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Effect of Horizontal Convection on the Natural Cooling Process at Water Surface in a Closed Water Body

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(*Received November 13, 2009 and accepted November 19, 2009*)

The characteristics of thermal convection in a closed water body with compound cross section were examined by experiment for measuring water temperature and flow visualization experiment, and considered the influence of the horizontal convection based on the horizontal density difference. A experiment for measuring water temperature revealed that there was little difference in the water temperature in each part though there was a difference of the heat flux on the water surface between the deep and shallow parts, that is, the fluid exchange was immediately done between both parts. Moreover, a flow visualization experiment and the theoretical analysis of the characteristics of the mixed layer found that the horizontal convection based on the density difference between the deep and shallow parts exerted an influence on the time change of the decreasing velocity of the water temperature in the mixed layer and also the developmental velocity of the mixed layer in the deep part.

INTRODUCTION

Water quality easily deteriorates in such closed water bodies as creeks and reservoirs because of poor circulation efficiency of the water due to low inflow and outflow. Water temperature stratification continuously occurs in such water bodies, particularly in the summer, as does oxygen deprivation due to the fact that dissolved oxygen on or at the vicinity of the water surface is not carried to the bottom layer. The disappearance of thermal stratification in a closed water body is thus a important problem which relates to water utilization or the water environment.

The behavior of water environmental substances such as dissolved oxygen in a closed water body is greatly influenced by advection and diffusion, as well as other physical processes. The driving powers of the flow are thermal convection based on solar radiation during the day, radiation cooling at night (thermal disturbance) and wind–induced flow based on the action of the wind (mechanical disturbance). Since about 1980, when water supplies were upgraded and a rise in concern about environmental problems began, experimental and theoretical research has been conducted on the mixing process in the density stratification field (for example, Fernando, 1991). In researching the thermal stratified closed water body, Asaeda and Tamai (1982) examined the structure of thermal convection in the continuous stratification field from the formation limit of the water temperature mixed layer and the hydraulic quantity after establishment. In addition, Mori (1989) conducted a detailed examination of the turbulent structure and entrainment velocity of the density interface by wind–induced flow in the two–layer stratification field. The results from these prior studies show that the disappearance of thermal stratification is greatly dependent on the scale of the convection based on disturbance by the action of the wind or heat. It is thus important to clarify the relation between the convection characteristics and process of stratification disappearance.

Here, when thinking about a field in which thermal disturbance is predominant, the characteristics of the convections which should be considered are the vertical convection, which develops downward by cooling at water surface, and the horizontal convection, based on the horizontal density difference which is caused by the shape of the bottom in the water body. It is presumed that a water body with compound cross section has complex characteristics of thermal convection because the interaction between the vertical and horizontal convections is large in the thermal convection field, where the driving power of the flow is small compared with wind–induced flow. Compound channel flow is a representative example of a flow field which has a depth difference, and it has been reported that a horizontal whirlpool which occurs secondarily by mutual interference of the low–water channel flow and the flood channel flow exerts an influence on the flow field. Regarding the process of cooling at water surface in a water body with compound cross section, there are two factors. One, the vertical convection based on settling of the cold water mass, that is, thermal generated by an unstable density at the water surface. Two, the horizontal convection is caused by the density difference based on the heat capacity difference between the deep and shallow parts. It is necessary to consider the influence of this horizontal convection on the developmental process of the water temperature mixed layer in order to examine the flow characteristic in a closed water body with compound cross section.

In this study, we examined the influence of the

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Table 1. The experimental conditions for the water temperature measurement

Case	Wate depth (cm)		Measurement interval (cm)	Temperatur (°C)		Rayleigh number		
	H_1	H_2		Water	Air	Section D	Section F	Section S
1–1	24	8	1 (0~8cm depth)	29.2	23.9	3.11×10^{10}	2.76×10^{10}	2.56×10^{8}
1–2			2 (8~22cm depth)	26.6	22.1	8.99×10^{10}	3.46×10^{9}	7.68×10^{8}
2–1	24	8	1 (0~8cm depth)	26.7	19.6	3.46×10^{9}	4.84×10^{10}	8.54×10^{8}
2–2			2 (8~22cm depth)	27.3	21.0	4.49×10^{10}	6.91×10^{10}	1.28×10^{8}

shape of the water body on the characteristic of thermal convection, especially the developmental process of the layer of uniformed water temperature, in other words a mixed layer. Firstly, we conducted an experiment for measuring water temperature, and then examined the characteristics of the formation of the thermal stratification and development of the mixed layer in the inhomogeneous cooling field based on the depth difference of the water body. Moreover, to examine the characteristics of the mixed layer in detail, a theoretical analysis was conducted on the time change of the decreasing velocity of the water temperature in the mixed layer, and the developmental velocity of the mixed layer. In addition, the pressure difference between the deep and shallow parts and the horizontal flow based on it were calculated in order to examine the scale of the horizontal convection.

METHODS AND RESULTS

Experimental equipment and methods

Water temperature measurement

The experimental equipment was a test tank with compound cross section made from an acrylic board 0.5 cm in thickness, with a water surface area of 60×20 cm, and a depth of 30 cm in the deep part and 10 cm in the shallow part (cf. Fig. 1). Moreover, we conducted comparative experiments with a water tank in which the shallow part and deep part were separated. The air temperature, water temperature, humidity and water level were measured to obtain the amount of heat transportation at the water surface (amount of evaporation). The experimental conditions are shown in Table 1. The Rayleigh number is a dimensionless number which shows the scale of convection, and the flux type of Rayleigh number, which uses heat flux at the water surface Q_s is shown by Eq. (1) as

$$R_a = \frac{\alpha g Q_s H^4}{\rho_r c \kappa^2 v} \tag{1}$$

where α is the coefficient of thermal expansion (1/°C); H, water depth (cm); κ, thermal diffusivity (cm²/s); v, the coefficient of kinematic viscosity (cm²/s); Q_s heat flux (J/cm²·s); ρ_r, reference density (g/cm³); c, isopiestic specific heat (J/g·°C); and g, gravitational acceleration (cm/s²). The parameters in Eq. (1) were defined as $\alpha = 2.5 \times 10^{-4} = 2.5$ and $\kappa = 1.4 \times 10^{-4}$. The experiment was conducted indoors. The water was heated for 5 hours with
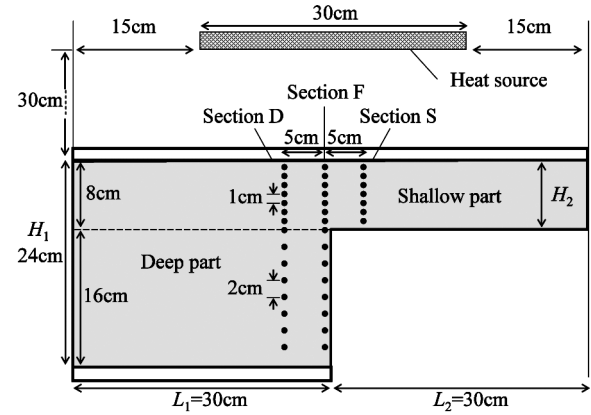
Fig. 1. The experimental equipment.

a heat source and then the cooling at water surface began without air temperature control. In Case 1, tap water was used as the fluid for the experiment, and an electrothermal board was used as a heat sources to create thermal stratification. In Case 2, however, the colored tap water was used as the fluid for the experiment in order to change the transparency, and an infrared ray lamp was used as the heat source to create thermal stratification. Each heat source was positioned at the height of 30 cm above the water surface.

Flow visualization experiment

The flow visualization experiment using the colored dye was conducted in the same water tank as the experiment for measuring water temperature and with the same experimental methodology, and the scale of the horizontal convection based on the horizontal density difference was examined. That is, Aniline blue was vertically dropped to the water surface at the step part of the water body, and the vertical distribution of the velocity of horizontal flow was obtained by following the track of the Aniline blue. The flow visualization experiment condition is shown in Table 2.

Table 2. The experimental conditions for the water temperature measurement

Case	Type of fluid field	Water depth (cm)		Rayleigh number
		H_1	H_2	
3–1	thermal stratification	24	8	2.08×10^{8}
3–2	thermal uniformity			2.59×10^{8}

Experimental results and discussions

Time changes of water temperature distribution and heat flux at the water surface

Figs. 2 and 3 show the time changes of the vertical distribution of the water temperature in the heating period and the cooling period in Case 1–2. At the heating period, it was understood that the water temperature in the upper layer rose from receiving heat through the water surface after heating begins, and then the thermal stratification was formed in each section. The water temperature hardly rose in the lower layer, and a stable density field was formed in the deep part. The water temperature in the vicinity of the water surface decreased rapidly in the immediate aftermath of cooling, and the inversion layer of the water temperature was formed. The water mass in the vicinity of the surface was transported downward by this unstable density, and the r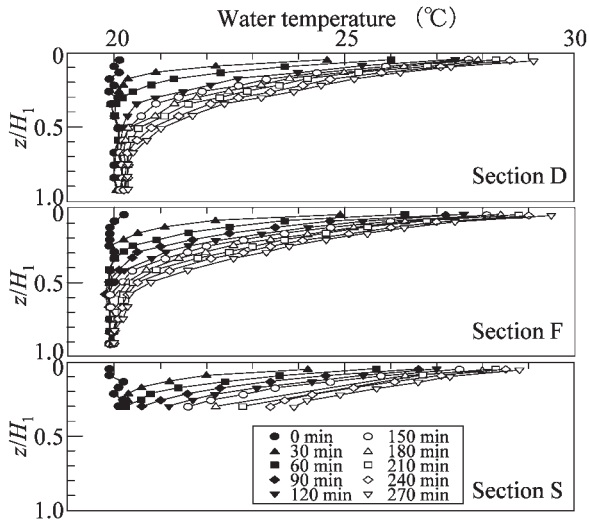eturn flow was generated, that is, the mixed layer was developed. The thickness of the mixed layer reached the bottom layer of the shallow part after about 90 minutes during the cooling period, and the water temperature in the shallow part decreased uniformly in the vertical direction afterwards. The depth of the mixed layer did not reach the bottom of the deep part during the measurement period, that is, the thermal stratification did not completely disappear in the deep part. It is thought that the increase of the water temperature in the lower layer was based on conduction of heat from the upper layer. Moreover, a remarkable water temperature difference was not seen on average although there was a great difference in heat capacity between the deep and shallow parts depending on the depth difference. Thus, it is guessed that the water temperature became horizontally uniform by the fluid exchange in both parts.

Fig. 4 shows the time change of the heat flux at the water surface in Case 1–2. At the heating period, the heat flux in section D (deep part) was high, which shows that this section received more heat quantity because the water body in the deep part had a larger heat capacity. Moreover, the heat flux in section S was large in the initial stage of the cooling period, and it is understood that the amount of heat radiation was also large. However, as the cooling progresses, the heat flux in the shallow part was either the same as the deep part or had become smaller.

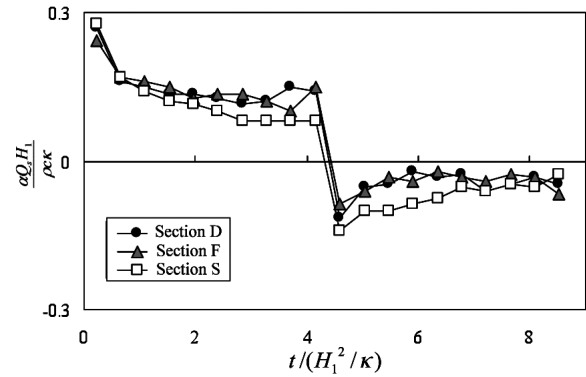**Fig. 2.** The time change of the water temperature during the heating period (Case 1–2).

Fig. 4. The time change of the heat flux at the water surface.
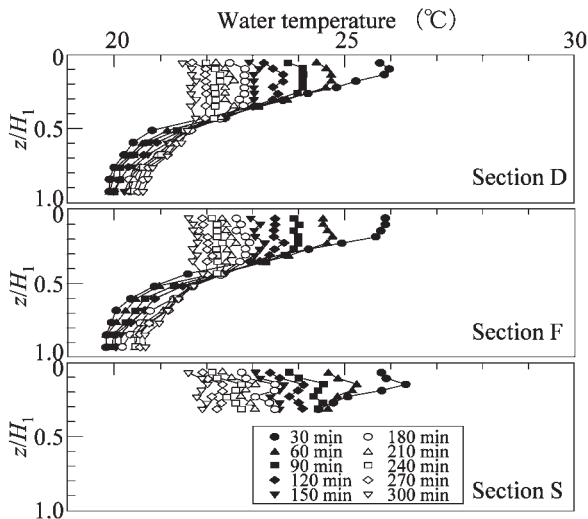
Fig3. The time change of the water temperature during the cooling period (Case 1–2).

From the above–mentioned result, it was shown that there was little difference in the water temperature in each part although there was some difference between the amount of heat accumulated in the deep part and that in the shallow part; the fluid exchange was done by horizontal convection based on the density difference between both parts. It is thought that the characteristics of thermal convection in a closed water body with compound cross section change due to the combination of the vertical convection based on the cooling at water surface and the horizontal convection caused by the density difference between the deep and shallow parts. As a result, it is thought that the development process of the mixed layer changes when the depth is even and it is thus necessary to examine the characteristics of the mixed layer. We theoretically and experimentally exam-

ined important parameters of the mixed layer, specifically the decreasing velocity of the water temperature in the mixed layer, the developmental velocity of the mixed layer, and the horizontal convection at the step part.

Characteristics of the mixed layer

In order to examine the characteristics of the mixed layer in the deep part, various factors must be taken into consideration, including heat flux at the water surface, heat mixing with the lower layer, and heat transportation based on the horizontal flow between the deep and shallow part. Assuming Boussinesq fluid motion in two dimensions, the governing equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{\rho}{\rho_0}g + v\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

where u and w are the velocity in the x and z directions; ρ is the density at water temperature T, ρ_0 is the reference density, p is the pressure deflection from hydrostatic pressure at constant density $\rho = 1$ g/cm³.

Firstly, we examined the decreasing velocity of the water temperature in the mixed layer in the deep part. The water temperature of the mixed layer in the deep part is T_1, and the water temperature difference compared to the lower layer is ΔT_1 (cf. Fig. 5). When the cold water mass subsides and reaches the density interface, the upper and lower layers mix, and a middle layer with a temperature of $T_1 - \Delta T_1/2$ is formed. At the same time, an upward flow which compensates for the downward flow is generated, and then the fluid in the middle layer is transported upwards. By this process, the thickness of the mixed layer h increases and the water temperature of the mixed layer T_1 decreases. The decreasing process of water temperature based on the cooling at water surface in the closed water body with compound cross section is classified into three types depending on the depth of thermocline h: Type–1 ($h < H_2$) : the decreasing process of water temperature based on only the vertical convections, Type–2 ($h = H_2$) : the decreasing process of water temperature based on the vertical convection and weak horizontal convections, and Type–3 ($h > H_2$): the decreasing process of water temperature based on both of the vertical and horizontal convections. The boundary conditions at the water surface and density interface are

$$\left\{ \begin{array}{l} z=0 \; ; \; (wT_1)_0 = -\dfrac{Q_1}{\rho c} \\[2mm] z=h \; ; \; (wT_1)_h = -(T_1 - \Delta T_1)\dfrac{dh}{dt} \end{array} \right. \tag{6}$$

where Q_1 is the heat flux at the water surface in the deep part, and subscripts 0, h show the value at $z=0$, h, respectively. Moreover, the boundary condition at the sidewalls and the boundary at the step part between the deep and shallow parts are

$$x=0 \; ; \; (uT_1)_0 = 0, \; x=L \; ; \; (uT_1)_L = 0 \tag{7}$$

$$x=0 \; ; \; (uT_1)_0 = 0, \; x=L \; ; \; (uT_1)_L = u_c \Delta T'' \tag{8}$$

where $\Delta T'' = T_1 - T_2$, and u_c are the referential velocity of the horizontal convection at the step part, and subscripts 0, L_1 show the value at $x=0$, L_1, respectively. Eq. (7) corresponds to the process both of Type 1 and Type 2, and Eq. (8) corresponds to the process of Type 3. When applying the boundary conditions given in Eqs. (6) and (7) to the case of Type 1, the decreasing velocity of the water temperature in the mixed layer is obtained as follows:

$$\frac{dT_1}{dt} = -\frac{Q_1}{\rho ch} - \Delta T_1 \frac{dh}{dt} \tag{9}$$

The case of Type 2 becomes similar to Type 1 when cold water passes by the slight horizontal convection from the bottom of the shallow part to the deep part. When applying the boundary conditions given in Eqs. (6) and (8) to the case of Type 3, the decreasing velocity of the water temperature in the mixed layer is obtained as follows:

$$\frac{dT_1}{dt} = -\frac{Q_1}{\rho ch} - \frac{\Delta T_1}{h}\frac{dh}{dt} - \frac{\Delta T''}{h}u_c\frac{H_2}{L_1} \tag{10}$$

Eq. (10) shows that the decreasing velocity of the water temperature in the mixed layer in the closed water body with compound cross section is determined by the balance of the heat flux at the water surface, the thickness and the developmental velocity of the mixed layer, and the velocity of the horizontal convection.
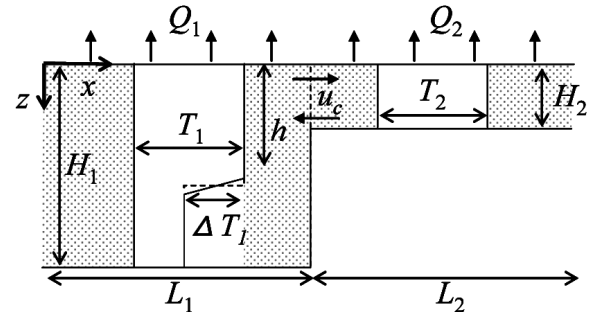

Fig. 5. The schematic diagram of the flow field.

Fig. 6 shows the time change of the decreasing velocity of the water temperature in the mixed layer in the deep part dT_1/dt obtained from the experiment for measuring water temperature. T_0 shows the water temperature of the mixed layer when cooling started. In the ini-

tial stage of the cooling at water surface, the temperature decreased rapidly because the heat flux at the water surface was at its largest. The difference between the water and air temperatures became smaller as cooling progresses, and the decreasing velocity of the water temperature in the mixed layer likewise decreased until it arrived at a state of equilibrium. It is thus understood that the term which contributes to the decreasing velocity of the water temperature in the mixed layer had changed as time passes. Fig. 7 shows the time change of the contribution rate of each term in Eq. (10) obtained by the result in Case 2–1. The figure shows that the contribution of the second term on the right–hand side was large at the initial stage of cooling (Type 1), the first term was large at the transient stage (Types 1 to 3), and the third term was large at the last stage (Type 3). In other words, it is understood that the contribution of the horizontal convection was based on the density difference between the deep and shallow parts when cooling advances.

Next, we examined the developmental velocity of the mixed layer. When the convection mass formed by the cooling at water surface reaches the density interface
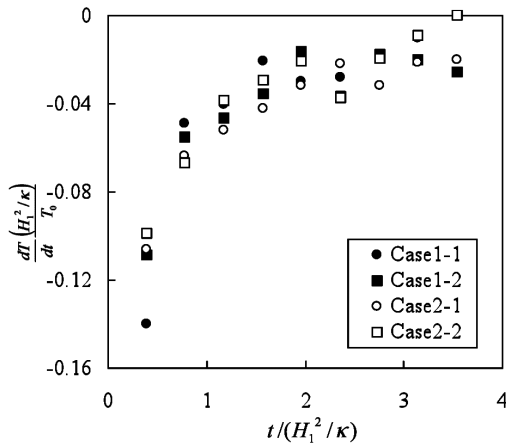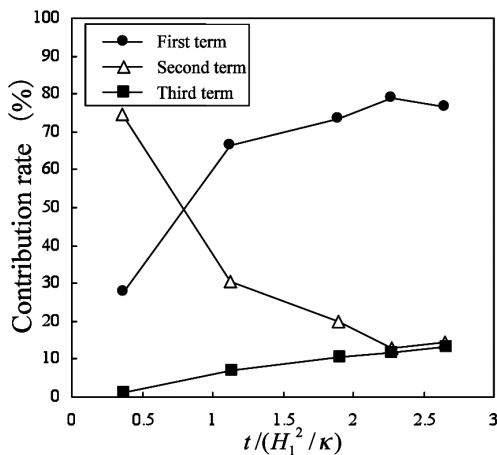
from Eq. (9), the velocity is

(Type 1 and Type 2)

$$w(h) = \left\{ -2\alpha g h \left(h\, \frac{dT_1}{dt} + \frac{Q_1}{\rho c} \right) \right\}^{1/3} \quad (11)$$

(Type 3)

$$w(h) = \left\{ -2\alpha g h \left(h\, \frac{dT_1}{dt} + \frac{Q_1}{\rho c} + \Delta T'' u_c \frac{H_2}{L_1} \right) \right\}^{1/3} \quad (12)$$

Moreover, the relationship between w and the variation in the thickness of the mixed layer dh/dt is given by the following formula from the energy conservation law:

$$(C_T w^2 + \alpha\, \Delta T g h)\frac{dh}{dt} = C_k w^3 \quad (13)$$

where C_T and C_k are constants, and the values 0.5 and 0.13, respectively, are used. Eqs. (11) and (12) show that the developmental velocity of the mixed layer dh/dt is determined by the balance of the heat flux at the water surface, the water temperature and its decreasing velocity in the mixed layer, and the horizontal convection velocity. Thus, the values of the decreasing velocity of the water temperature in the mixed layer and the developmental velocity of the mixed layer are determined while they influence each other.

Fig. 8 shows the time change of the developmental velocity of the mixed layer obtained from the experimental results. The developmental velocity of the mixed layer was greatest immediately after cooling starts, and decreased as cooling progresses. From the figure, it can be understood that the developmental velocity of the mixed layer accelerates from the boundary of $h=H_2$. It is thought that the developmental velocity of the mixed layer became fast because the decreasing process of the water temperature in the mixed layer transfer from Type 1 to Type 3.

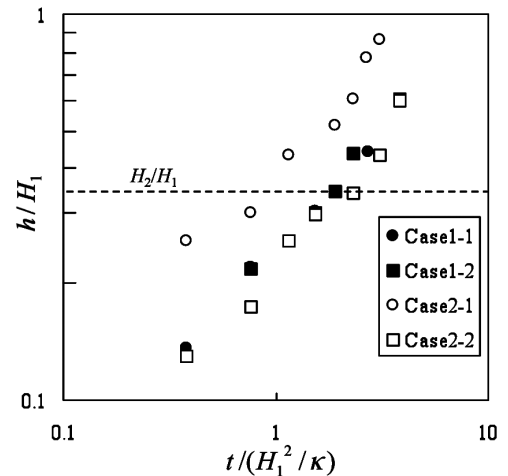In the previous paragraph, it was shown that the

Fig. 6. The time change of the decreasing velocity of the water temperature in the mixed layer in the deep part.

Fig. 7. The time change of the contribution rate of each term in Eq. (10).

Fig. 8. The time change of the developmental velocity of the mixed layer.

horizontal convection is caused at the step part when the depth of the mixed layer is larger than the depth of the shallow part. As mentioned above, this is based on the pressure gradient in the horizontal direction generated by the density difference between the deep and shallow parts. So, the pressure difference between the deep and shallow parts was obtained from the vertical distribution of the water temperature in order to examine the scale of this horizontal convection. The density of water ρ_z at water temperature T_z is as follows:

$$\rho_z = \{1 + \alpha (T_z - 4)\}^{-1} \tag{14}$$

Since pressure in depth z is $\rho_z gz$, the total water pressure P in basic depth d is as follows:

$$P = \int_0^d \rho_z gz dz \tag{15}$$

In this research, the depth of the shallow part is used for the basic depth d. Fig. 9 shows the time change of the pressure difference between the D and S sections in Case 2–1. In this figure, P_0 shows the pressure in the initial water temperature as the reference pressure and ΔP shows the pressure difference between the D and S sections. When cooling begins, the pressure of the deep part (D section) is high, and the shallow part (S section) is low, that is, the value of ΔP is positive. However, the value of ΔP is negative when cooling advances. It is thought that the density difference is horizontally generated because the shallow part is cooled earlier than the deep part due to the difference of the decreasing velocity of the water temperature in the mixed layer when the depth of the mixed layer is larger than the depth of the shallow part.

The scale of the horizontal convection generated by the horizontal difference of density is thus estimated as follows. Assuming that the flow field is steady and that the movement is linear because the phenomenon is extremely slow, Eqs. (3) and (4) are rewritten as follows, respectively:

$$\frac{\partial P}{\partial x} = \rho_0 v \; \frac{\partial^2 u_1}{\partial z^2} \tag{16}$$

$$\frac{\partial p}{\partial z} = \rho_1 g \tag{17}$$

where u_1 is the velocity of the horizontal convection in the deep part; ρ_1 is the density of the water body of the mixed layer in the deep part. p is deleted from the above formulas;

$$\frac{\partial^2}{\partial z^2} \left(\frac{\partial u_1}{\partial z} \right) = \frac{g}{\rho_0 v} \; \frac{\partial \rho_1}{\partial x} \tag{18}$$

The flow velocities at the water surface and the density interface at the step part are assumed to be 0;

$$z = 0 \; ; \; u_1 = 0$$
$$z = h \; ; \; u_1 = 0 \tag{19}$$

Moreover, assuming that there is no flow below the density interface, the vertical integration value of the horizontal flow velocity in the mixed layer becomes 0 because it is a closed space;

$$\int_0^h u_1 dz = 0 \tag{20}$$

Here, when assuming that ρ_1 shifts linearly into the x direction, and integrating Eq. (18), the velocity distribution in the step part is described as flow;

$$\frac{u_1}{U} = 2 \left(\frac{z}{h} \right)^3 - 3 \left(\frac{z}{h} \right)^2 + \left(\frac{z}{h} \right) \tag{21}$$

where U is defined as follows;

$$U = \frac{gh^3}{12 \rho_0 v} \; \frac{\partial \rho_1}{\partial x} \tag{22}$$

Fig. 10 shows the vertical distribution of the horizontal flow velocity obtained from the flow visualization experiment in Case 3. It is understood that the flow from the deep part to the shallow part occurred in the upper layer and the turned flow occurred in the lower layer.
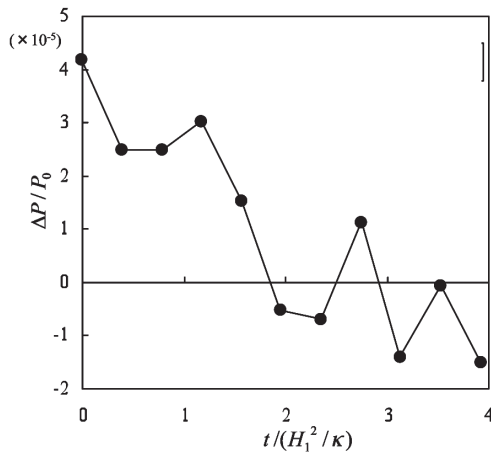

Fig. 9. The time change of the pressure difference between the D and S sections.
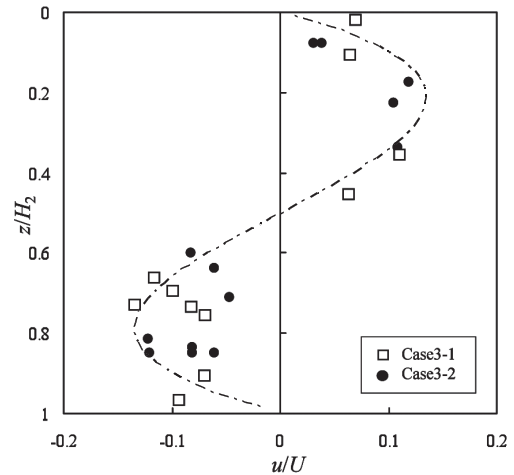

Fig. 10. The vertical distribution of the horizontal flow velocity.

The velocity of horizontal convection in the thermal uniformity was larger than the thermal stratification; the developmental velocity of the thermal convection was large in the thermal uniformity because of the absence of negative buoyancy by the stratification. Moreover, the calculation result obtained by setting the value of U for satisfying the experimental result is shown in the figure. From the figure, it was understood that Eq. (21) can roughly reproduce the tendency of the vertical distribution of the horizontal flow.

As the result of considering the characteristics of the mixed layer from the experiment for measuring water temperature and theoretical analysis, it was shown that the horizontal convection based on the density difference between the deep and shallow parts exerted an influence on the time change of the decreasing velocity of the water temperature in the mixed layer and the developmental velocity of the mixed layer in the deep part, and that the contribution rate increased with the passage of time.

CONCLUSION

We examined the characteristics of thermal convection in a closed water body with compound cross section from experiment for measuring water temperature and flow visualization experiment. The results are mentioned bellow.

As first, from the experiment for measuring water temperature, it was shown that there was little difference in the water temperature in each part though there was difference of the heat flux at the water surface between the deep and shallow parts. That is, the fluid exchange was done rapidly in both parts. Next, from the flow visualization experiment and the theoretical analysis of the characteristics of the mixed layer, it was shown that the horizontal convection based on the density difference between the deep and shallow parts exerted an influence on the time change of the decreasing velocity of the water temperature in the mixed layer and the

developmental velocity of the mixed layer in the deep part.

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