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<https://doi.org/10.15017/17888>

出版情報 : 九州大学大学院システム情報科学紀要. 15 (1), pp.13-18, 2010-03-26. 九州大学大学院システム情報科学研究所

バージョン :

権利関係 :

Characterization of All Robust PID Controllers for Belt Conveyor System via Corrected Polynomial Stabilization

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(Received Desember 11, 2009)

Abstract: Polynomial stabilization method has important feature in PID tuning. It computationally characterizes the entire set of admissible PID gains for various control system configurations. This paper shows a correction is needed in order to find all robust PID region controllers that satisfy a given robust performance. We also provides selection procedure for searching the best PID gains controllers in the obtained PID gains region. Then, the corrected polynomial stabilization algorithm is applied on a short DC servo-driven belt conveyor system. Here, we suggest a simple model for the system. The admissible PID gains are showed both in 2D plot at specified value of k_p , and in 3D plot for various values of k_p . Hence, this paper provides a viable and practical means for modeling and robust PID tuning for a short DC servo-driven belt conveyor system.

Keywords: PID tuning, Robust performance, Short DC servo-driven belt conveyor system, Modeling, Selection procedure, Correction of polynomial stabilization algorithm.

1. Introduction

The majority of control systems in this world are operated by Proportional-Integral-Derivative (PID) controllers. Meanwhile, belt conveyor systems have been used in many industrial applications, especially in manufacture industries for transporting material. Because of these reasons, we are interested on doing research in modeling and PID controllers synthesis for belt conveyor system, by a simple method and computationally efficient so that it can be applied in real applications.

The studies of belt conveyor system can be found in the following works. The paper¹⁾ derived the dynamical equations of a double conveyor system. They considered the belt displacement as a simple linear proportion to the motor angle. More complicated model was described in 2). The belt was first divided into N section and each them was modeled as a spring-mass-damper system. All spring-mass-damper parameters are assumed to be constants. In this paper, we consider a model for DC short servo-driven belt conveyor system.

We will use the Polynomial Stabilization (PS) method to find PID region that meet our control problem setup. This method, based on generalized Hermite-Biehler theorem^{3),4)}, can provide computational characterization of all admissible PID gains

controller for various control problems. PS algorithm can cover the problem of stabilizing nominal PID controller in continuous time domain, discrete time domain, system with time delay⁵⁾; as well as finding all robust PID controllers that satisfy a given robust performance⁶⁾. Such a characterization for all admissible PID gains controller involves the solution of sets linear programming problem.

The contributions of this paper are stated in the follows. First, we make correction on the PS algorithm for solving a given robust performance in 6). This correction is based on the empirical observation and tests within the admissible PID gains region, created by the original and corrected PS algorithm. We provides the selection procedure to find the best PID controller from the admissible PID controllers. We will apply the corrected PS algorithm for a short DC servo-driven belt conveyor system. We suggest a simple model for the system. We will also model the uncertainties with conditions that we may meet in practical application.

This paper is organized as follows. In Sec. 2, we will show the control objectives and its solution approach. The correction and selection procedure of PS method are given in Sec. 3. The nominal model of belt conveyor system and its uncertainty model is described in Sec. 4. Synthesis PID gains controller for the belt conveyor system by using the corrected PS method is given in Sec. 5. Finally, conclusion is given in Sec. 6.

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2. Objectives and Solution Approach

Consider Single-Input Single-Output (SISO) feedback control system in **Fig. 1**. Here, r is reference input, y is output, and d is energy-bounded disturbance. $G(s)$ is nominal plant model that we want to control. The plant model is in form of $G(s) = N(s)/D(s)$, where $N(s)$ and $D(s)$ are coprime polynomial. $\Delta(s)$ is any stable and proper transfer function with $\|\Delta\|_\infty < 1$. $W_1(s) = N_1(s)/D_1(s)$ and $W_2(s) = N_2(s)/D_2(s)$ are weighting functions for describing frequency-domain characteristics of the performance and model uncertainty of the system, respectively. Moreover, $D_1(s)$ and $D_2(s)$ are Hurwitz polynomial.

The controller $C(s)$ is PID controller, formalize as

$$C(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}.$$

Then, the complementary sensitivity function ($T(s)$) and the sensitivity function ($S(s)$) are,

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}, \quad S(s) = \frac{1}{1 + C(s)G(s)}.$$

Define the characteristic equation of nominal feedback system as

$$\alpha(s, k_p, k_i, k_d) \triangleq sD(s) + (k_d s^2 + k_p s + k_i)N(s).$$

The control objectives are:

1. Find all stabilizing PID controller for nominal feedback system.
2. For the system with multiplicative uncertainty, we want the plant satisfy the robust performance specification, that is

$$\| |W_1(s)S(s)| + |W_2(s)T(s)| \|_\infty \leq 1. \quad (1)$$

The solution approach for PID stabilization of nominal feedback system is equivalent with placing all the closed loop characteristic polynomial poles to

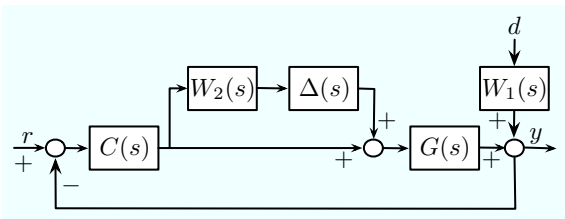


Fig. 1 The feedback control system configuration.

the open left half plane of s plane. In other word, the following requirement has to be fulfilled.

$$\alpha(s, k_p, k_i, k_d) \quad (2)$$

is Hurwitz.

In order to solve robust performance specification in Eq. (1), the paper⁶⁾ suggests the following lemma to convert Eq. (1) into PS.

Lemma 1 Let

$$\frac{A(s)}{B(s)} = \frac{a_0 + a_1 s + \dots + a_x s^x}{b_0 + b_1 s + \dots + b_x s^x}$$

and

$$\frac{E(s)}{F(s)} = \frac{e_0 + e_1 s + \dots + e_y s^y}{f_0 + f_1 s + \dots + f_y s^y}$$

be stable and proper rational functions with $b_x \neq 0$ and $f_y \neq 0$. Then

$$\left\| \left| \frac{A(s)}{B(s)} \right| + \left| \frac{E(s)}{F(s)} \right| \right\|_\infty \leq 1,$$

if and only if

- $B(s)F(s) + e^{j\theta} A(s)F(s) + e^{j\phi} E(s)B(s)$ is Hurwitz for all θ and $\phi \in [0, 2\pi)$.
- $|a_x/b_x| + |e_y/f_y| \leq 1$.

■

If we fit the robust performance specification into the result of **Lemma 1**, the conversion from robust performance condition into PS, can be written as

$$\begin{aligned} \beta(s, k_p, k_i, k_d, \theta, \phi) \triangleq & sD_1(s)D_2(s)D(s) \\ & + e^{j\theta} sN_1(s)D_2(s)D(s) \\ & + (k_d s^2 + k_p s + k_i)[D_1(s)D_2(s)N(s) \\ & + e^{j\phi} D_1(s)N_2(s)N(s)] \end{aligned} \quad (3)$$

is Hurwitz for all θ and $\phi \in [0, 2\pi)$, and

$$|W_1(\infty)S(\infty)| + |W_2(\infty)T(\infty)| \leq 1. \quad (4)$$

The solution of the control objectives on PID tuning, is the combination of Eq. (2), (3), and (4). Equivalently, we have to solve the following condition (a), (b), and (c).

- (a) $\alpha(s, k_p, k_i, k_d)$ is Hurwitz.
- (b) $\beta(s, k_p, k_i, k_d)$ is Hurwitz for all θ and $\phi \in [0, 2\pi)$.
- (c) $|W_1(\infty)S(\infty)| + |W_2(\infty)T(\infty)| \leq 1$.

Condition (c) is easy to solve while for condition (a) and (b), we have to stabilize the polynomials using to PS which are described in 5) and 6).

3. Correction and Selection Procedure

The **Lemma 1** is just mathematical tool to convert infinity norm expression into polynomial form. Hence, **Lemma 1** does not preserve the properties within the robust performance. In other word, we can not just apply PS to solve robust performance criterion in nominal model only, and expect that the obtained PID gains controller will hold robust performance specification for every designed perturbed systems. Therefore, in order to find the PID gain controllers that satisfy Eq. (1) via PS, we have to solve the condition (b) and (c) for nominal and its perturbations plant. Then, we seek for intersection of PID gains regions which is created by PS algorithm for nominal parameter and its perturbation. This intersection region is solution region that satisfy robust performance criterion, given in Eq. (1). This statement has been validated through frequency test on Eq. (1), for the controller regions that are obtained through the original and the corrected PS algorithm.

PS algorithm provides all robust PID controllers for a system. Now there is a question on how to choose the best PID parameter from those admissible robust PID controllers. The best robust PID controller is the PID parameter that gives minimum value of the infinity norm of robust performance in Eq. (1). For a simple bounded polygon region of admissible PID controller, the following results is found from empirical observation. In the boundary of admissible region, the infinity norm in Eq. (1) is equal one. This norm will be reduced as we move away from the boundary of region. For such kind of controller region shape, the farthest point from the boundary is centroid of polygon. Therefore, in order to find the best PID controller, we have to find the largest admissible region at a specific value of k_p , and find the centroid in that region. This point will give us the lowest infinity norm in Eq. (1).

4. Belt Conveyor System Model

In this section, we develop mathematical models of a short DC servo-driven belt conveyor system. First, the nominal model of the belt conveyor system is presented in Sec. 4.1. Then, the procedure of finding the weighting function W_2 for describing uncertainties of the belt conveyor model, is given in Sec 4.2. We will use those models to apply the corrected PS method that solve the control objectives in Sec. 2.

Table 1 Parameters of the belt conveyor system.

Parameter	Value	Unit
Rotor inertia (J)	1.2×10^{-6}	$\text{Kg} \cdot \text{m}^2$
Torque constant (K_m)	13.3×10^{-1}	$\text{N} \cdot \text{m}/\text{A}$
Back emf. constant (K_b)	13.3×10^{-1}	Volt/rad
Resistance (R)	2.17	Ω
Inductance (L)	1.17×10^{-2}	H
Friction torque (B)	2.5×10^{-3}	$\text{N} \cdot \text{m} \cdot \text{s}$
Motor radius (r)	17.4	mm
Belt elastic modulus (k)	4.5×10^{-2}	N/m
Belt mass (M_b)	0.2	kg
Material mass (M_m)	1	kg

4.1 Nominal Model

We suggest mathematical model of DC servo-driven belt conveyor system can be divided into two primary parts, i.e., DC motor and conveyor load, as depicted in **Fig. 2**. The conveyor load transfer function G_{lo} , consists of the transfer function of the conveyor belt and the load mass which is transported by the belt. Here, we consider the G_{lo} is the feedback loop from the motor angle to the motor torque. The output of G_{lo} is torque force τ_{lo} , that has the effect of reducing torque τ produce by the motor. The input of the belt conveyor system u is the voltage, and the output y is the motor angle θ . We neglect friction within the belt conveyor system since its considerably produce small effect for the model. The other model parameter in the nominal model are explained in **Table 1**.

For a short belt conveyor system, we model G_{lo} as single spring-mass system, as shown in **Fig. 3**. The spring represents the tension or elasticity of the belt. The mass (M) consists of belt mass (M_b), and material mass carried (M_m) by the system. Based on this model, the load transfer function is

$$G_{lo} = \frac{\tau_{lo}(s)}{\theta(s)} = s^2(M_m + M_b)r + kr.$$

Combine the transfer function of the load and DC motor, we obtain mathematical model of the nominal belt conveyor system as the following.

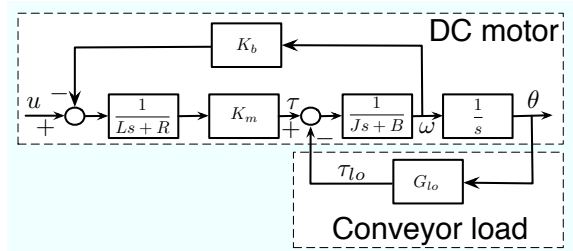


Fig. 2 The nominal model of the belt conveyor system.

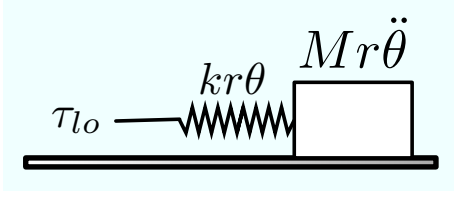


Fig. 3 The conveyor load model.

$$G_0 = \frac{N_0(s)}{D_0(s)} \quad (5)$$

$$= \frac{K_m}{G_{l_0}(Ls + R) + K_m K_b s + s(Js + B)(Ls + R)}$$

4.2 Model with Uncertainty

Multiplicative uncertainty can be calculated by the following relative error between nominal plant $G_0(j\omega)$ and its all possible perturbations $\hat{G}(j\omega)$,

$$l_I(j\omega) = \left| \frac{\hat{G}(j\omega) - G_0(j\omega)}{G_0(j\omega)} \right|.$$

Uncertainty weight function $W_2(j\omega)$ is chosen such that

$$|W_2(j\omega)| \geq l_I(j\omega), \quad \forall \omega.$$

5. Simulation Results

Before we synthesizes the admissible robust PID controller for the system, we decides the $W_1(s)$ and $W_2(s)$. The $W_1(s)$ is formulated as suggested in 7), in the form of

$$W_1(s) = \frac{s/M_s + \omega_b}{s + \omega_b \epsilon}.$$

Here, ω_b is closed loop bandwidth and ϵ is steady state error. M_s is peak sensitivity written as

$$M_s := \|S\|_\infty = |S(j\omega_{max})| = \frac{\alpha \sqrt{\alpha^2 + 4\xi^2}}{\sqrt{(1 - \alpha^2)^2 + 4\xi^2 \alpha^2}},$$

where $\alpha = \sqrt{0.5 + 0.5\sqrt{1 + 8\xi^2}}$ and $\omega_{max} = \alpha\omega_n$, ω_n is natural frequency.

We define our nominal performance specifications as the following.

- Steady state error is 0.001 [rad].
- In order to have damping ratio around 0.5, we set peak sensitivity (M_s) less than 1.5.
- Closed loop bandwidth frequency is 2 [rad/sec].

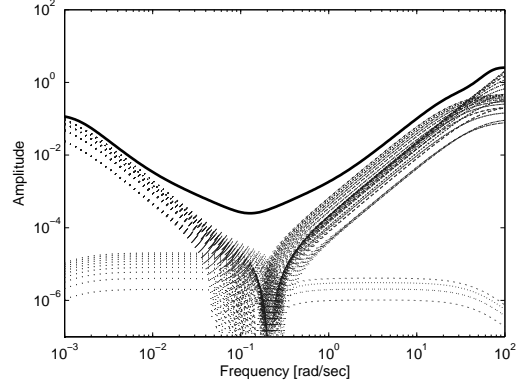


Fig. 4 Uncertainty weighting function W_2 plot (solid) and its relative error l_I plot (dotted).

Therefore, $W_1(s)$ can be formalized as

$$W_1(s) = \frac{s/1.5 + 2}{s + 0.002}. \quad (6)$$

The model uncertainties are determined under various condition as follow.

- Material mass (M_m) is varied by $\pm 100\%$.
- Belt elastic modulus (k) is varied by $\pm 20\%$.
- Time delay within the system is 0.1 [sec].

Using parameters in **Table 1**, we plot relative error l_I of the nominal belt conveyor system with its determined perturbations in **Fig. 4**. Then, we can choose the weighting function W_2 for describing the uncertainties as

$$W_2 = \frac{N_2}{D_2} = \frac{2.209s^6 + 122.6s^5 + 4199s^4 + b}{s^6 + 112.7s^5 + 9186s^4 + c}$$

$$b = 6375s^3 + 1063s^2 + 102s + 0.8992$$

$$c = (2.883 \times 10^5)s^3 + (3.758 \times 10^6)s^2 + 7241s + 6.807.$$

5.1 The Stabilizing PID Controllers

In this section, we conduct the PS algorithm to find the stabilizing PID controller for the system, or the solution of condition (a) in Sec. 2. For the belt conveyor system with an input delay = 0.1s, the nominal system in Eq. 5 is changed into

$$G_d(s) = e^{-0.1s} \cdot \frac{N_0(s)}{D_0(s)}.$$

Before we can apply the PS algorithm for finding the stabilizing PID gains controller for the system, we need to make an approximation for the input delay into polynomial form, by using Padè approximation. Using \mathcal{H}_∞ model matching method⁸⁾ with maximum error is 10^{-3} , the first order Padè approximation will suffice to model the input delay. Then, G_d can be approximated as

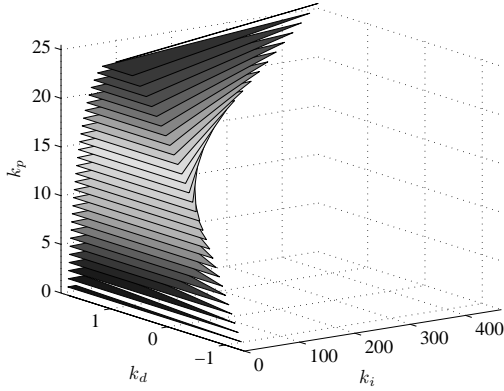


Fig. 5 The all stabilizing (k_i, k_d) gains of the belt conveyor system with delay time $0.1s$, at various value of k_p .

$$G_d(s) = \frac{(-s + 20) \cdot N_0(s)}{(s + 20) \cdot D_0(s)}.$$

Using this nominal model approximation, We are ready to using the PS algorithm for obtaining the set of stabilizing PID gains. The all stabilizing PID gains controllers for the nominal system at various values of k_p , is shown in **Fig. 5**.

5.2 The Robust PID Tuning

In order to solve the problem, we have to attain the conditions (a) and (b) simultaneously, as mentioned in Sec. 2. It is easy to see that condition (c) always gives us true inequality for the nominal system and its perturbations, and therefore give us no additional constraint. Hence, the robust PID gains region depend only by condition (b), that can be solved by using PS algorithm.

Based on Sec. 3, we need to find the intersection region of nominal system and its perturbation for obtaining the true robust PID controller. The example of the intersection region at $k_p = 3.44$ is given in **Fig. 6**. Therefore, this intersection region is the robust PID controllers at $k_p = 3.44$. Sweeping at the various values of k_p , the all robust PID tuning at various values of k_p is shown in **Fig. 7**.

In order to find the minimum value of infinity norm of a given robust performance, we select centroid of the largest PID region from whole admissible region in **Fig. 7**. The largest admissible PID region is when k_p equal 3.44 . The centroid point at this k_p is $(k_i = 0.580, k_d = 0.093)$. The position of centroid is shown in **Fig. 8**.

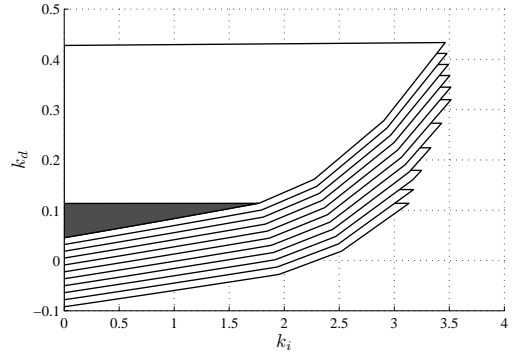


Fig. 6 The intersection region (dark) of solution condition (b) for the nominal system and its perturbations.

We perform robustness test in frequency domain for the nominal model of belt conveyor system and its perturbations, using the centroid point. The frequency test result in **Fig. 9** shows that our admissible PID gains controller passes the test.

Addition observation can be made by using the results. Our nominal belt conveyor system with an input delay 0.1 [sec] has ultimate gain $K_u = 19.1173$, and ultimate period $T_u = 0.4565$. Hence, by using Ziegler-Nichols formula, the Ziegler-Nichols PID gain is $k_p = 11.4704, k_i = 50.2557, k_d = 0.6545$. The PID gain obtained from the Ziegler-Nichols method is lies outside the robust PID showed in **Fig. 7**. Therefore, we can say that PID controller via ziegler-Nichols for the belt conveyor system, is fragile (opposite of robust) controller for our designed robust performance.

The comparison of step responses for the nominal system and its perturbation using Ziegler-Nichols

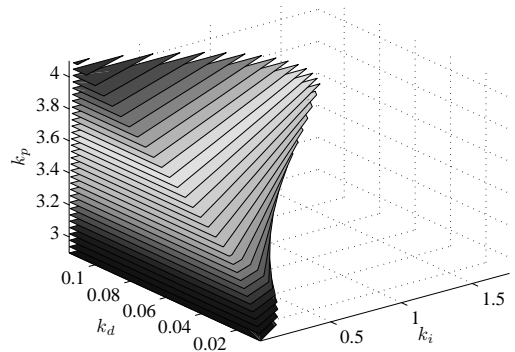


Fig. 7 The all robust (k_i, k_d) gains of the belt conveyor system, at various value of k_p .

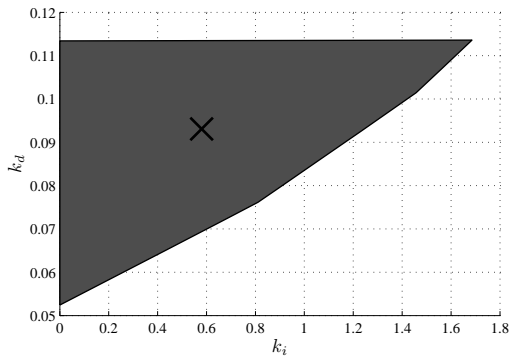


Fig. 8 All PID gains controller satisfied robust performance specification at $k_p = 3.44$, and its best PID gain controller (cross).

PID controller, with the best PID gain controller obtained by corrected PS, is shown in the **Fig. 10**. Our PID controller via PS method shows better performance both for nominal and perturbation plant comparing the one via Ziegler-Nichols method, and relatively follows the designed performance.

6. Conclusion

We have presented the PID tuning process to find the all stabilizing and robust PID controllers, based on corrected PS algorithm. We applied the method on a short DC servo-driven belt conveyor system. A simple nominal and uncertainty model is given to cover the practical aspect of the belt conveyor system. Based on those models, we have synthesized the all PID gains controller for the belt conveyor system.

A best candidate of robust PID gains controller was picked from the admissible region that satisfy our design objectives. We conducted robustness tests in frequency domain and step response test,

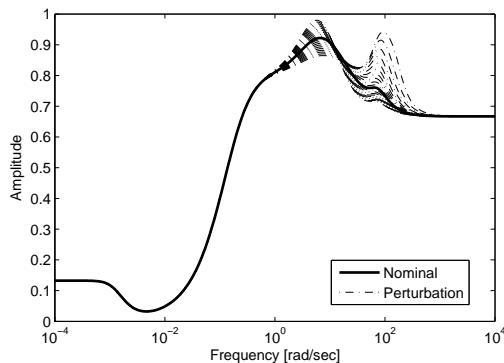


Fig. 9 Frequency tests for the nominal and its perturbation systems.

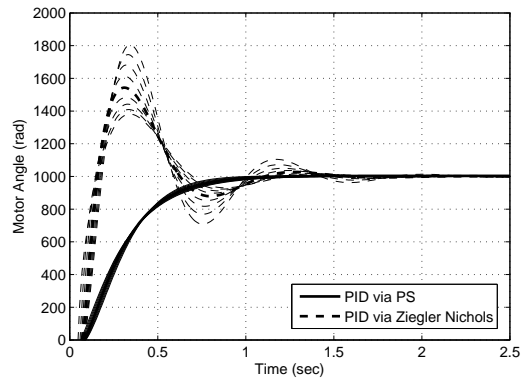


Fig. 10 Step responses for nominal (bolt) the belt conveyor system and its perturbations, using PID tuning via PS and via Ziegler-Nichols.

both for its nominal and perturbed systems. The frequency response test indicates that robust performance condition is achieved, whereas the step responses show that the performances are reasonably follows the designed performance criterion. In addition, we have shown our admissible robust PID gains controller tuning has better performance than PID gain controller tuning via Ziegler-Nichols method.

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