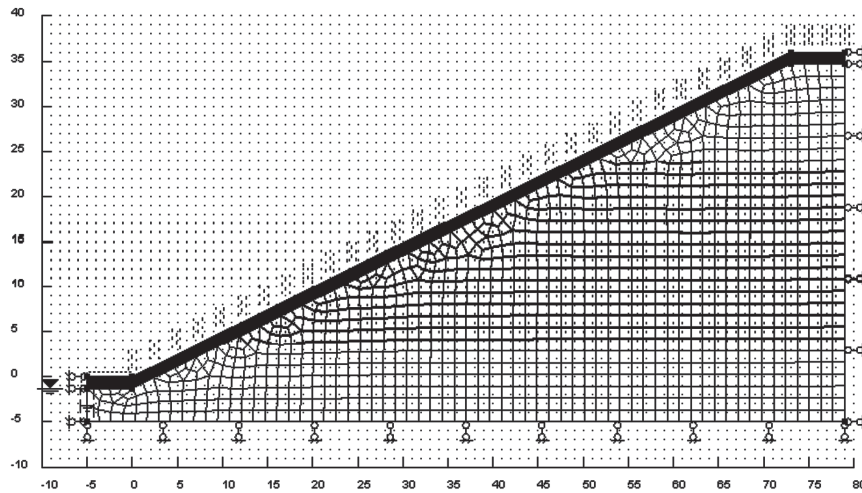

Fig. 4. Slope model used for the FEM analysis.

Table 1. Soil properties of the studied slope

Layers/parameters	γ (kN/m³)	C (kN/m²)	Φ (degree)	E (kN/m²)	υ	k (cm/s)
Surface	18	3	35	20000	0.3	$8 \times 10–4$
Bedrock	26	1000	40	400000	0.22	$2.5 \times 10–10$

Table 2. FEM results of analyzed slope in the different scenarios

Species	F_s (R)	$\rightarrow$	$F_s \Delta S$ (R)	F_s (NR)	$\rightarrow$	$F_s \Delta S$ (NR)
Quercus rysophylla	1.10	$\rightarrow$	1.14	1.75	$\rightarrow$	1.78
Pinus pseudostrobus	1.10	$\rightarrow$	1.12	1.75	$\rightarrow$	1.76
Acacia berlandieri	1.10	$\rightarrow$	1.17	1.75	$\rightarrow$	1.82
Ligustrum lucidum	1.10	$\rightarrow$	1.11	1.75	$\rightarrow$	1.75

F_s (R) = Factor of safety without roots presence in rainfall conditions; $F_s \Delta S$ (R) = Factor of safety with roots presence in rainfall conditions; F_s (NR) = Factor of safety without roots presence in dry conditions; $F_s \Delta S$ (NR) = Factor of safety with roots presence in dry conditions.

0.01 to 0.22 kN/m² (*Quercus rysophylla*), from 0.006 to 0.08 kN/m² (*Pinus pseudostrobus*), from 0.01 to 0.46 kN/m² *(Acacia berlandieri)* and from 0.01 to 0.05 kN/m² (*Ligustrum lucidum*). The results of the finite element method (FEM) analysis in the different scenarios indicate that the slope is in a stable condition. However, the presence of roots on the slope have a bigger positive effect during rainfall conditions, the F_s increment is especially notable in *Acacia berlendieri* (6.36%) and *Quercus rysophylla* (3.63%). In other hand, in no–rainfall conditions the Fs increment for the same species is *Acacia berlendieri* (4%) and *Quercus rysophylla* (1.71%). The species that contributed the best to the stability of the studied slope was *Acacia berlendieri* followed by *Quercus rysophylla*, *Pinus pseudostrobus* and *Ligustrum lucidum*. The root area ratio in the applied model was observed as the factor that influences the coefficient of additional cohesion provided by roots on the slope. All species considered for this study contributed to the stability of slope in a positive way. The F_s improvement due the roots of the species living in Chipinque National Park of Sierra Madre Oriental, Nuevo Leon is a very important information to understand the potential contribution to slope stabilization of each forest species in the study area.

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