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Identification of Hammerstein Systems with Wavelet Expansion

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Abstract: This paper addresses identification of Hammerstein systems using wavelet expansion from noise corrupted data. When the static memoryless nonlinear part of Hammerstein systems can be considered as the sum of some known functions, the linear part can be estimated as a multi-input single-output (MISO) system with the numerical algorithm for subspace state space system identification method (N4SID). Simulation results illustrate this approach is effective.

Keywords: Hammerstein systems, Nonlinear system identification, ARMAX model, Parameter estimation, Subspace identification method

1. Introduction

Most systems are nonlinear in practice. The identification of nonlinear systems has been an active research topic. Nonlinear dynamical system can be described with many forms. As a special case, Hammerstein model is the most widely applied nonlinear dynamic system, for example, distillation columns, heat exchangers and continuous stirred-tank reactor (CSTR), etc. It consists of a static memoryless nonlinear block followed by a linear dynamic block. Many researchers have proposed lots of methods, which can be roughly divided into two categories: the parametric and nonparametric identification of Hammerstein systems. The nonparametric identification schemes are based on that the linear part is an FIR or IIR model, therefore, the system is a nonlinear ARX model. A number of approaches are proposed on the parametric identification of Hammerstein systems. They include: the iterative method[1)], the over-parameterization method[2)], the blind method[3)], the frequency domain method[4)], the least-squares method[5)], the subspace identification method[6)] and the wavenet method[10)], etc.

The iterative method is the basic approach of parameter estimation. The estimate results show that it is a good algorithm. However, the analysis of convergence is hard. The over-parameterization method is often applied to estimate the linear systems. For the Hammerstein systems, the unknown nonlinear block is parameterized linearly with the unknown coefficients. Order of the new linear system will be so large. The blind method is based on using the output measurements. The linear part is estimated by fast sampling at the output. Identification of the nonlinear part can be carried out in a number of ways[3)]. By means of Fourier transform, the frequency domain method makes the nonlinear part expand to a series of Fourier function. The linear part and the nonlinear part can be identified. The convergence rate of Fourier series is a topic[4)]. The nonlinear part can be used as input of the linear ARMAX model. All parameters can be estimated by the least-squares method[5)]. The parameters of the nonlinear part can be obtained by the averaged method. The numerical algorithm for subspace state space system identification method (N4SID) and least squares support vector machines (LS-SVM) method are used to identify Hammerstein systems[6)]. However, the linear part is an ARX model. The wavelet neural network can be used to estimate Hammerstein systems[10)]. The nonparametric model can be obtained.

In this paper, we consider identification of the discrete-time Hammerstein systems. We assume that the nonlinear part is the sum of some known functions with the unknown coefficients. Every known function can be expanded with wavelet function. We select the Daubechies wavelets which are a family of orthogonal wavelets with compactly support. The linear part of Hammerstein systems is described as an ARMAX model. For the linear part, the identification problem can be considered as identification of multi-input single-output linear system. We make use of the numerical algorithm for subspace state space system identification method (N4SID) to obtain the estimated state space model of system. The Hammerstein nonlinear ARMAX model can be identified by the model transformation.

Subspace identification methods (SIM) have a

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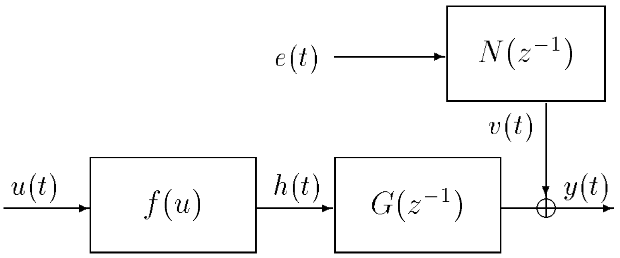

Fig. 1 The discrete Hammerstein systems.

perfect development in practice and theory over ten years[8]. As the non-iterative method, there is no problem about the convergence of algorithm and with a better numerical reliability. Especially, the N4SID algorithm is widely utilized for identification of multi-input multi-output systems.

Briefly, this paper is organized as follows: Section 2 presents the problem formulation related to the Hammerstein systems, and in Section 3, we analyze the feasibility of subspace identification algorithm. Section 4 provides an illustrative example to prove the effectiveness of the algorithm. Finally, some conclusions are obtained in Section 5.

2. Problem Formulation

2.1 The Discrete Hammerstein Systems

Consider the Hammerstein model shown in **Fig.1**. The nonlinear block of the systems can be represented by the sum of some known functions in the input signal as follows

$$f(u) = p_1 f_1(u(t)) + ... + p_m f_m(u(t)) \tag{1}$$

The linear dynamic block can be described as

$$\alpha(z^{-1})y(t) = \beta(z^{-1})f(u(t)) + \gamma(z^{-1})e(t) \tag{2}$$

$u(t)$, $y(t)$ and $e(t)$ are input, output and noise, respectively. The $h(t)$ is an unmeasured internal signal, which is the output of the nonlinear block $h(t) = f(u)$, and just as the input of the linear block. $G(z^{-1}) = \frac{\beta(z^{-1})}{\alpha(z^{-1})}$, and $N(z^{-1}) = \frac{\gamma(z^{-1})}{\alpha(z^{-1})}$. The z^{-1} is the shift operator $[z^{-1}y(t) = y(t-1)]$, $\alpha(z^{-1})$, $\beta(z^{-1})$ and $\gamma(z^{-1})$ are scalar polynomials in z^{-1}:

$$\begin{aligned}\alpha(z^{-1}) &= 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + ... + \alpha_n z^{-n}\\ \beta(z^{-1}) &= \beta_1 z^{-1} + \beta_2 z^{-2} + ... + \beta_n z^{-n}\\ \gamma(z^{-1}) &= 1 + \gamma_1 z^{-1} + \gamma_2 z^{-2} + ... + \gamma_n z^{-n}\end{aligned} \tag{3}$$

Parameter estimation is to identify all coefficients of the nonlinear block and linear block by measuring the inputs and outputs of the system. For a nonzero and finite constant k, any pair $(kf(u), G(z^{-1})/k)$ will produce the same input and output measurements. Therefore, either gain of $f(u)$ or $G(z^{-1})$ must be fixed[3]. the first coefficient p_1 of function $f(u)$ must be one. The Hammerstein model can be described as follows

$$\sum_{i=0}^{n} \alpha_i y(t-i) = \sum_{i=1}^{n}\sum_{j=1}^{m} \beta_i p_j f_j(u(t-i)) + \sum_{i=0}^{n} \gamma_i e(t-i) \tag{4}$$

A MISO system can be written in the following form

$$\begin{aligned}\mathbf{x}_{t+1} &= \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t + w_t\\ y_t &= \mathbf{C}\mathbf{x}_t + \mathbf{D}\mathbf{u}_t + e_t\end{aligned} \tag{5}$$

where $\mathbf{x}_t \in R^n$, $\mathbf{u}_t \in R^r$, $y_t \in R^1$, $\boldsymbol{w}_\mathbf{t} \in \mathbf{R^n}$, and $e_t \in R^1$ are the system state, input, output, state noise, and output measurement noise, respectively. the system output, $\mathbf{A} \in R^{n\times n}$, $\mathbf{B} \in R^{n\times r}$, $\mathbf{C} \in R^{1\times n}$ and $\mathbf{D} \in R^{1\times r}$ are matrices of the system. One can design a Kalman filter for the system to estimate the state variables.

$$\hat{\mathbf{x}}_{t+1} = \mathbf{A}\hat{\mathbf{x}}_t + \mathbf{B}\mathbf{u}_t + \mathbf{K}(y_t - \mathbf{C}\mathbf{x}_t - \mathbf{D}\mathbf{u}_t) \tag{6}$$

where $\mathbf{K} \in R^{n\times 1}$ is the Kalman gain matrix that can be obtained from the Ricatti equation. Therefore, we have the following innovation form

$$\begin{aligned}\mathbf{x}_{t+1} &= \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t + \mathbf{K}e_t\\ y_t &= \mathbf{C}\mathbf{x}_t + \mathbf{D}\mathbf{u}_t + e_t\end{aligned} \tag{7}$$

We introduce the following assumption:

A1: $(\mathbf{A}, \mathbf{C})$ is observable.

A2: $(\mathbf{A}, [\mathbf{B}\ \mathbf{K}])$ is controllable.

A3: The $e(t)$ is independent of input $u(t)$.

Based on these assumptions, by the Z transforms, and we can obtain

$$\begin{aligned}y_t = &[\mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}]\mathbf{u}_t\\ &+[\mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{K} + \mathbf{I}]e_t\end{aligned} \tag{8}$$

$$\begin{aligned}y_t = &[\frac{\mathbf{C}\mathrm{adj}[\mathbf{I} - \mathbf{A}z^{-1}]\mathbf{B}}{z^{-n}\det[z\mathbf{I} - \mathbf{A}]} + \mathbf{D}]\mathbf{u}_t\\ &+[\frac{\mathbf{C}\mathrm{adj}[\mathbf{I} - \mathbf{A}z^{-1}]\mathbf{K}}{z^{-n}\det[z\mathbf{I} - \mathbf{A}]} + \mathbf{I}]e_t\end{aligned} \tag{9}$$

From the above equation, the state space model in the innovation form can be derived to be an ARMAX model. We will introduce the wavelet transformation and Daubechies wavelets. The nonlinear part can be expanded with Daubechies wavelets.

2.2 Wavelet Transformation and Daubechies Wavelets

Wavelet means 'small wave', which oscillates and satisfies

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

where ψ is the wavelet function. A signal can be expressed as a linear decomposition,

$$f(t) = \sum_k a_k \psi_k(t) \tag{10}$$

a_k are the real-valued expansion coefficients and ψ_k are a set of real-valued functions called the expansion set. If the set is unique, the set can be called the basis for the functions that can be expressed. If the inner product among basis functions is often written as:

$$< \psi_j(t), \psi_k(t) >= \delta_{j,k}$$

we call the wavelet function is the orthogonal basic function. Every square-integrable function can be shown as the equation (10), the form is called discrete wavelet transform (DWT). The coefficients of DWT can be obtained using the inner product:

$$a_k =< f(t), \psi_k >= \int f(t)\psi(t)dt$$

Wavelets are defined by the wavelet function $\psi_{(t)}$ and scaling function $\phi_{(t)}$ in the time domain. Assume that a wavelet ψ and the corresponding scaling function ϕ constitute an orthogonal wavelet system. When a function $f(t) \in L_2(R)$, it means all square integrable function f can be expressed as the following wavelet expansion with some levels

$$f(t) = \sum_{k=-\infty}^{\infty} c_k \phi(2^j t - k) + \sum_{k=-\infty}^{\infty} \sum_{j=0}^{\infty} d_{j,k} \psi(2^j t - k)$$

For the orthogonal wavelets, the Daubechies wavelets are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support[11) 12)]. However, The Daubechies wavelets are not defined in terms of the resulting scaling and wavelet functions, they are not possible to write down in mathematic equation. The coefficients of the wavelet and scale sequence can be given. While the wavelet with high order of vanishing moments, its smoothness feature is very fine.

3. Identification Algorithm

At first, we know that the nonlinear part is a sum of some known functions of the input, the input is the finite sequence $\{u(t)\}_1^N$, where N is the sample number. Taking these known functions expand to the sum of some wavelet basis functions $\psi(t)$ and a scale function $\phi(t)$, respectively, where $\psi(t) = 2^{j/2}\psi(2^j t - k)$ and $\phi(t) = 2^{j/2}\phi(2^j t - k)$. Then we can obtain the wavelet expansion of one-dimension function.

$$f_i(u) = c^i_{n,k}\phi_{n,k}(u) + \sum_{j=0}^{n} d^i_{j,k}\psi_{j,k}(u) = DWT\{f_i(u)\}$$

Where $\{c^i_{n,k}\}$ and $\{d^i_{n,k}\}$ are the coefficients of the discrete wavelet transform (DWT). The nonlinear part is

$$f(u) = DWT\{f_1(u)\} + ... + p_m DWT\{f_m(u)\}$$

For the white noise sequence $\{e(t)\}_1^N$, we utilize the discrete wavelet transform (DWT) and can obtain the follows.

$$E(W) = DWT\{e(t)\} = c^e_{n,k}\phi_{n,k}(t) + \sum_{j=0}^{n} d^e_{j,k}\psi_{j,k}(t)$$

The DWT of output can be showed as follows

$$Y(W) = DWT\{y(t)\} = c^y_{n,k}\phi_{n,k}(t) + \sum_{j=0}^{n} d^y_{j,k}\psi_{j,k}(t)$$

Where the wavelet and scale functions that we used, are the same orthogonal wavelet and scale functions with the compact support, the levels of expansion are same. It makes that the length of data of input is the same as output. For the following linear system, it can be considered a multiple input single output system. The input can be showed the following vectors.

$$U_i = \begin{pmatrix} DWT\{f_1(u_i)\} \\ DWT\{p_2 f_2(u_i)\} \\ DWT\{p_3 f_3(u_i)\} \\ \vdots \\ DWT\{p_m f_m(u_i)\} \end{pmatrix} \tag{11}$$

We will perform the subspace identification algorithm. It always consists of two basic steps. The first step is to utilize a projection of certain subspaces of the data, the QR and singular value decompositions, so we can obtain an estimate of the extended observability matrix. The second step recovers the system matrices from the extended observability matrix.

From the equation (7), the following input-output matrix equation can be obtained.

$$\mathbf{Y}_f = \Gamma_i \mathbf{X}_i + \mathbf{H}_i^d \mathbf{U}_f + \mathbf{H}_i^s \mathbf{E}_f \tag{12}$$

First the input and output block Hankel matrices are defined as:

$$\mathbf{U}_{0|i-1} := \begin{pmatrix} U_0 & U_1 & \dots & U_{j-1} \\ U_1 & U_2 & \dots & U_j \\ \vdots & \vdots & \ddots & \vdots \\ U_{i-1} & U_i & \dots & U_{i+j-2} \end{pmatrix}$$

$$\mathbf{Y}_{0|i-1} := \begin{pmatrix} Y_0 & Y_1 & \dots & Y_{j-1} \\ Y_1 & Y_2 & \dots & Y_j \\ \vdots & \vdots & \ddots & \vdots \\ Y_{i-1} & Y_i & \dots & Y_{i+j-2} \end{pmatrix} \tag{13}$$

$\mathbf{U}_p := \mathbf{U}_{0|i-1}, \mathbf{U}_f := \mathbf{U}_{i|2i-1}$,

$\mathbf{Y}_p := \mathbf{Y}_{0|i-1}, \mathbf{Y}_f := \mathbf{Y}_{i|2i-1}$.

The subscript p and f denote, respectively, the past and the future. $\mathbf{W}_p =: [\mathbf{Y}_p^T \mathbf{U}_p^T]^T$, The extended observability matrix Γ_i:

$$\Gamma_i := \begin{pmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{i-1} \end{pmatrix} \tag{14}$$

The state sequence $\mathbf{X}_i$ is denoted as

$$\mathbf{X}_i := \begin{pmatrix} X_i & X_{i+1} & X_{i+2} & \dots & X_{i+j-1} \end{pmatrix} \tag{15}$$

We give the definition of orthogonal projection with the matrices $\mathcal{A} \in R^{p\times j}$ and $\mathcal{B} \in R^{q\times j}$, The orthogonal projection of the row space of $\mathcal{A}$ into the row space of $\mathcal{B}$ is denoted by $\mathcal{A}/\mathcal{B} = \mathcal{A}\mathcal{B}^+\mathcal{B}$ ($\bullet^+$ denote the Moore-Penrose pseudo-inverse of the matrix $\bullet$). $\mathcal{A}/\mathcal{B}^\perp$ is the projection of the row space of $\mathcal{A}$ into $\mathcal{B}^\perp$, the orthogonal complement of the row space of $\mathcal{B}$, and the projection $\mathcal{A}/\mathcal{B}^\perp = \mathcal{A} - \mathcal{A}/\mathcal{B}$.

From the equation (12), the projection of the future outputs $\mathbf{Y}_f$ onto the past data $\mathbf{W}_p$ is

$$\mathbf{Y}_f/\mathbf{W}_p = \Gamma_i \mathbf{X}_i/\mathbf{W}_p + \mathbf{H}_i^d \mathbf{U}_f/\mathbf{W}_p + \mathbf{H}_i^s \mathbf{E}_f/\mathbf{W}_p$$

We know that $\mathbf{U}_f/\mathbf{W}_p = 0$ and $\mathbf{E}_f/\mathbf{W}_p = 0$, the equation can be obtained as follow.

$$\mathbf{Y}_f/\mathbf{W}_p = \Gamma_i \mathbf{X}_i/\mathbf{W}_p = \Gamma_i \tilde{\mathbf{X}}_i$$

Where the $\tilde{\mathbf{X}}_i$ can be shown to correspond to an estimate for the state $\mathbf{X}_i$. In order to calculate the projection of the row space of the input-output data, we performing the following LQ factorization:

$$\begin{pmatrix} \mathbf{U}_{0|i-1} \\ \mathbf{U}_{i|i} \\ \mathbf{U}_{i+1|2i-1} \\ \mathbf{Y}_{0|i-1} \\ \mathbf{Y}_{i|i} \\ \mathbf{Y}_{i+1|2i-1} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_{11} & 0 & 0 & 0 & 0 & 0 \\ \mathbf{L}_{21} & \mathbf{L}_{22} & 0 & 0 & 0 & 0 \\ \mathbf{L}_{31} & \mathbf{L}_{32} & \mathbf{L}_{33} & 0 & 0 & 0 \\ \mathbf{L}_{41} & \mathbf{L}_{42} & \mathbf{L}_{43} & \mathbf{L}_{44} & 0 & 0 \\ \mathbf{L}_{51} & \mathbf{L}_{52} & \mathbf{L}_{53} & \mathbf{L}_{54} & \mathbf{L}_{55} & 0 \\ \mathbf{L}_{61} & \mathbf{L}_{62} & \mathbf{L}_{63} & \mathbf{L}_{64} & \mathbf{L}_{65} & \mathbf{L}_{66} \end{pmatrix} \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \mathbf{Q}_3 \\ \mathbf{Q}_4 \\ \mathbf{Q}_5 \\ \mathbf{Q}_6 \end{pmatrix}$$

From the above equation, we can obtain

$$\mathbf{Y}_f/\mathbf{W}_p = \mathbf{L}_{\mathbf{U}_p} \mathbf{L}_{11} \mathbf{Q}_1 + \mathbf{L}_{\mathbf{Y}_p} \begin{pmatrix} \mathbf{L}_{41} & \mathbf{L}_{42} & \mathbf{L}_{43} & \mathbf{L}_{44} \end{pmatrix} \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \mathbf{Q}_3 \\ \mathbf{Q}_4 \end{pmatrix}$$

We denote $O_i = \mathbf{Y}_f/\mathbf{W}_p$, and obtain the projection for past input data $\mathbf{L}_{\mathbf{U}_p}$ and the projection for past output data $\mathbf{L}_{\mathbf{Y}_p}$ from the following equation

$$\begin{pmatrix} \mathbf{L}_{\mathbf{U}_p} & \mathbf{L}_{\mathbf{U}_f} & \mathbf{L}_{\mathbf{Y}_p} \end{pmatrix} \begin{pmatrix} \mathbf{L}_{11} & 0 & 0 & 0 \\ \mathbf{L}_{21} & \mathbf{L}_{22} & 0 & 0 \\ \mathbf{L}_{31} & \mathbf{L}_{32} & \mathbf{L}_{33} & 0 \\ \mathbf{L}_{41} & \mathbf{L}_{42} & \mathbf{L}_{43} & \mathbf{L}_{44} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_{51} & \mathbf{L}_{52} & \mathbf{L}_{53} & \mathbf{L}_{54} \\ \mathbf{L}_{61} & \mathbf{L}_{62} & \mathbf{L}_{63} & \mathbf{L}_{64} \end{pmatrix}$$

We can also calculate the O_{i+1} in the samily way, where $\mathbf{W}_{0|i} = (\mathbf{U}_{0|i}^T \ \mathbf{Y}_{0|i}^T)$, $O_{i+1} = \mathbf{Y}_{i+1|2i-1}/\mathbf{W}_{0|i}$.

$$O_{i+1} = \mathbf{L}_{\mathbf{U}_p}^+ \begin{pmatrix} \mathbf{L}_{11} & 0 \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix}$$

$$+\mathbf{L}_{\mathbf{Y}_p}^+ \begin{pmatrix} \mathbf{L}_{41} & \mathbf{L}_{42} & \mathbf{L}_{43} & \mathbf{L}_{44} & 0 \\ \mathbf{L}_{51} & \mathbf{L}_{52} & \mathbf{L}_{53} & \mathbf{L}_{54} & \mathbf{L}_{55} \end{pmatrix} \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \mathbf{Q}_3 \\ \mathbf{Q}_4 \\ \mathbf{Q}_5 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \mathbf{L}_{\mathbf{U}_p^+} & \mathbf{L}_{\mathbf{U}_f^+} & \mathbf{L}_{\mathbf{Y}_p^+} \end{pmatrix} \begin{pmatrix} \mathbf{L}_{11} & 0 & 0 & 0 & 0 \\ \mathbf{L}_{21} & \mathbf{L}_{22} & 0 & 0 & 0 \\ \mathbf{L}_{31} & \mathbf{L}_{32} & \mathbf{L}_{33} & 0 & 0 \\ \mathbf{L}_{41} & \mathbf{L}_{42} & \mathbf{L}_{43} & \mathbf{L}_{44} & 0 \\ \mathbf{L}_{51} & \mathbf{L}_{52} & \mathbf{L}_{53} & \mathbf{L}_{54} & \mathbf{L}_{55} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_{61} & \mathbf{L}_{62} & \mathbf{L}_{63} & \mathbf{L}_{64} & \mathbf{L}_{65} \end{pmatrix}$$

We calculate the singular value decomposition of the matrix O_i, and determine the order by inspecting the singular values.

$$O_i = \mathbf{U}\Sigma\mathbf{V}^T \simeq \begin{pmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{pmatrix} \begin{pmatrix} \mathbf{S}_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{pmatrix} \tag{17}$$

By means of the singular value decomposition of matrix O_i, the extended observability matrix Γ_i and Γ_{i-1} can be obtained.

$$\Gamma_i = \mathbf{U}_1\mathbf{S}_1^{1/2},\ \Gamma_{i-1} = \Gamma_i(1:l(i-1),:) \tag{18}$$

The state sequence can be estimated as follows

$$\tilde{\mathbf{X}}_i = \Gamma_i^{+} O_i,\quad \tilde{\mathbf{X}}_{i+1} = \Gamma_{i-1}^{+} O_{i+1} \tag{19}$$

By minimizing the least-squares errors $\mathbf{e}_w$ and e_v, we can obtain the extract estimates of the system matrices **A**, **B**, **C** and **D** from the following equation

$$\begin{pmatrix} \tilde{\mathbf{X}}_{i+1} \\ \mathbf{Y}_{i|i} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{X}}_i \\ \mathbf{U}_{i|i} \end{pmatrix} + \begin{pmatrix} \mathbf{e}_w \\ e_v \end{pmatrix} \tag{20}$$

The Kalman gain matrix $\mathbf{K} = \mathbf{e}_w/e_v$. Then we can derive the ARMAX model from the estimated system matrices. Finally, we can obtain all parameters of the ARMAX model by the model transformation.

In the process of identification, we know the parameter $p_1 = 1$, the parameters of linear part of Hammerstein systems can be estimated directly. There is a large amount of redundancy in estimating the parameters of the nonlinear part of system. By means of the average method[5], we can figure out the parameters $\hat{p_j}$ as follows

$$\hat{p_j} = \frac{1}{n}\sum_{i=1}^{n} \frac{\widehat{p_j\beta_i}}{\hat{\beta_i}},\quad j = 2, 3, \ldots, m \tag{21}$$

4. Simulation Examples

The linear dynamic part is described by an ARMAX model as follows.

$$\alpha(z^{-1}) = 1 - 1.50z^{-1} + 0.60z^{-2} + 0.10z^{-3}$$
$$\beta(z^{-1}) = 0.80z^{-1} - 0.68z^{-2} + 0.22z^{-3}$$
$$\gamma(z^{-1}) = 1 - 1.64z^{-1} + 0.75z^{-2} + 0.12z^{-3}$$

The nonlinear static part is represented a sum of some known functions of the input.

$$\begin{aligned} h(t) &= p_1 f_1(u(t)) + p_2 f_2(u(t)) + p_3 f_3(u(t)) \\ &= u(t) - 0.6\sin(u^2(t)) + 0.2u(t)e^{-u(t)} \end{aligned}$$

We will estimate all coefficients, which includes two parts of the linear and nonlinear. These coefficients can be expressed as a vector form

$$\theta = [\alpha_1\ \alpha_2\ \alpha_3\ \beta_1\ \beta_2\ \beta_3\ p_2\ p_3\ \gamma_1\ \gamma_2\ \gamma_3]$$

In order to estimate the parameters, the input signal must be a persistent excitation signal[7]. We select the standard Gaussian white noise as input

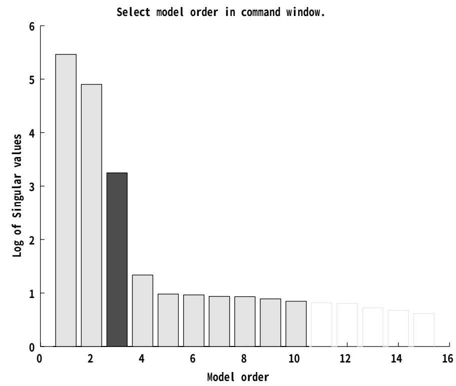

Fig. 2 The singular value decompositions map ($\sigma_{e1}^2 = 0.36$).

$u(t)$ and the data length $N = 2500$. The horizon is chosen as $n = 10$, 20 Monte Carlo simulations will be run. The disturbance $e(t)$ is also a white noise with zero mean and finite variance σ_e^2. The $e(t)$ is independent of input $u(t)$. For the linear part of the system, there are the three input sequences $u(t), \sin(u^2(t)), u(t)e^{-u(t)}$ and one output $y(t)$. It is a 3-inputs and 1-output system. The orthogonal compactly supported wavelet that we select is the Daubechies wavelet db3, and the level of wavelet is 3. We will identify the MISO system by means of **MATLAB** in two different noise cases. At first, we take the three input signals to expand to the sum of wavelet basis function with the wavelet db3. The approximation and detail quantities can be obtained. The total length of the data after the wavelet decomposition is the same as the original data. We apply the subspace identification method to estimate it.

The order of the system is recommended $n = 3$ from the singular value decomposition map shown as **Fig.2**. Based on the order, we obtain the estimated state space model of the MISO systems, furthermore, the model is a canonical form. By means of the model transform from the state space model to the polynomial model, we can obtain all coefficients of the system. When the variance of noise is $\sigma_{e2}^2 = 0.36$, The estimated coefficients $\hat{\theta}_1$ can be given, and we can obtain the estimated coefficients $\hat{\theta}_2$ under the noise's variance $\sigma_{e2}^2 = 4$. The final estimated parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ are in **Table 1**. The estimated nonlinear part and linear part are shown as **Fig.3** and **Fig.4**. Finally, we calculate the mean square error (MSE). $e_{MSE} = \frac{1}{K}\sum_{i=1}^{K}(\hat{\theta}_i - \theta_i)^2$, and K is the total number of parameters. Put the MSEs into the **Table 1**. According to the MSEs, the pro-

Table 1 The parameter estimates with the db3 wavelet ($\hat{\theta}_1 : \sigma_{e1}^2 = 0.36,\ \hat{\theta}_2 : \sigma_{e2}^2 = 4$).

θ	α_1	α_2	α_3	β_1
true	-1.50000	0.60000	0.10000	0.80000
$\hat{\theta}_1$	-1.49605	0.59508	0.10207	0.79849
$\hat{\theta}_2$	-1.43270	0.48753	0.15176	0.80370
θ	β_2	β_3	p_2	p_3
true	-0.68000	0.22000	-0.60000	0.20000
$\hat{\theta}_1$	-0.67315	0.21603	-0.57784	0.20672
$\hat{\theta}_2$	-0.63501	0.17767	-0.43863	0.16415
θ	γ_1	γ_2	γ_3	MSE
true	-1.64000	0.75000	0.12000	0.00000
$\hat{\theta}_1$	-1.58840	0.69428	0.12010	0.00058
$\hat{\theta}_2$	-1.52920	0.58708	0.17439	0.00840

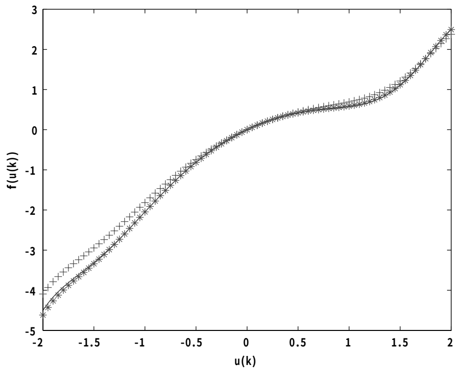

Fig. 3 Nonlinear function with the db3 wavelet: actual (solid), estimated (*:$\sigma_{e1}^2 = 0.36$) and estimated (+:$\sigma_{e2}^2 = 4.00$).

posed algorithm can be proved to be effective.

5. Conclusion

In this paper, a method for identification of Hammerstein nonlinear ARMAX model was proposed. When the nonlinear part of Hammerstein systems is described as a sum of some known functions. With wavelet expansion, we can take every known function expand to the sum of the orthogonal wavelet basis function with many levels. The Daubechies wavelets are selected as the basis function. The approximation and detail of those known functions can be obtained by means of the wavelet decomposition. In other words, the data of signals in wavelet domain are given. For the multi-input single-output system, We can obtain the state space model in innovation form of the system by the N4SID method, and then use the model transformation, all parameters of Hammerstein nonlinear ARMAX model can be estimated. The simulation indicates the effectiveness of the proposed method.

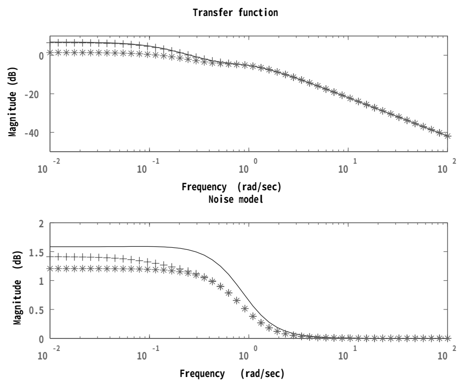

Fig. 4 Frequency response of linear part with the *db*3 wavelet: actual (solid), estimated (*:$\sigma_{e1}^2 = 0.36$) and estimated (+:$\sigma_{e2}^2 = 4.00$).

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