A Circular Pattern Recognition and the Probability Tables : Multiple Circular Runs Test

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A Circular Pattern Recognition and the Probability Tables
——Multiple Circular Runs Test——

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Abstract

A nonparametric method of pattern recognition is proposed for directional observations, or for observations on a cyclic time series. Such observations, like birds migration, pollens' movement, bio-rhythm of living et al, can be rearranged directionally from an origin on a circumference of a unit circle. Then the authors suggest a possibility to apply a multiple circular runs test whether several sets of observations on a circle are drawn from the same distribution as a methodology of pattern recognition. Thus the present paper gives firstly the well-explained mathematical formulation of the generalized problem on the basis of combinatorial theory, secondly the numerical tables of critical numbers of runs with upper and lower significance levels and finally a compact FORTRAN program of proto-type for further cases of practical application of the present method proposed.

1. Introduction

In some cases, observational subjects may be at first recognized on their directions, locating in a space with a certain number of dimensions. These observations for distributing locations of subjects are only limited in their orientations or angular data, as seen in birds migration, pollens' movement and so on. Even if such observations may be obtained with the coordinates on the dimensional axes, it may be possible to say that multivariate observations may be also regarded as a kind of directional data, although there exist further inference problems on locations and dispersions of the observations.

It occurs also that a number of events is observed on a cyclic time series, like a bio-rhythm of living things, an ecology of insects and so on. That is to say, these observations are arranged directionally from an origin on a circumference of a unit circle.

In such situations on a cyclic circumstance, there exist some statistical theories for the circular distributions, *e.g.* Mardia [6], and for the circular runs tests, e.g. Whitworth [10], Jablonski [5], Barton and David [3, 4], Asano [1], Stephens [8] and also Mardia [6]. However, these papers have not realized immediately the availability for practical applications, because of the shortage of numerical tables of probabilities with the number of sets of observations and the respective sizes.

The purpose of the present paper is firstly to show well-explained mathematical formulation for circular runs test

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with multi-sample case, secondly to give extensively the numerical table of critical number of runs for upper and lower significance levels of critical probabilities for the sake of instant applications, and finally to present the compact FORTRAN program for more general cases in practice.

2. Jablonski runs test with multiple colors

The present paper is suggested in pattern recognition of circular distributions for testing whether several sets of observations on a circle are drawn from the same distribution. This concerns itself with a randomization test for an arrangement of several observation sets, belonging to an identical population.

The fundamental theories of runs test were originally studied by Whitworth [10] and Jablonski [5], and from the viewpoint of pattern recognition, Jablonski runs are proposed in the present paper.

Let several sets of observations, say k sets, be given on a circumference, and let the respective size of observations be r_i, where $i=1, 2, \cdots, k$, and $r\equiv\sum_{i=1}^{k} r_i$. Then, basing on all the common divisors $\{d_i\}$ of $r_1, r_2, \ldots, r_k$ such that $d_1\equiv 1<d_2<\ldots<d_n\equiv h$, the number N of total circular arrangements, omitted identical arrangements by (d_i/r)-rotation, is given by

$$N=\sum_{i=1}^{n} Q_{d_i}/(r/d_i), \tag{1}$$

where $Q_{d_i}/(r/d_i)$ is the number of circular orderings, any one of which consists of the juxtaposition of just d_i similar orderings of (r/d_i) samples, and where Q_{d_i} is obtained by solving the following linear simultaneous equations,

$$\sum_{\{d|d_i, h\}} Q_d=(r/d_i)!/\prod_{j=1}^{k}(r_j/d_i)!, \quad \text{for } i=1, \ldots, n \tag{2}$$

where the sum is over all Q_d for which d_i is a factor of d, and d is a factor of h. Then the total number (1) of circular orderings is rewritten as

$$N=\frac{1}{r}\sum_{i=1}^{n}\phi(d_i)\cdot(r/d_i)!/\prod_{j=1}^{k}(r_j/d_i)! \tag{3}$$

where $\phi(d_i)$ is Euler's totient function of d_i.

In order to obtain the probability function of observing an arbitrary t runs, where $k\leq t\leq r$, let $J_k(r_1, r_2, \ldots, r_k|t)$ be the number of the linear orderings of $\{r_i\}$ observations with just t runs, when the line is bent untill the two ends of the line join. Then the number $N(t)$ of observing t circular runs under the present condition is

$$N(t)=\sum_{i=1}^{n} Q_{d_i}(t)/(r/d_i) \tag{4}$$

where $Q_{d_i}(t)/(r/d_i)$ is the number of circular orderings, any one of which consists of the juxtaposition of just d_i similar orderings of (r/d_i) samples and has just t runs, and where $Q_{d_i}(t)$ is also obtained by solving the following linear simlutanuous equations.

$$\sum Q_d(t)=J_k\left(\frac{r_1}{d_i}, \ldots, \frac{r_k}{d_i}\middle|\frac{t}{d_i}\right) \tag{5}$$

where the sum follows the rule as described above, and where

$$J_k(r_1, \ldots, r_k|t)=\sum_{(t_1,\cdots,t_k)}\left\{\prod_{i=1}^{k}\binom{r_i-1}{t_i-1}\sum_{\substack{x=k-1\\ t_k\geq t'-x}}^{t'}\left[\binom{x}{t_k+x-t'}\cdot J_{k-1}(t_1, \ldots, t_{k-1}|x)\cdot r/t'\right]\right\} \tag{6}$$

where the outer sum is over all possible k-partitions of t, say, $(t_1, t_2, \ldots, t_k)$, and $t'=\sum_{i=1}^{k-1} t_i$. Since

$$J_2(r_1, r_2 | t) = r\binom{r_1-1}{s-1}\binom{r_2-1}{s-1}/s, \qquad (7)$$

where $s=t/2$ for $k=2$ in (6), J_3 $(r_1, r_2, r_3 | t)$ is obtained by using (7) and (6). Therefore, for an arbitrary integer k, $J_k(r_1, ..., r_k | t)$ is successively obtained on the basis of (6). Then the total number (4) is rewritten as

$$N(t) = \frac{1}{r}\sum_{i=1}^{n}\phi(d_i)J_k\left(\frac{r_1}{d_i}, \ldots, \frac{r_k}{d_i}\middle|\frac{t}{d_i}\right). \qquad (8)$$

Thus the distribution function for t circular runs, say $F(t | r_1, r_2, \ldots, r_k)$, is given by

$$F(t | r_1, r_2, \ldots, r_k) = P_r\{T \leq t\} \qquad (9)$$

$$= \sum_{T=k}^{t} N(T)/N. \qquad (10)$$

3. Complete computer program for probabilities on multiple circular runs

To obtain values of a distribution function of multiple circular runs, and to see the critical values for testing hypothesis whether K sets of observations on a circle belong to a similar distribution function, a FORTRAN program is completed with a simple specification to apply by using a minimum equipment of FACOM U-400.

The inputs are only the number K of colors and the respective sizes $IR(I)$ of samples on a circle, where $I=1, 2, \cdots, K$, and $K \geqslant 3$, because of the existence of the wide table for $K=2$ in Asano [1]. The outputs are the cumulative probabilities of the distribution function of the respective numbers T's of circular runs, where $T=K, K+1, \ldots, \sum_{I=1}^{K} IR(I)$. Figure 1 gives an example of the output.

The program, however, is shown as a proto-type, and therefore the program may be more improved on the computing time. About ten steps from the top are to store values of combinations and to use at the subsequent steps. The sizes of the dimensions must be expanded for the use of further cases, not including in the Table. For the sake of saving computing time, an intermediate result, if any, is available by a small change in view of formula (7) in section 2.

```
A PATTERN RFCOGNITION FOR CIRCULAR DISTRIBUTIONS WITH  6 COLORS

        4 = N 1    4 = N 2    5 = N 3    5 = N 4    5 = N 5    5 = N 6

     NO. OF RUNS        CUMULATIVE PROBABILITIES

          1                 0.0000000E 00
          2                 0.0000000E 00
          3                 0.0000000E 00
          4                 0.0000000E 00
          5                 0.0000000E 00
          6                 0.1316278E-14
          7                 0.5923251E-13
          8                 0.1593354E-11
          9                 0.3113326E-10
         10                 0.4772178E-09
         11                 0.5957887E-08
         12                 0.6192033E-07
         13                 0.5432704E-06
         14                 0.4061969E-05
         15                 0.2604305E-04
         16                 0.1437688E-03
         17                 0.6847701E-03
         18                 0.2815159E-02
         19                 0.9982683E-02
         20                 0.3046366E-01
         21                 0.7974535E-01
         22                 0.1784058E 00
         23                 0.3398724E 00
         24                 0.5505167E 00
         25                 0.7613694E 00
         26                 0.9135765E 00
         27                 0.9841735E 00
         28                 0.9999879E 00

TOTAL NO. OF CIRCULAR ORDFRINGS =    0.9116614E 17
```

Fig. 1. An example of the output of a computer program proposed.

4. Table of critical numbers of runs and an illustration

A pattern recognition proposed here for a circular arrangement of several sets of samples, observed directly or after adjusting the observations, is statistically to test whether several sets of observations are drawn from the same population, *e.g.* an information source. Such a testing hypothesis depends upon a probability to obtain such an observed number of circular runs caused randomly from the same population. That is to say, this means that the proposed method is concerned with a randomness test under a null hypothesis of belonging to the similar distribution.

From (10) of section 2, critical numbers of circular runs for the respective sizes of k sample sets are tabulated at

LIST OF PROGRAM

```
ISN    STNO.   SOURCE STATEMENT

        C          A PATTERN RECOGNITION FOR CIRCULAR DISTRIBUTIONS
        C               ---  CIRCULAR MULTIPLE RUNS TEST  ---
        C
        C
        C
        C              K --- NO. OF COLORS
        C             IR --- SAMPLE SIZE FOR EACH COLOR ,
        C                       WHERE IR(1).LE.IR(2).LE. ... .LE.IR(K)
        C
        C
        C               SUBROUTINE REQUIRED --- KAIJO AND JKT
        C
        C
                 DIMENSION IR(10),IRT(10),ID(10),ITRT(10)
                 DIMENSION QKAI(10),QJUNK(30,10),REN(30),RJUN(30),COMB(1275)
                 DO 1 KAZ=1,1275
               1 COMB(KAZ)=1.0
                 DO 4 I1=2,50
                 DO 4 I2=2,I1
                 J1=I1-1
                 J2=I2-1
                 KAZ=I1-I2+1
                 DO 2 M=2,I2
               2 KAZ=KAZ+52-M
                 DO 3 I=1,J2
               3 COMB(KAZ)=COMB(KAZ)*FLOAT(J1-I+1)/FLOAT(I)
               4 CONTINUE
               5 READ(5,100,END=99) K,(IR(I),I=1,10)
                 ITR=0
                 DO 6 I=1,K
               6 ITR=ITR+IR(I)
                 III=IR(1)
                 NO=0
                 DO 8 I=1,III
                 IAA=0
                 DO 7 J=1,K
               7 IAA=IAA+MOD(IR(J),I)
                 IF(IAA.NE.0) GO TO 8
                 NO=NO+1
                 ID(NO)=I
               8 CONTINUE
                 DO 14 I=1,NO
                 NNO=NO-I+1
                 DO 9 KK=1,K
               9 IRT(KK)=IR(KK)/ID(NNO)
                 ITRT(NNO)=ITR/ID(NNO)
                 CALL KAIJO(K,ITRT(NNO),IRT,QKAI(NNO))
                 DO 10 IT=1,ITR
                 J0=MOD(IT,ID(NNO))
                 QJUNK(IT,NNO)=0.0
                 RJUN(IT)=0.0
                 REN(IT)=0.0
                 IF(J0.NE.0) GO TO 10
                 IREN=IT/ID(NNO)
              10 CALL JKT(K,ITRT(NNO),IRT,IREN,QJUNK(IT,NNO),COMB)
                 IF(I.EQ.1) GO TO 14
                 NNN=NNO+1
                 QI=0.0
                 DO 12 K1=NNN,NO
                 N1=MOD(ID(K1),ID(NNO))
                 IF(N1.NE.0) GO TO 12
                 QI=QI+QKAI(K1)
                 DO 11 IT=1,ITR
              11 REN(IT)=REN(IT)+QJUNK(IT,K1)
              12 CONTINUE
                 QKAI(NNO)=QKAI(NNO)-QI
                 DO 13 IT=1,ITR
              13 QJUNK(IT,NNO)=QJUNK(IT,NNO)-REN(IT)
              14 CONTINUE
                 TOQ=0.0
                 DO 15 I=1,NO
                 Q=ITRT(I)
                 QKAI(I)=QKAI(I)/Q
                 TOQ=TOQ+QKAI(I)
                 DO 15 IT=1,ITR
              15 RJUN(IT)=RJUN(IT)+QJUNK(IT,I)/Q
                 CHEK=0.0
                 DO 16 IT=1,ITR
                 CHEK=CHEK+RJUN(IT)
              16 REN(IT)=CHEK/TOQ
                 WRITE(6,200) K,((IR(I),I),I=1,K)
                 WRITE(6,201)
                 DO 17 I=1,ITR
              17 WRITE(6,202) I,REN(I)
                 WRITE(6,203) TOQ
                 WRITE(6,204)
                 GO TO 5
              99 STOP
             100 FORMAT(11I5)
             200 FORMAT(/,16X,'A PATTERN RECOGNITION FOR CIRCULAR DISTRIBUTIONS WIT
                1H',I3,' COLORS',//,27X,10(I2,' = R',I2,3X),//)
             201 FORMAT(1H0,25X,'NO. OF RUNS',7X,'CUMULATIVE PROBABILITIES',//)
             202 FORMAT(1H ,28X,I3,13X,E14.7)
             203 FORMAT(/,20X,'TOTAL NO. OF CIRCULAR ORDERINGS = ',E16.7,/)
             204 FORMAT(1H1)
                 END
```

LIST OF PROGRAM (continued)

```
      SUBROUTINE JKT(K,ITR,IR,IJ,SJUN,COMB)
      DIMENSION KIR(10,10),KITR(10),KIJ(10),ITLES(10),ISUMT(10)
     1,IX(10),IT(10,10),IR(10)
      DIMENSION Q(10),RJUN(10),SEKC(10),RJA(10),RJB(10),COMB(1275)
      DO 1 I=1,K
    1 KIR(I,K)=IR(I)
      KIJ(K)=IJ
      KITR(K)=ITR
      NSP=K
    2 RJUN(K)=0.0
      IF(KIJ(K).LT.K) GO TO 25
      IT(1,K)=0
    3 IT(1,K)=IT(1,K)+1
      IT(2,K)=0
    4 IT(2,K)=IT(2,K)+1
      IT(3,K)=0
    5 IT(3,K)=IT(3,K)+1
      IF(K.EQ.3) GO TO 9
      IT(4,K)=0
    6 IT(4,K)=IT(4,K)+1
      IF(K.EQ.4) GO TO 9
      IT(5,K)=0
    7 IT(5,K)=IT(5,K)+1
      IF(K.EQ.5) GO TO 9
      IT(6,K)=0
    8 IT(6,K)=IT(6,K)+1
      IF(K.EQ.6) GO TO 9
    9 ITLES(K)=0
      LL=K-1
      DO 10 I=1,LL
   10 ITLES(K)=ITLES(K)+IT(I,K)
      ISUMT(K)=ITLES(K)+IT(K,K)
      IF(ISUMT(K).NE.KIJ(K)) GO TO 21
      SE=1.0
      DO 12 I=1,K
      I1=KIR(I,K)
      J1=IT(I,K)
      IF((I1-1)*(J1-1).EQ.0) GO TO 12
      KAZ=I1-J1+1
      DO 11 M=2,J1
   11 KAZ=KAZ+52-M
      SE=SE*COMB(KAZ)
   12 CONTINUE
      SEKC(K)=SE
      RJA(K)=0.0
      IX(K)=LL-1
   13 IX(K)=IX(K)+1
      IF(IT(K,K).LT.ITLES(K)-IX(K)) GO TO 20
      I1=IX(K)+1
      J1=IT(K,K)+I1-ITLES(K)
      IF(I1.LT.J1) GO TO 20
      Q(K)=1.0
      IF((I1-1)*(J1-1).EQ.0) GO TO 15
      KAZ=I1-J1+1
      DO 14 M=2,J1
   14 KAZ=KAZ+52-M
      Q(K)=COMB(KAZ)
   15 IF(K.EQ.3) GO TO 17
      MM=K-1
      DO 16 I=1,MM
   16 KIR(I,MM)=IT(I,K)
      KITR(MM)=ITLES(K)
      KIJ(MM)=IX(K)
      K=K-1
      GO TO 2
   17 RJB(K)=0.0
      IJ=IX(K)
      IA=MOD(IJ,2)
      IF(IA.NE.0) GO TO 19
      II=0.5*IJ
      I1=IT(1,K)
      I2=IT(2,K)
      IF(I1.LT.II) GO TO 19
      IF(I2.LT.II) GO TO 19
      RJB(K)=FLOAT(ITLES(K))/FLOAT(II)
      IF(II.EQ.1) GO TO 19
      KAZ1=I1-II+1
      KAZ2=I2-II+1
      DO 18 M=2,II
      KAZ1=KAZ1+52-M
   18 KAZ2=KAZ2+52-M
      RJB(K)=RJB(K)*COMB(KAZ1)*COMB(KAZ2)
   19 RJB(K)=Q(K)*RJB(K)*FLOAT(KITR(K))/FLOAT(ITLES(K))
      RJA(K)=RJA(K)+RJB(K)
   20 IF(IX(K).LT.ITLES(K)) GO TO 13
      RJUN(K)=RJUN(K)+SEKC(K)*RJA(K)
   21 IF(K.LT.6) GO TO 22
      IF(IT(6,K).LT.KIR(6,K)) GO TO 8
   22 IF(K.LT.5) GO TO 23
      IF(IT(5,K).LT.KIR(5,K)) GO TO 7
   23 IF(K.LT.4) GO TO 24
      IF(IT(4,K).LT.KIR(4,K)) GO TO 6
   24 IF(IT(3,K).LT.KIR(3,K)) GO TO 5
      IF(IT(2,K).LT.KIR(2,K)) GO TO 4
      IF(IT(1,K).LT.KIR(1,K)) GO TO 3
   25 IF(K.EQ.NSP) GO TO 26
      MM=K
      K=K+1
      RJB(K)=RJUN(MM)
      GO TO 19
   26 SJUN=RJUN(K)
 1000 RETURN
      END
```

```
      SUBROUTINE KAIJO(K,ITOTAL,IT,Q)
      DIMENSION IT(10),R(10)
      JO=0
      I1=K-1
      DO 1 I=1,I1
    1 JO=JO+IT(I)
      Q=1.0
      DO 2 I=1,JO
    2 Q=Q*FLOAT(ITOTAL-I+1)
      SEKR=1.0
      DO 4 I=1,I1
      R(I)=1.0
      K1=IT(I)
      DO 3 L=1,K1
    3 R(I)=R(I)*FLOAT(L)
    4 SEKR=SEKR*R(I)
      Q=Q/SEKR
      RETURN
      END
```

six critical probabilities from both tails of the distribution of runs, in the cases that $r_i=1(1)10$ for $k=3$, 4 and $r_i=1(1)5$ for $k=5$, 6, where $i=1, 2, \cdots, k$.

To illustrate the use of the Table of critical values in testing for randomness of an arrangement under the null hypothesis that four distributions are identical, where $k=4$, $r_1=6$, $r_2=6$, $r_3=8$ and $r_4=8$. The observations are presented by degrees of angles as follows:

Π_1: 4, 90, 102, 189, 277, 284

Π_2: 71, 133, 145, 163, 174, 253

Π_3: 16, 23, 52, 113, 122, 265, 338, 351

Π_4: 37, 207, 218, 231, 244, 303, 315, 324,

and the arrangement is shown in Figure 2.

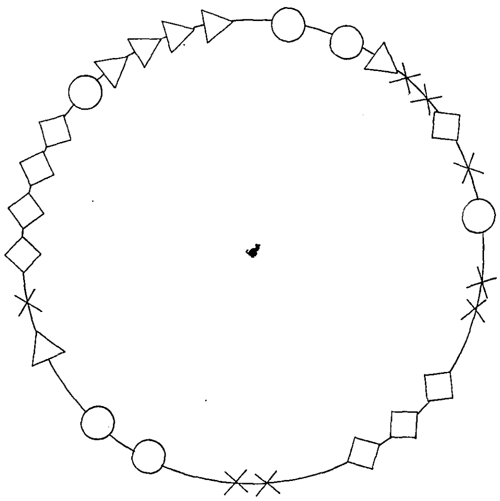

Fig. 2. An illustration of multiple circular runs test, in case of $k=4$, $r_1=6$, $r_2=6$, $r_3=8$ and $r_4=8$.

In this illustration, the number of circular runs may be easily counted as fifteen by Figure 2. Thus, from the Table of critical numbers for $r_1=r_2=6$ and $r_3=r_4=8$, it is shown for $\alpha=0.05$ that

$$\Pr\{16 \leqq t \leqq 26\} \geqq 1-0.05 \tag{11}$$

or

$$\Pr\{t \leqq 25\} \geqq 1-0.05 \tag{12}$$

The difference between (11) and (12) depends only upon an alternative hypothesis, which should be given *a priori* by the physical situation of research. Therefore, in the present illustration, the observed arrangement is significant in (11) for current two-sided test. If the test is concerned with a one-sided test, the observed difference of arrangements is non-significant in the situation of (12).

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Table I. Critical Values for the Number of Circular Runs Test, $k=3$

K=3 Sample size			PERCENT POINTS Lower side						Upper side					
R1	R2	R3	0.1	0.5	1.0	2.5	5.0	10.	10.	5.0	2.5	1.0	0.5	0.1
2	2	2	•	•	•	•	•	•	6	6	6	6	6	6
2	2	3	•	•	•	•	•	3	7	7	7	7	7	7
2	2	4	•	•	•	•	3	3	7	7	8	8	8	8
2	2	5	•	•	•	3	3	4	7	8	8	8	8	8
2	2	6	•	•	•	3	3	4	8	8	8	8	8	8
2	2	7	•	•	•	3	3	4	8	8	8	8	8	8
2	2	8	•	•	3	3	4	4	8	8	8	8	8	8
2	2	9	•	•	3	3	4	4	8	8	8	8	8	8
2	2	10	•	3	3	3	4	4	8	8	8	8	8	8
2	3	3	•	•	•	•	3	4	7	8	8	8	8	8
2	3	4	•	•	•	3	3	4	8	8	9	9	9	9
2	3	5	•	•	3	3	4	4	9	9	9	9	10	10
2	3	6	•	3	3	4	4	4	9	9	9	10	10	10
2	3	7	•	3	3	4	4	5	9	9	10	10	10	10
2	3	8	•	3	3	4	4	5	9	10	10	10	10	10
2	3	9	3	3	4	4	5	5	9	10	10	10	10	10
2	3	10	3	3	4	4	5	5	10	10	10	10	10	10
2	4	4	•	•	3	3	4	4	9	9	10	10	10	10
2	4	5	•	3	3	4	4	5	9	10	10	11	11	11
2	4	6	•	3	4	4	5	5	10	10	11	11	11	12
2	4	7	3	3	4	4	5	5	10	11	11	11	12	12
2	4	8	3	4	4	5	5	6	11	11	11	12	12	12
2	4	9	3	4	4	5	5	6	11	11	12	12	12	12
2	4	10	3	4	4	5	6	6	11	11	12	12	12	12
2	5	5	3	3	4	4	5	5	10	11	11	11	12	12
2	5	6	3	4	4	5	5	6	11	11	12	12	12	13
2	5	7	3	4	4	5	5	6	11	12	12	13	13	13
2	5	8	3	4	5	5	6	6	12	12	13	13	13	14
2	5	9	4	4	5	5	6	7	12	12	13	13	14	14
2	5	10	4	5	5	6	6	7	12	13	13	13	14	14
2	6	6	3	4	4	5	5	6	11	12	12	13	13	14
2	6	7	4	4	5	5	6	6	12	12	13	13	14	14
2	6	8	4	5	5	6	6	7	12	13	13	14	14	15
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10	10	10	11	12	13	14	15	16	23	24	25	25	26	27

Table. Ⅱ. Critical Values for the Number of Circular Runs Test, $k=4$

K=4				PERCENT POINTS											
Sample size				Lower side						Upper side					
R1	R2	R3	R4	0.1	0.5	1.0	2.5	5.0	10.	10.	5.0	2.5	1.0	0.5	0.1
2	2	2	2	•	•	•	4	4	5	8	8	8	8	8	8
2	2	2	3	•	•	4	4	5	5	9	9	9	9	9	9
2	2	2	4	•	4	4	5	5	5	9	10	10	10	10	10
2	2	2	5	•	4	4	5	5	6	10	10	10	11	11	11
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2	5	6	8	9	9	10	11	11	12	18	19	19	20	20	20
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2	5	6	10	9	10	10	11	11	12	19	20	20	21	21	22
2	5	7	7	9	10	10	11	11	12	18	19	19	20	20	21
2	5	7	8	9	10	11	11	12	13	19	19	20	20	21	21
2	5	7	9	10	10	11	12	12	12	19	20	21	21	21	22
2	5	7	10	10	11	11	12	13	14	20	21	21	22	22	23
2	5	8	8	10	11	11	12	13	13	19	20	21	21	22	22
2	5	8	9	10	11	12	12	13	14	20	21	21	22	22	23
2	5	8	10	10	11	12	13	13	14	21	21	22	23	23	24
2	5	9	9	10	11	12	13	14	14	21	21	22	23	23	24
2	5	9	10	11	12	12	13	14	15	21	22	23	23	24	24
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2	6	6	6	8	9	10	10	11	12	17	18	19	19	19	20
2	6	6	7	9	10	10	11	12	12	18	19	19	20	20	21
2	6	6	8	9	10	11	11	12	13	19	19	20	21	21	21
2	6	6	9	10	11	11	12	13	13	19	20	21	21	22	22
2	6	6	10	10	11	11	12	13	14	20	21	21	22	22	23
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2	6	7	8	10	11	11	12	13	13	20	20	21	21	22	22
2	6	7	9	10	11	12	12	13	14	20	21	21	22	22	23
2	6	7	10	11	12	12	13	14	14	21	21	22	23	23	24
2	6	8	8	10	11	12	13	13	14	20	21	22	22	23	23
2	6	8	9	11	12	12	13	14	15	21	22	22	23	23	24
2	6	8	10	11	12	13	14	14	15	22	22	23	24	24	25
2	6	9	9	11	12	13	14	14	15	22	22	23	24	24	25
2	6	9	10	12	13	13	14	15	16	22	23	24	24	25	25
2	6	10	10	12	13	14	15	15	16	23	24	24	25	25	26
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2	7	7	8	10	11	12	13	13	14	20	21	22	22	23	23
2	7	7	9	11	12	12	13	14	15	21	22	22	23	23	24
2	7	9	9	12	13	13	14	15	16	23	23	24	25	25	26
2	7	7	10	11	12	13	14	14	15	22	22	23	24	24	25
2	7	8	8	11	12	12	13	14	15	21	22	22	23	23	24
2	7	8	9	11	12	13	14	14	15	22	23	23	24	24	25
2	7	8	10	12	13	13	14	15	16	22	23	24	24	25	26
2	7	9	10	12	13	14	15	16	16	23	24	25	25	26	26
2	7	10	10	13	14	14	15	16	17	24	25	25	26	26	27
2	8	8	8	11	12	13	14	15	15	22	23	23	24	24	25
2	8	8	9	12	13	13	14	15	16	23	23	24	25	25	26
2	8	8	10	12	13	14	14	15	16	23	24	25	25	26	26
2	8	9	9	12	13	14	15	16	17	23	24	25	25	26	27
2	8	9	10	13	14	15	15	16	17	24	28	25	26	26	27
2	8	10	10	13	14	15	16	17	18	25	26	26	27	27	28
2	9	9	9	13	14	15	16	16	17	24	25	26	26	27	27
2	9	9	10	13	14	18	16	17	18	25	26	26	27	27	29
2	9	10	10	14	15	16	17	17	18	26	26	27	28	28	29
2	10	10	10	15	16	16	17	18	19	26	27	28	29	29	30
3	3	3	3	4	5	5	6	7	7	11	12	12	12	12	12

Table II (continued). Critical Values for the Number of Circular Runs Test, $k=4$

K=4				PERCENT POINTS											
Sample size				Lower side						Upper side					
R1	R2	R3	R4	0.1	0.5	1.0	2.5	5.0	10.	10.	5.0	2.5	1.0	0.5	0.1
3	3	3	4	5	6	6	7	7	8	12	13	13	13	13	13
3	3	3	5	5	6	6	7	8	8	13	13	14	14	14	14
3	3	3	6	6	6	7	7	8	9	13	14	14	15	15	15
3	3	3	7	6	7	7	8	8	9	14	14	15	15	15	16
3	3	3	8	6	7	7	8	9	9	14	15	15	16	16	16
3	3	3	9	7	7	8	8	9	10	15	15	16	16	16	17
3	3	3	10	7	8	8	9	9	10	15	15	16	16	17	17
3	3	4	4	5	6	7	7	8	8	13	13	14	14	14	14
3	3	4	5	6	7	7	8	8	9	14	14	15	15	15	15
3	3	4	6	6	7	7	8	9	9	14	15	15	16	16	16
3	3	4	7	7	7	8	9	9	10	15	15	16	16	16	17
3	3	4	8	7	8	8	9	9	10	15	16	16	17	17	17
3	3	4	9	7	8	9	9	10	10	16	16	17	17	17	18
3	3	4	10	8	8	9	10	10	11	16	17	17	18	18	18
3	3	5	5	6	7	8	8	9	9	15	15	15	16	16	16
3	3	5	6	7	8	8	9	9	10	15	16	16	16	17	17
3	3	5	7	7	8	8	9	10	10	16	16	17	17	17	18
3	3	5	8	8	8	9	10	10	11	16	17	17	18	18	18
3	3	5	9	8	9	9	10	11	11	17	17	18	18	19	19
3	3	5	10	8	9	10	10	11	12	17	18	18	19	19	19
3	3	6	6	7	8	9	9	10	11	16	16	17	17	18	18
3	3	6	7	8	9	9	10	10	11	17	17	18	18	18	19
3	3	6	8	8	9	9	10	11	11	17	18	18	19	19	19
3	3	6	9	8	9	10	11	11	12	18	18	19	19	20	20
3	3	6	10	9	10	10	11	12	12	18	19	19	20	20	21
3	3	7	7	8	9	10	10	11	12	17	18	18	19	19	20
3	3	7	8	9	9	10	11	11	12	18	18	19	19	20	20
3	3	7	9	9	10	10	11	12	13	18	19	20	20	20	21
3	3	7	10	9	10	11	11	12	13	19	20	20	21	21	22
3	3	8	8	9	10	10	11	12	13	19	19	20	20	21	21
3	3	8	9	9	10	11	12	12	13	19	20	20	21	21	22
3	3	8	10	10	11	11	12	13	14	20	20	21	22	22	23
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3	3	9	10	10	11	12	13	13	14	20	21	22	22	23	23
3	3	10	10	11	12	12	13	14	15	21	22	22	23	23	24
3	4	4	4	6	7	7	8	8	9	14	14	15	15	15	15
3	4	4	5	6	7	8	8	9	10	15	15	15	16	16	16
3	4	4	6	7	8	8	9	9	10	15	16	16	17	17	17
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3	4	4	10	8	9	10	10	11	12	17	18	18	19	19	20
3	4	5	5	7	8	8	9	10	10	15	16	16	17	17	17
3	4	5	6	7	8	9	9	10	11	16	17	17	17	18	18
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3	4	5	10	9	10	10	11	12	12	18	19	19	20	20	21
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3	4	6	7	8	9	10	11	11	12	18	18	19	19	19	20
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3	4	6	9	9	10	11	11	12	13	19	19	20	20	21	21
3	4	6	10	10	11	11	12	12	13	19	20	20	21	21	22
3	4	7	7	9	10	10	11	12	12	18	19	19	20	20	21
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3	4	7	9	10	11	11	12	13	13	19	20	21	21	22	22
3	4	7	10	10	11	12	12	13	14	20	21	21	22	22	23
3	4	8	8	10	11	11	12	13	14	20	20	21	21	22	22
3	4	8	9	10	11	12	13	13	14	20	21	21	22	22	23
3	4	8	10	11	12	12	13	14	14	21	21	22	23	23	24
3	4	9	9	11	12	12	13	14	15	21	22	22	23	23	24
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3	5	6	9	10	11	11	12	13	14	20	20	21	21	22	22
3	5	6	10	10	11	12	13	13	14	20	21	21	22	22	23
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3	5	7	8	10	11	12	12	13	14	20	20	21	22	22	22
3	5	7	9	11	11	12	13	13	14	20	21	22	22	23	23
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3	5	8	8	11	12	12	13	14	14	21	21	22	22	23	23
3	5	8	9	11	12	13	13	14	15	21	22	22	23	23	24
3	5	8	10	11	12	13	14	15	15	22	22	23	24	24	25
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3	5	10	10	12	13	14	15	16	16	23	24	25	25	26	26
3	6	6	6	9	10	11	11	12	13	19	19	20	20	20	21
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3	6	6	8	10	11	12	12	13	14	20	21	21	22	22	23
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3	6	6	10	11	12	13	13	14	15	21	22	22	23	23	24
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3	6	7	8	11	12	12	13	14	15	21	21	22	22	23	23
3	6	7	9	11	12	13	14	14	15	21	22	23	23	24	24
3	6	7	10	12	13	13	14	15	16	22	23	23	24	24	25
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3	6	8	9	12	13	13	14	15	16	22	23	23	24	24	25
3	6	8	10	12	13	14	15	15	16	23	23	24	25	25	26
3	6	9	9	12	13	14	15	15	16	23	24	24	25	25	26
3	6	9	10	13	14	14	15	16	17	23	24	25	25	26	27
3	6	10	10	13	14	15	16	16	17	24	25	26	26	27	27
3	7	7	7	11	12	12	13	14	15	21	21	22	23	23	23
3	7	7	8	11	12	13	14	14	15	22	22	23	23	24	24
3	7	7	9	12	13	13	14	15	16	22	23	23	24	24	25
3	7	7	10	12	13	14	15	15	16	23	24	24	25	25	26
3	7	8	8	12	13	13	14	15	16	22	23	24	24	24	25
3	7	8	9	12	13	14	15	16	16	23	24	24	25	25	26
3	7	8	10	13	14	14	15	16	17	24	24	25	26	26	27
3	7	9	9	13	14	14	15	16	17	24	24	25	26	26	27
3	7	9	10	13	14	15	16	17	18	24	25	26	26	27	27
3	7	10	10	14	15	16	16	17	18	25	26	26	27	28	28
3	8	8	8	12	13	14	15	16	16	23	24	24	25	25	26
3	8	8	9	13	14	15	15	16	17	24	24	25	26	26	27
3	8	8	10	13	14	15	16	17	18	24	25	26	26	27	28
3	8	9	9	13	14	15	16	17	18	25	25	26	27	27	28
3	8	9	10	14	15	16	17	17	18	25	26	27	27	28	28
3	8	10	10	15	16	16	17	18	19	26	27	27	28	28	29
3	9	9	9	14	15	16	17	17	18	25	26	27	27	28	28
3	9	9	10	15	16	16	17	18	19	26	27	27	28	29	29
3	9	10	10	15	16	17	18	19	20	27	28	28	29	29	30
3	10	10	10	16	17	17	18	19	20	27	28	29	30	30	31
4	4	4	4	7	7	8	8	9	10	15	15	16	16	16	16
4	4	4	5	7	8	8	9	10	10	16	16	17	17	17	17
4	4	4	6	8	8	9	10	10	11	16	17	17	18	18	18
4	4	4	7	8	9	9	10	11	11	17	17	18	18	18	19
4	4	4	8	8	9	10	10	11	12	17	18	19	19	19	20
4	4	4	9	9	10	10	11	12	12	18	18	19	19	20	20
4	4	4	10	9	10	10	11	12	13	18	19	19	20	20	21
4	4	5	5	8	9	9	10	10	11	16	17	17	18	18	18
4	4	5	6	8	9	10	10	11	12	17	18	18	18	19	19
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4	4	6	6	9	10	10	11	11	12	18	18	19	19	20	20
4	4	6	7	9	10	11	11	12	13	19	19	20	20	20	21
4	4	6	8	10	11	11	12	13	13	19	20	20	21	21	22
4	4	6	9	10	11	12	12	13	14	20	20	21	21	22	22
4	4	6	10	10	11	12	13	13	14	20	21	21	22	22	23
4	4	7	7	10	11	11	12	13	15	19	20	20	21	21	22
4	4	7	8	10	11	12	12	13	14	20	21	21	22	22	22
4	4	7	9	11	12	12	13	14	14	21	21	22	22	23	23
4	4	7	10	11	12	13	13	14	15	21	22	22	23	23	24
4	4	8	8	11	12	12	13	14	14	21	21	22	22	23	23
4	4	8	9	11	12	13	13	14	15	21	22	22	23	23	24
4	4	8	10	12	13	13	14	15	15	22	23	23	24	24	25
4	4	9	9	12	13	13	14	15	16	22	23	23	24	24	25
4	4	9	10	12	13	14	14	15	16	23	23	24	25	25	26
4	4	10	10	13	14	14	15	16	17	23	24	25	25	26	26
4	5	5	5	8	9	10	10	11	12	17	18	18	19	19	19
4	5	5	6	9	10	10	11	12	12	18	18	19	19	20	20
4	5	5	7	9	10	11	11	12	13	19	19	20	20	20	21
4	5	5	8	10	11	11	12	13	13	19	20	20	21	21	22
4	5	5	9	10	11	12	12	13	14	20	20	21	21	22	22
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4	5	6	6	9	10	11	12	12	13	19	19	20	20	20	21
4	5	6	7	10	11	11	12	13	14	19	20	20	21	21	22
4	5	6	8	10	11	12	13	13	14	20	21	21	22	22	23
4	5	6	9	11	12	12	13	14	15	21	21	22	22	23	23
4	5	6	10	11	12	13	14	14	15	21	22	22	23	23	24
4	5	7	7	10	11	12	13	13	14	20	21	21	22	22	23
4	5	7	8	11	12	12	13	14	15	21	22	22	23	23	23
4	5	7	9	11	12	13	14	14	15	21	22	23	23	24	24
4	5	7	10	12	13	13	14	15	15	22	23	23	24	24	25
4	5	8	8	11	12	13	14	14	15	22	22	23	23	24	24
4	5	8	9	12	13	13	14	15	16	22	23	23	24	24	25
4	5	8	10	12	13	14	15	16	16	23	24	24	25	25	26
4	5	9	9	12	13	14	15	16	16	23	24	24	25	25	26

Table Ⅱ (continued). Critical Values for the Number of Circular Runs Test, $k=4$

K=4				PERCENT POINTS											
Sample size				Lower side						Upper side					
R1	R2	R3	R4	0.1	0.5	1.0	2.5	5.0	10.	10.	5.0	2.5	1.0	0.5	0.1
4	5	9	10	13	14	14	15	16	17	24	24	25	26	26	27
4	5	10	10	13	14	15	16	17	18	24	25	26	26	27	27
4	6	6	6	10	11	11	12	13	14	20	20	21	21	21	22
4	6	6	7	10	11	12	13	13	14	20	21	21	22	22	23
4	6	6	8	11	12	12	13	14	15	21	22	22	23	23	23
4	6	6	9	11	12	13	14	15	15	22	22	23	23	24	24
4	6	6	10	12	13	13	14	15	16	22	23	23	24	24	25
4	6	7	7	11	12	13	13	14	15	21	22	22	23	23	24
4	6	7	8	12	13	13	14	15	15	22	22	23	24	24	24
4	6	7	9	12	13	14	14	15	16	22	23	24	24	25	25
4	6	7	10	12	13	14	15	16	17	23	24	24	25	25	26
4	6	8	8	12	13	14	15	15	16	22	23	24	24	25	25
4	6	8	9	13	14	14	15	16	17	23	24	24	25	25	25
4	6	8	10	13	14	15	16	16	17	24	24	25	26	26	27
4	6	9	9	13	14	15	16	16	17	24	25	25	26	26	27
4	6	9	10	14	15	15	16	17	18	25	25	26	27	27	28
4	6	10	10	14	15	16	17	17	18	25	26	27	27	28	28
4	7	7	7	12	13	13	14	15	16	22	22	23	24	24	24
4	7	7	8	12	13	14	15	15	16	23	23	24	24	25	25
4	7	7	9	13	14	14	15	16	17	23	24	24	25	25	26
4	7	7	10	13	14	15	16	16	17	24	25	25	26	26	27
4	7	8	8	13	14	14	15	16	16	23	24	25	25	26	26
4	7	8	9	13	14	15	16	17	17	24	25	25	26	26	27
4	7	8	10	14	15	15	16	17	18	25	25	26	27	27	28
4	7	9	9	14	15	15	16	17	18	25	25	26	27	27	28
4	7	9	10	14	15	16	17	18	19	25	26	27	27	28	29
4	7	10	10	15	16	17	17	18	19	26	27	28	28	29	29
4	8	8	8	13	14	15	16	17	17	24	25	25	26	26	27
4	8	8	9	14	15	16	16	17	18	25	26	26	27	27	28
4	8	8	10	14	15	16	17	18	19	26	26	27	28	28	29
4	8	9	9	14	15	16	17	18	19	26	26	27	28	28	29
4	8	9	10	15	16	17	18	18	19	26	27	28	28	29	29
4	8	10	10	15	17	17	18	19	20	27	28	28	29	30	30
4	9	9	9	15	16	17	18	18	19	26	27	28	28	29	30
4	9	9	10	16	17	17	18	19	20	27	28	28	29	30	30
4	9	10	10	16	17	18	19	20	21	28	29	29	30	30	31
4	10	10	10	17	18	18	19	20	29	29	30	31	31	32	32
5	5	5	5	9	10	10	11	12	12	18	19	19	19	20	20
5	5	5	6	9	10	11	12	12	13	19	19	20	20	21	21
5	5	5	7	10	11	11	12	13	14	20	20	21	21	21	22
5	5	5	8	10	11	12	13	13	14	20	21	21	22	22	23
5	5	5	9	11	12	12	13	14	15	21	21	22	22	23	23
5	5	5	10	11	12	13	14	14	15	21	22	22	23	23	24
5	5	6	6	10	11	12	12	13	14	20	20	21	21	21	22
5	5	6	7	11	12	12	13	14	14	20	21	21	22	22	23
5	5	6	8	11	12	13	13	14	15	21	22	22	23	23	24
5	5	6	9	12	13	13	14	15	15	22	22	23	23	24	24
5	5	6	10	12	13	14	14	15	16	22	23	23	24	24	25
5	5	7	7	11	12	13	13	14	15	21	22	22	23	23	24
5	5	7	8	12	13	13	14	15	16	22	22	23	24	24	24
5	5	7	9	12	13	14	15	15	16	22	23	24	24	25	25
5	5	7	10	13	14	14	15	16	17	23	24	24	25	25	26
5	5	8	8	12	13	14	15	15	16	23	23	24	24	25	25
5	5	8	9	13	14	14	15	16	17	23	24	24	25	25	26
5	5	8	10	13	14	15	16	16	17	24	25	25	26	26	27
5	5	9	9	13	14	15	16	16	17	24	25	25	26	26	27
5	5	9	10	14	15	15	16	17	18	25	25	26	27	27	28
5	5	10	10	14	15	16	17	18	18	25	26	27	27	28	28
5	6	6	6	11	12	12	13	14	14	20	21	22	22	22	23
5	6	6	7	11	12	13	14	14	15	21	22	22	23	23	24
5	6	6	8	12	13	13	14	15	16	22	23	23	24	24	24
5	6	6	9	12	13	14	15	15	16	23	23	24	24	25	25
5	6	6	10	13	14	14	15	16	17	23	24	24	25	25	26
5	6	7	7	12	13	13	14	15	16	22	23	23	24	24	25
5	6	7	8	12	13	14	15	15	16	23	23	24	24	25	25
5	6	7	9	13	14	14	15	16	17	23	24	25	25	26	26
5	6	7	10	13	14	15	16	17	17	24	25	25	26	26	27
5	6	8	8	13	14	15	15	16	17	23	24	25	25	26	26
5	6	8	9	13	14	15	16	17	18	24	25	25	26	26	27
5	6	8	10	14	15	16	16	17	18	25	26	26	27	27	28
5	6	9	9	14	15	16	17	17	18	25	26	26	27	27	28
5	6	9	10	14	15	15	16	17	18	26	29	27	28	28	29
5	6	10	10	15	16	17	18	18	19	26	27	28	28	29	29
5	7	7	7	12	13	14	15	16	16	23	23	24	25	25	25
5	7	7	8	13	14	15	15	16	17	24	24	25	25	26	26
5	7	7	9	13	14	15	16	17	18	24	25	25	26	26	27
5	7	7	10	14	15	16	17	17	18	25	26	26	27	27	28
5	7	8	8	14	15	15	16	17	18	24	25	26	26	27	27
5	7	8	9	14	15	16	17	17	18	25	26	26	27	27	28
5	7	8	10	15	16	16	17	18	19	26	26	27	28	28	29
5	7	9	9	15	16	16	17	18	19	26	26	27	28	28	29

K=4				PERCENT POINTS											
Sample size				Lower side						Upper side					
R1	R2	R3	R4	0.1	0.5	1.0	2.5	5.0	10.	10.	5.0	2.5	1.0	0.5	0.1
5	7	9	10	15	16	17	18	19	20	26	27	28	28	29	30
5	7	10	10	16	17	17	18	19	20	27	28	29	29	30	30
5	8	8	8	14	15	16	17	17	18	25	26	26	27	27	28
5	8	8	9	15	16	16	17	18	19	29	27	27	28	28	29
5	8	8	10	15	16	17	18	19	20	27	27	28	29	29	30
5	8	9	9	15	16	17	18	19	20	27	27	28	29	29	30
5	8	9	10	16	17	18	19	19	20	27	28	29	29	30	31
5	8	10	10	16	17	18	19	20	21	28	29	29	30	31	31
5	9	9	9	16	17	18	19	19	20	27	28	29	29	30	31
5	9	9	10	16	18	18	19	20	21	28	29	30	30	31	31
5	9	10	10	17	18	19	20	21	22	29	30	30	31	31	32
5	10	10	10	18	19	19	20	21	22	30	30	31	32	32	33
6	6	6	6	11	12	13	14	14	15	21	22	22	23	23	24
6	6	6	7	12	13	13	14	15	16	22	23	23	24	24	25
6	6	6	8	12	13	14	15	16	16	23	23	24	25	25	25
6	6	6	9	13	14	15	15	16	17	23	24	25	25	26	26
6	6	6	10	13	14	15	16	17	18	24	25	25	26	26	27
6	6	7	7	12	13	14	15	16	17	23	24	24	25	25	26
6	6	7	8	13	14	15	16	16	17	24	24	25	25	26	26
6	6	7	9	14	15	15	16	17	18	24	25	26	26	27	27
6	6	7	10	14	15	16	17	17	18	25	26	26	27	27	28
6	6	8	8	14	15	15	16	17	18	24	25	26	26	27	27
6	6	8	9	14	15	16	17	18	18	25	26	26	27	27	28
6	6	8	10	15	16	16	17	18	19	26	26	27	28	28	29
6	6	9	9	15	16	16	17	18	19	26	27	27	28	28	29
6	6	9	10	15	16	17	18	19	20	27	27	28	29	29	30
6	6	10	10	16	17	18	18	19	20	27	28	29	29	30	30
6	7	7	7	13	14	15	16	16	17	24	24	25	25	26	26
6	7	7	8	14	15	15	16	17	18	24	25	26	26	27	27
6	7	7	9	14	15	16	17	18	18	25	26	26	27	27	28
6	7	7	10	15	16	16	17	18	19	26	27	27	28	28	29
6	7	8	8	14	15	16	17	18	19	25	26	26	27	27	28
6	7	8	9	15	16	17	17	18	19	26	27	27	28	28	29
6	7	8	10	15	16	17	18	19	20	27	27	28	29	29	30
6	7	9	9	15	16	17	18	19	20	27	27	28	29	29	30
6	7	9	10	16	17	18	19	19	20	27	28	29	29	30	31
6	7	10	10	17	18	18	19	20	21	28	29	30	30	31	31
6	8	8	8	15	16	17	18	18	19	26	27	27	28	28	29
6	8	8	9	15	17	17	18	19	20	27	28	28	29	29	30
6	8	8	10	16	17	18	19	20	20	27	28	29	30	30	31
6	8	9	9	16	17	18	19	20	21	28	28	29	30	30	31
6	8	9	10	17	18	18	19	20	21	28	29	30	30	31	31
6	8	10	10	17	18	19	20	21	22	29	30	30	31	32	32
6	9	9	9	17	18	18	19	20	21	28	29	30	30	31	32
6	9	9	10	17	18	19	20	21	22	29	30	30	31	32	32
6	9	10	10	18	19	20	21	22	22	30	31	31	32	32	33
6	10	10	10	18	20	20	21	22	23	31	31	32	33	33	34
7	7	7	7	14	15	15	16	17	18	25	25	26	26	27	27
7	7	7	8	14	15	16	17	18	19	25	26	27	27	28	28
7	7	7	9	15	16	17	18	19	19	26	27	27	28	28	29
7	7	7	10	15	16	17	18	19	20	27	27	28	29	29	30
7	7	8	8	15	16	17	18	18	19	26	27	27	28	28	29
7	7	8	9	16	17	17	18	19	20	27	28	28	29	29	30
7	7	8	10	16	17	18	19	20	21	28	28	29	30	30	31
7	7	9	9	16	17	18	19	20	21	28	28	29	30	30	31
7	7	9	10	17	18	18	19	20	21	28	29	30	30	31	32
7	7	10	10	17	18	19	20	21	22	29	30	30	31	32	32
7	8	8	8	16	17	17	18	19	20	27	28	28	29	29	30
7	8	8	9	16	17	18	19	20	21	28	28	29	30	30	31
7	8	8	10	17	18	19	19	20	21	28	29	30	30	31	32
7	8	9	9	17	18	19	20	20	21	28	29	30	31	31	32
7	8	9	10	17	18	19	20	21	22	29	30	31	31	32	32
7	8	10	10	18	19	20	21	22	23	30	31	31	32	33	33
7	9	9	9	17	19	19	20	21	22	29	30	31	31	32	33
7	9	9	10	18	19	20	21	22	23	30	31	31	32	33	33
7	9	10	10	19	20	20	21	22	23	31	32	32	33	33	34
7	10	10	10	19	20	21	22	23	24	32	32	33	34	34	35
8	8	8	8	16	17	18	19	20	21	28	28	29	30	30	31
8	8	8	9	17	18	19	20	20	21	29	29	30	31	31	32
8	8	8	10	17	19	19	20	21	22	29	30	31	31	32	33
8	8	9	9	17	19	19	29	21	22	29	30	31	31	32	32
8	8	9	10	18	19	20	21	22	23	30	31	31	32	33	33
8	8	10	10	19	20	21	22	22	23	31	32	32	33	33	34
8	9	9	9	18	19	20	21	22	23	30	31	32	32	33	33
8	9	9	10	19	20	21	22	22	23	31	32	32	33	34	34
8	9	10	10	19	21	21	22	23	24	32	32	33	34	34	35
8	10	10	10	20	21	22	23	24	25	32	33	34	35	35	36
9	9	9	9	19	20	21	22	23	24	31	32	32	33	34	34
9	9	9	10	19	21	21	22	23	24	32	33	33	34	34	35
9	9	10	10	20	21	22	23	24	25	32	33	34	35	35	36
9	10	10	10	21	22	23	24	25	26	33	34	35	36	36	37
10	10	10	10	21	22	23	24	25	26	33	34	35	36	36	37

Table Ⅲ. Critical values for the Number of the Circular Runs Test, k=5, 6

K=5 — PERCENT POINTS

Sample size R1	R2	R3	R4	R5	Lower side 0.1	0.5	1.0	2.5	5.0	1.0	Upper side 1.0	5.0	2.5	1.0	0.5	0.1
2	2	2	2	2	•	5	5	6	6	7	10	10	10	10	10	10
2	2	2	2	3	5	5	6	6	7	7	11	11	11	11	11	11
2	2	2	2	4	5	6	6	7	7	8	12	12	12	12	12	12
2	2	2	2	5	6	6	7	7	8	8	12	12	13	13	13	13
2	2	2	3	3	5	6	6	7	7	8	12	12	12	12	12	12
2	2	2	3	4	6	6	7	7	8	8	13	13	13	13	13	13
2	2	2	3	5	6	7	7	8	8	9	13	13	14	14	14	14
2	2	2	4	4	6	7	7	8	8	9	13	14	14	14	14	14
2	2	2	4	5	7	7	8	8	9	9	14	14	15	15	15	15
2	2	2	5	5	7	8	8	9	9	10	15	15	16	16	16	16
2	2	3	3	3	6	7	7	8	8	9	13	13	13	13	13	13
2	2	3	3	4	6	7	8	8	9	9	13	14	14	14	14	14
2	2	3	3	5	7	8	8	9	9	10	14	14	15	15	15	15
2	2	3	4	4	7	8	8	9	9	10	14	15	15	15	15	15
2	2	3	4	5	7	8	9	9	10	10	15	15	16	16	16	16
2	2	3	5	5	8	9	9	10	10	11	16	16	17	17	17	17
2	2	4	4	4	8	8	9	9	10	10	15	16	16	16	16	16
2	2	4	4	5	8	9	9	10	10	11	16	16	17	17	17	17
2	2	4	5	5	9	9	10	10	11	12	17	17	17	18	18	18
2	2	5	5	5	9	10	10	11	12	12	17	18	18	19	19	19
2	3	3	3	3	7	7	8	8	9	9	14	14	14	14	14	14
2	3	3	3	4	7	8	8	9	9	10	14	15	15	15	15	15
2	3	3	3	5	8	8	9	9	10	10	15	15	16	16	16	16
2	3	3	4	4	8	8	9	9	10	11	15	16	16	16	16	16
2	3	3	4	5	8	9	9	10	11	11	16	16	17	17	17	17
2	3	3	5	5	9	9	10	11	11	12	17	17	18	18	18	18
2	3	4	4	4	8	9	9	10	11	11	16	16	17	17	17	17
2	3	4	4	5	9	10	10	11	11	12	17	17	18	18	18	18
2	3	4	5	5	9	10	11	11	12	13	18	18	18	19	19	19
2	3	5	5	5	10	11	11	12	13	13	18	19	19	20	20	20
2	4	4	4	4	9	10	10	11	11	12	17	17	18	18	18	18
2	4	4	4	5	9	10	11	11	12	13	18	18	19	19	19	19
2	4	4	5	5	10	11	11	12	13	13	19	19	19	20	20	20
2	4	5	5	5	11	12	12	13	13	14	19	20	20	21	21	21
2	5	5	5	5	11	12	13	13	14	15	20	21	21	22	22	22
3	3	3	3	3	7	8	8	9	9	10	14	15	15	15	15	15
3	3	3	3	4	8	9	9	10	10	11	15	16	16	16	16	16
3	3	3	3	5	8	9	10	10	11	11	16	16	17	17	17	17
3	3	3	4	4	8	9	10	10	11	11	16	17	17	17	17	17
3	3	3	4	5	9	10	10	11	11	12	17	17	18	18	18	18
3	3	3	5	5	9	10	11	11	12	13	18	18	19	19	19	19
3	3	4	4	4	9	10	10	11	12	12	17	17	18	18	18	18
3	3	4	4	5	10	10	11	12	12	13	18	18	19	19	19	19
3	3	4	5	5	10	11	11	12	13	13	19	19	19	20	20	20
3	3	5	5	5	11	12	12	13	13	14	19	20	20	21	21	21
3	4	4	4	4	10	11	11	12	12	13	18	19	19	19	19	19
3	4	4	4	5	10	11	12	12	13	14	19	19	20	20	20	20
3	4	4	5	5	11	12	12	13	14	14	20	20	20	21	21	21
3	4	5	5	5	12	12	13	14	14	15	20	21	21	22	22	22
3	5	5	5	5	12	13	14	14	15	16	21	22	22	23	23	23
4	4	4	4	4	10	11	12	12	13	14	19	19	20	20	20	20
4	4	4	4	5	11	12	12	13	14	14	20	20	20	21	21	21
4	4	4	5	5	12	13	13	14	14	15	21	21	21	22	22	22
4	4	5	5	5	12	13	14	14	15	16	21	22	22	23	23	23
4	5	5	5	5	13	14	14	15	16	17	22	23	23	24	24	24
5	5	5	5	5	14	15	15	16	17	17	23	24	24	25	25	25

K=6 — PERCENT POINTS

Sample size R1	R2	R3	R4	R5	R6	Lower side 0.1	0.5	1.0	2.5	5.0	1.0	Upper side 1.0	5.0	2.5	1.0	0.5	0.1
2	2	2	2	2	2	6	7	7	8	8	9	12	12	12	12	12	12
2	2	2	2	2	3	7	7	8	8	9	9	13	13	13	13	13	13
2	2	2	2	2	4	7	8	8	9	9	10	14	14	14	14	14	14
2	2	2	2	2	5	8	8	9	9	10	10	14	15	15	15	15	15
2	2	2	2	3	3	7	8	8	9	9	10	14	14	14	14	14	14
2	2	2	2	3	4	8	9	9	10	10	11	15	15	15	15	15	16
2	2	2	2	4	4	8	9	10	10	11	11	15	16	16	16	16	16
2	2	2	2	4	5	9	10	10	11	11	12	16	17	17	17	17	17
2	2	2	2	5	5	9	10	11	11	12	12	17	17	18	18	18	18
2	2	2	3	3	3	8	9	9	10	10	11	15	15	15	15	15	15
2	2	2	3	3	4	9	9	10	10	11	11	16	16	16	16	16	16
2	2	2	3	3	5	9	10	10	11	11	12	16	17	17	17	17	17
2	2	2	3	4	4	9	10	10	11	11	12	16	17	17	17	17	17
2	2	2	3	4	5	10	10	11	11	12	13	17	18	18	18	18	18
2	2	2	3	5	5	10	11	11	12	13	13	18	18	19	19	19	19
2	2	2	4	4	4	10	10	11	12	12	13	17	18	18	18	18	18
2	2	2	4	4	5	10	11	11	12	13	13	18	18	19	19	19	19
2	2	2	4	5	5	11	12	12	13	13	14	19	19	20	20	20	20
2	2	2	5	5	5	11	12	13	13	14	15	20	20	20	21	21	21
2	2	3	3	3	3	9	9	10	10	11	11	16	16	16	16	16	16
2	2	3	3	3	4	9	10	10	11	12	12	16	17	17	17	17	17
2	2	3	3	3	5	10	11	11	12	12	13	17	18	18	18	18	18
2	2	3	3	4	4	10	11	11	12	12	13	17	18	18	18	18	18
2	2	3	3	4	5	10	11	12	12	13	13	18	18	19	19	19	19
2	2	3	3	5	5	11	12	12	13	13	14	19	19	20	20	20	20
2	2	3	4	4	4	10	11	12	12	13	14	18	19	19	19	19	19
2	2	3	4	4	5	11	12	12	13	14	14	19	19	20	20	20	20
2	2	3	4	5	5	12	12	13	14	14	15	20	20	21	21	21	21
2	2	3	5	5	5	12	13	14	14	15	16	21	21	21	22	22	22
2	2	4	4	4	4	11	12	12	13	14	14	19	20	20	20	20	20
2	2	4	4	4	5	12	13	13	14	14	15	20	20	21	21	21	21
2	2	4	4	5	5	12	13	14	14	15	16	21	21	22	22	22	22
2	2	4	5	5	5	13	14	14	15	16	16	22	22	22	23	23	23
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