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Kinetic Theory of Surface Carrier Waves in a Semibounded Semiconductor

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(Abstract)

This paper investigates the propagation of surface carrier waves which are associated with drifting carriers in a semibounded semiconductor. The description is done on the basis of a kinetic theory under the specular reflection for the carriers at the semiconductor surface. The dispersion equation is derived and compared with that which is obtained from the hydrodynamic theory.

1. Introduction

The space charge waves associated with a drifting stream of carriers in a semiconductor are called carrier waves. Recently the surface carrier waves supported by a finite semiconductor have received a great deal of attention, mainly on account of the possibilities of constructing a new type of travelling-wave amplifiers by using an active interaction between the carrier waves and an external curcuit.

Kino[1] has discussed at great length the loss properties of the surface carrier waves with zero-temperature approximation and has refered to much of work of this problem. Sumi[2], Dean and Robinson[3], and Wang[4] have used the hydrodynamic theory for the carrier stream to study the effect of carrier temperature on the surface carrier waves. However the hydrodynamic theory, which describes the average properties of a system of carriers, does not always reflect the thermal motion of carriers properly. As in the case of surface plasma waves, a kinetic theory based on the Boltzmann equation for the carrier stream is expected to provide a more detailed information about the propagation of the surface carrier waves. The kinetic theory has been used so far by Pines and Schrieffer[5], Robinson and Swartz[6], and Kobayashi and Fujisawa[7] in work on the one-dimensional carrier waves (bulk carrier waves) in an unbounded semiconductor.

In this paper we shall present an investigation of the surface carrier waves by using the kinetic theory. To simplify the theoretical treatment, we consider an isotropic, homogeneous semiconductor bounded by a dielectric half-space and assume that the carriers are specularly reflected from the boundary surface. We can use the electrostatic approximation since the average drift velocity of the carriers and the phase velocity of the surface carrier waves are sufficiently small compared with the velocity of light. We shall derive a general expression for the dispersion equation of the surface carrier waves. In order to evaluate the dispersion equation explicitly,

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we shall make an approximation which is valid for the carrier waves of microwave range in an usual semiconductor at room temperature. An approximate dispersion equation is obtained and briefly compared with the corresponding equation which has been derived by using the hydrodynamic theory.

2. The solution of the Boltzmann equation

We consider a homogeneous and isotropic semiconductor with one species of mobile carriers, which is confined to the half-space $y>0$ and bounded by a dielectric for $y<0$ (**Fig. 1**). We assume that a

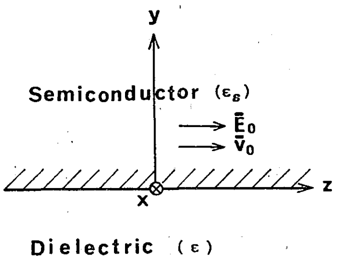

Fig. 1. Semibounded semiconductor and dielectric

static electric field E_0 is applied in the z direction and the carriers are drifting in this direction with an average velocity v_0 through a uniform background of stationary lattice ions. The dynamics of the carriers in the electrostatic approximation may be described by the linearized Boltzmann and Poisson equations:

$$\left(\frac{\partial}{\partial t}+\bar{v}\cdot\nabla+\frac{q}{m^*}E_0\frac{\partial}{\partial v_z}\right)f(\bar{r},\bar{v},t)$$

$$=\frac{q}{m^*}\bar{E}\cdot\frac{\partial}{\partial\bar{v}}f_0+\left(\frac{\partial}{\partial t}f\right)_c \tag{1}$$

$$\bar{E}=-\nabla\varphi \tag{2}$$

$$\nabla^2\varphi=\frac{\rho}{\varepsilon_s} \tag{3}$$

$$\rho=q\iiint_{-\infty}^{\infty}f(\bar{r},\bar{v},t)d^3\bar{v} \tag{4}$$

where q, m^*, and $\bar{v}$ are the charge, the effective mass, and the velocity variable of the carriers, ε_s is the permittivity of the semiconductor, $\bar{E}$ and φ are the electric field and the potential of the perturbation, f and ρ are the perturbations of the velocity distribution function and the charge density of the carriers, f_0 is the stationary velocity distribution function, respectively, and $(\partial f/\partial t)_c$ represents the collision term of the carriers.

We assume that f_0 is a displaced Maxwellian:

$$f_0=\frac{n_0}{(\sqrt{2\pi}v_T)^3}\exp\left[-\frac{(\bar{v}-v_0\hat{z})^2}{2v_T^2}\right] \tag{5}$$

where v_T and n_0 are the thermal velocity and the equilibrium number density of the carriers, and $\hat{z}$ is the unit vector in the z direction.

The simplified collision models of a relaxation type are commonly used for the collision term $(\partial f/\partial t)_c$. Following three models have been introduced in work on the one-dimensional carrier waves.

(i) Pines' model[5]

$$\left(\frac{\partial}{\partial t}f\right)_c=-\nu f \tag{6}$$

(ii) Robinson's model[6]

$$\left(\frac{\partial}{\partial t}f\right)_c=-\nu\left(f-\frac{n}{n_0}f_0\right) \tag{7}$$

(iii) Kobayashi's model[7]

$$\left(\frac{\partial}{\partial t}f\right)_c=-\nu\left(f-\frac{n}{n_0}F_0\right) \tag{8}$$

where ν and $n=\rho/q$ are the average carrier-lattice collision frequency and the perturbed number density of the carriers, and F_0 is a equilibrium Maxwellian:

$$F_0=\frac{n_0}{(\sqrt{2\pi}v_T)^3}\exp\left[-\frac{v^2}{2v_T{}^2}\right] \tag{9}$$

We discuss briefly the physical differences among these models. It is obvious that Pines' model (6) does not conserve the crriers. Robinson's model (7) conserves the carriers, but implies that the velocity distribution tends to relax toward the displaced Maxwellian (5). As was pointed out by Kobayashi and Fujisawa[7], this model would be appropriate for a plasma stream composed of a set of electrons and ions drifting with the same velocity v_0. In the present case, however, the lattice ions are stationary and the velocity distribution tends to relax toward the equilibrium Maxwelian (9) due to the carrier-lattice collision. On the other hand, Kobayashi's model (8) conserves the carriers and reflects such a relaxation process properly. Accordingly, we will adopt Kobayashi's model (8) as the collision term.

We must specify a boundary condition for the distribution function f at $y=0$. To simplify an analytic treatment, we assume an idealized boundary surface where the carriers are reflected specularly. Then the boundary condition can be expressed as follows:

$$f(v_y, y=0)=f(-v_y, y=0). \tag{10}$$

It should be noted that the boundary condition (10) can be applied in spite of the presence of a current in the semiconductor, since the average drift velocity of the carriers is parallel to the boundary surface $y=0$.

Following the method proposed by Guernsey[8], we can resolve the system (1)-(4) with (8) and (10). Let the perturbation be of the form $\exp[-i(\omega t-\beta z)]$ and independent of x, where ω and β are the angular frequency and the propagation constant of the surface carrier waves. This implies that we consider the response of the system far enough away from an initial perturbation. Bearing in mind a Fourier-Laplace transform with respect to z and t, we assume that β is real and ω is complex with a small positive imaginary part.

In order to be able to apply a Fourier transform with respect to y, we conceive the semiconductor to extend into the region $y<0$ and continue f and $\bar{E}$ specularly to this region as follows:

$$f(v_y, -y)=f(-v_y, y) \tag{11}$$

$$E_z(-y)=E_z(y) \tag{12}$$

$$E_y(-y)=-E_y(y). \tag{13}$$

By virtue of (11)-(13), the boundary condition (10) can be achieved and the Boltzmann equation (1) is satisfied for all y. We define the Fourier transform and its inverse transform by

$$\tilde{\varphi}(\zeta)=\int_{-\infty}^{\infty}\varphi(y)e^{-i\zeta y}\,dy, \tag{14}$$

$$\varphi(y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{\varphi}(\zeta)e^{i\zeta y}\,d\zeta. \tag{15}$$

Substituting the explicit collision term (8) in (1) and applying the Fourier transform to (1)-(4), we obtain the following relationships between the Fourier components:

$$\frac{q}{m^*}E_0\frac{\partial}{\partial v_z}\tilde{f}-i[(\omega+i\nu)-(\zeta v_y+\beta v_z)]\tilde{f}$$
$$=\frac{\nu F_0}{n_0 q}\tilde{\rho}-i\frac{qf_0}{m^*v_T^2}[\zeta v_y+\beta(v_z-v_0)]\tilde{\varphi}, \tag{16}$$

$$(\zeta^2+\beta^2)\tilde{\varphi}=\frac{\tilde{\rho}}{\varepsilon_s}-2\frac{\partial}{\partial y}\varphi(+0), \qquad (17)$$

$$\tilde{\rho}=q\iiint_{-\infty}^{\infty}\tilde{f}\,d^3\bar{v}, \qquad (18)$$

where $\varphi(+0)$ and $\partial\varphi(+0)/\partial y$ denote the values of $\varphi(y)$ and $\partial\varphi(y)/\partial y$ just inside the semiconductor. In the above, we have suppressed the common factor exp $[-i(\omega t-\beta z)]$. The differential equation (16) with respect to v_z can be solved in a straightforward manner under the condition that $\tilde{f}$ tends to zero as $|v_z|\to\infty$. Substituting the result in (18) and integrating over the whole range of $\bar{v}$, after somewhat tedious calculations, we obtain the following expression for the charge density $\tilde{\rho}$ (see Appendix A):

$$\frac{\tilde{\rho}}{\varepsilon_s}=\frac{\dfrac{\omega_P^2}{2\Omega^2}Z'(\alpha_2)}{1+i\dfrac{\nu Z(\alpha_1)}{\sqrt{2}\,\Omega}}(\zeta^2+\beta^2)\tilde{\varphi} \qquad (19)$$

where

$$Z(\alpha)=i2e^{-\alpha^2}\int_{-\infty}^{i\alpha}e^{-\theta^2}\,d\theta \qquad (20)$$

$$\alpha_1=\frac{\omega+i\nu}{\sqrt{2}\,\Omega} \qquad (21)$$

$$\alpha_2=\frac{\omega-\beta v_0+i\nu}{\sqrt{2}\Omega} \qquad (22)$$

$$\Omega=\left[v_T^2(\zeta^2+\beta^2)+i\frac{qE_0}{m^*}\beta\right]^{1/2}, \qquad (23)$$

$\omega_P=(n_0q^2/m^*\varepsilon_s)^{1/2}$ is the plasma frequency of the carriers, $Z(\alpha)$ is the usual plasma dispersion function[9], $Z'(\alpha)$ denotes its derivative, and Ω is defined such that the real part of Ω is positive when ζ and β are real.

Substituting (19) in (17) and applying the Fourier inverse transform (15), we can deduce the expression for φ in the semibounded ($y>0$) semiconductor as follows:

$$\varphi=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1+i\dfrac{\nu Z(\alpha_1)}{\sqrt{2}\Omega}}{(\zeta^2+\beta^2)D(\zeta)}e^{i\zeta y}\,d\zeta\frac{\partial}{\partial y}\varphi(+0) \qquad (24)$$

where

$$D(\zeta)=1+i\frac{\nu Z(\alpha_1)}{\sqrt{2}\,\Omega}-\frac{\omega_P^2}{2\Omega^2}Z'(\alpha_2). \qquad (25)$$

The value of $\varphi(+0)$ is obtained by setting $y=0$ in (24). This gives a relation between $\varphi(+0)$ and $\partial\varphi(+0)/\partial y$, which will be used to derive the dispersion equation of the surface carrier waves. However, it should be noted that the value of $\partial\varphi(+0)/\partial y$ is not correctly obtained by setting $y=0$ in the derivative of (24) with respect to y. This is because the Fourier representattion of $\partial\varphi/\partial y$ (regarded as defining a function in $-\infty<y<\infty$) is antisymmetric in y and jumps by amount of assumed surface charge in crossing the plane $y=0$.

3. The dispersion equation of the surface carrier waves

In the dielectric region ($y<0$), the Laplace equation holds and hence the potential is given by

$$\varphi=C\exp(\pm\beta y) \qquad (26)$$

where C is an arbitrary constant, the upper sign on β holds when β is positive, and the lower does when β is negative. The assumed dependence of the form exp $[-i(\omega t-\beta z)]$ has been suppressed in (26).

To determine the dispersion equation of the surface carrier waves we must specify the boundary conditions on the field vectors at $y=0$. The specular reflection condition (10) implies that there is no charge accumulation on the boundary surface. Therefore the normal displacement as well as the parallel electric field are continuous across the boundary. With the help of (26), the boundary conditions are expressed as follows:

$$\varphi(+0)=C \text{ and } \varepsilon_s\frac{\partial}{\partial y}\varphi(+0)=\varepsilon(\pm\beta)C \tag{27}$$

where ε is the permittivity of the dielectric. Setting $y=0$ in (24) and applying the boundary conditions (27), we obtain the dispersion equation of the surface carrier waves as follows:

$$\frac{\varepsilon_s}{\varepsilon}+(\pm)\frac{\beta}{\pi}\int_{-\infty}^{\infty}\frac{1+i\frac{\nu Z(\alpha_1)}{\sqrt{2}\Omega}}{(\zeta^2+\beta^2)D(\zeta)}d\zeta=0. \tag{28}$$

The dispersion equation (28) is general within the framework of the electrostatic treatment. It is noted that the effect of the first-order term $E_0\partial f/\partial v_z$ in the Boltzmann equation (1) is included in Ω given by (23). It we set $E_0=\nu=v_0=0$ in (28), then (28) reduces to the dispersion equation of the surface plasma waves in a collisionless plasma half-space [8]. We have adopted the collision model (8) on the ground of the arguments in Section 2, and restricted our attention to this collision model. If one uses the other two collision models (6) and (7), the dispersion equations different from (28) will be derived (see Appendix B).

4. Approximate dispersion equation

The dispersion equation (28) is complicated by the presence in the integrand of the plasma dispersion function $Z(\alpha)$. In order to evaluate (28) we might perform the numerical computations on a particular case. It is perhaps more instructive, however, to give an approximate dispersion equation which includes a correction to the result derived from the hydrodynamic theory. It is known that if $|\alpha|\gtrsim 5$ is satisfied, $Z(\alpha)$ can be evaluated in terms of its asymptote for $|\alpha|\gg 1$ with high enough precision. When the imaginary part of α is positive, the asymptote is given[9] by

$$Z(\alpha)\simeq-\frac{1}{\alpha}\left(1+\frac{1}{2\alpha^2}+\frac{3}{4\alpha^4}+\cdots\right). \tag{29}$$

Bearing this in mind, we shall derive the approximate dispersion equation for (28).

We consider the surface carrier waves of microwave frequencies in an usual semiconductor at room temperature. Then the appropriate values for the frequency, the propagation constant, and the other parameters can be taken in following ranges[10]:

$$\left.\begin{array}{l}\omega\lesssim 10^{11}\text{Hz},\ |\beta|\lesssim 10^6\text{m}^{-1},\ \nu\simeq 10^{12}\text{Hz},\\ v_T\lesssim 10^6\text{m}\cdot\text{sec}^{-1},\ |v_0|\lesssim 10^6\text{m}\cdot\text{sec}^{-1}.\end{array}\right\} \tag{30}$$

Taking (30) into account and assuming an phenomenological relation $qE_0/m^*=\nu v_0$ between E_0 and v_0, it is numerically confirmed from (21)-(23) that $|\alpha_1|\gtrsim 5$ and $|\alpha_2|\gtrsim 5$ are satisfied when $|\zeta|\lesssim|\beta|$. It can be readily shown that both the imaginary parts of α_1 and α_2 are always positive whenever $\omega<\nu$. In the region $|\zeta|\lesssim|\beta|$, therefore, we can use the asymptote (29) for $Z(\alpha_1)$ and $Z(\alpha_2)$ with enough precision and obtain the following asymptotic expressions for the integrand in (28):

$$D(\zeta)\simeq D_0-\frac{D_1}{\nu^2}\left[v_T^2(\zeta^2+\beta^2)+i\frac{\beta qE_0}{m^*}\right] \tag{31}$$

$$1+i\frac{\nu Z(\alpha_1)}{\sqrt{2}\,\Omega}\simeq\frac{\omega}{\omega+i\nu}$$

$$-i\frac{\nu[v_{\mathrm{T}}^{2}(\zeta^{2}+\beta^{2})+i\beta qE_{0}/m^{*}]}{(\omega+i\nu)^{3}} \qquad (32)$$

where

$$\left.\begin{aligned} D_{0}&=\frac{\omega}{\omega+i\nu}-\frac{\omega_{\mathrm{P}}^{2}}{(\omega-\beta v_{0}+i\nu)^{2}},\\ D_{1}&=\frac{i\nu^{3}}{(\omega+i\nu)^{3}}+\frac{3\nu^{2}\omega_{\mathrm{P}}^{2}}{(\omega-\beta v_{0}+i\nu)^{4}}. \end{aligned}\right\} \qquad (33)$$

We will use these expressions over the whole range of integration of ζ. Of course the asymptotes of $Z(\alpha_1)$ and $Z(\alpha_2)$ are invalid in the region $|\zeta|\gg|\beta|$, but we may expect the error to be small since the integrand in (28) becomes exceedingly small in this region.

Following the approximation mentioned above, the integral in (28) can be evaluated by means of the residue calculus. After straightforward calculations, we obtain the approximate dispersion equation as follows:

$$\left(1+\frac{\varepsilon_s}{\varepsilon}\right)+\frac{a\omega_{\mathrm{P}}^{2}}{(\omega-\beta v_{0}+i\nu)^{2}}+\frac{(\pm)\beta(b-a)\omega_{\mathrm{P}}^{2}}{\gamma(\omega-\beta v_{0}+i\nu)^{2}}=0 \qquad (34)$$

where

$$\gamma=\left[\beta^{2}-\frac{\nu^{2}}{v_{\mathrm{T}}^{2}}\left(\frac{D_{0}}{D_{1}}-i\frac{\beta qE_{0}}{\nu^{2}m^{*}}\right)\right]^{1/2}, \qquad (35)$$

$$\left.\begin{aligned} a&=\frac{1+i3\beta qE_{0}/m^{*}(\omega-\beta v_{0}+i\nu)^{2}}{D_{0}-i(\beta qE_{0}/m^{*}\nu^{2})D_{1}},\\ b&=-\frac{3\nu^{2}}{(\omega-\beta v_{0}+i\nu)^{2}D_{1}}, \end{aligned}\right\} \qquad (36)$$

and γ is defined such that the real part of γ is positive when β is real and ω is complex with a small positive imaginary part. Equation (34) has been derived on the assumption that β is real. This expression can be analytically continued, however, by requiring that the upper sign on β applies when the real part of β is positive and the lower does when the real part is negative. The complex roots of β for real ω give the propagation constants of the surface carrier waves, which are excited at a signal frequency of ω. Equation (34), after being rationalized, yields a polynomial equation and the roots for β can be determined numerically. Because of the algebraic complexity, it is difficult to compare (34) directly with the dispersion equation obtained by using the hydrodynamic theory.

In order to show clearly the difference between the kinetic and the hydrodynamic treatments, it is convenient to consider the collision-dominant ($|\omega-\beta v_0|\ll\nu$) situation which has been commonly assumed in the latter treatment. It this case, (34) is further simplified as follows:

$$1+\frac{\varepsilon_s}{\varepsilon}+i\frac{\omega_{\mathrm{R}}(\pm\beta/\gamma_{\mathrm{c}}-1)}{\omega-\beta v_{0}+i\omega_{\mathrm{R}}}+\frac{\pm\beta(3\omega_{\mathrm{R}}/\nu)}{\gamma_{\mathrm{c}}(1-3\omega_{\mathrm{R}}/\nu)}=0 \qquad (37)$$

where

$$\gamma_{\mathrm{c}}=\left[\beta^{2}-i\frac{\nu}{v_{\mathrm{T}}^{2}}\left(\frac{\omega+i\omega_{\mathrm{R}}}{1-3\,\omega_{\mathrm{R}}/\nu}-\beta v_{0}\right)\right]^{1/2}, \qquad (38)$$

and $\omega_{\mathrm{R}}=\omega_{\mathrm{P}}^2/\nu$ is the dielectric relaxation frequency. In the above we have used the phenomenological reation $qE_0/m^*=\nu v_0$. On the other hand, the corresponding dispersion equation obtained in the hydrodynamic treatment is given[2),11)] by

$$1+\frac{\varepsilon_s}{\varepsilon}+i\frac{\omega_{\mathrm{R}}(\pm\beta/\gamma_{\mathrm{h}}-1)}{\omega-\beta v_{0}+i\omega_{\mathrm{R}}}=0 \qquad (39)$$

where

$$\gamma_{\mathrm{h}}=\left[\beta^{2}-i\frac{\nu}{v_{\mathrm{T}}^{2}}(\omega-\beta v_{0}+i\omega_{\mathrm{R}})\right]^{1/2}. \qquad (40)$$

From a brief comparison (37) with (39), we find that the fourth term in (37), which depends on the ratio $3\omega_R/\nu$, gives a correction due to the finite spread of velocity distribution of the carrier stream. It is seen from (38) and (40) that a similar correction is also included in the expression for γ_c. It is noted that in our approximation, γ given by (35) provides the attenuation constant of the surface carrier waves in the transverse direction inside the semiconductor. Setting $\gamma=0$ or $\gamma_c=0$, therefore, from (35) or (38) we can obtain an approximate dispersion equation for the one-dimensional carrier waves with the same correction as mentioned above.

5. Concluding remarks

We have presented a kinetic treatment of the surface carrier waves supported by a semibounded semiconductor, by using the Boltzmann-Poisson equation and assuming the specular reflection of carriers at the boundary surface.

The principal results of this work are summarized as follows:

(i) A general expression for the dispersion equation of the surface carrier waves is given by (28). In the absence of collision and static electric field, this reduces to the dispersion equation of the surface plasma waves.

(ii) An approximate dispersion equation (34) is valid for the surface carrier waves of microwave frequencies in an usual semiconductor at room temperature.

(iii) A simplified dispersion equation (37) is useful in a collision-dominant situation. This includes a correction, which depends on the ratio of the dielectric relaxation frequency to the collision frequency, to the corresponding equation obtained in the hydrodynamic treatment.

Although for simplicity, we have confined ourselves to the system of one set of carriers, the present analysis can be easily extended to the system of two sets of carriers. It is expected that a similar analysis provides a more detailed information about the two-stream instabilities of the surface carrier waves.

Before concluding, we should point out that in a real semiconductor, there are several situations when the specular reflection condition is not necessary satisfied. However it is difficult to present an analytical method to take into account such situations. The only simple alternative is to assume a completely diffusive boundary where the carriers, arriving at the boundary, are scattered with a complete loss of their velocity, that is, $f(v_y>0, y=0)=0$. This boundary model is analogous to that of a completely absorbing boundary in the hydrodynamic treatment, which has been introduced by Barybin[12] for a carrier stream in the presence of depletion layer near the semiconductor surface. As distinct from the specular reflection, the diffuse scattering of carriers produces the macroscopic surface charge as a result of the charge accumulation on the boundary surface. The Boltzmann-Poisson equation in this situation can be solved by using the Wiener-Hopf technique, but we must evaluate the magnitude of the surface charge density in order to achieve the boundary value problem. An approximate solution method for the diffuse scattering condition will be presented in future studies.

Appendix A

We first consider the case where $qE_0/m^*>0$. Then the solution of the differential equation (16) with respect to v_z, which is bounded for $|v_z|\to\infty$, is given as follows:

$$\tilde{f}=\frac{m^*}{qE_0}\int_{-\infty}^{v_z}\Big[\frac{\nu\tilde{\rho}}{qn_0}F_0(v_z')-i\frac{q\tilde{\varphi}}{m^*v_T^2}\{\zeta v_y+\beta(v_z'-v_0)\}f_0(v_z')\Big]\exp[-i\{A(v_z)-A(v_z')\}]\,dv_z' \qquad (A1)$$

where

$$A(v_z)=\frac{m^*}{qE_0}\Big[\frac{\beta}{2}v_z^2-(\omega+i\nu-\zeta v_y)v_z\Big]. \qquad (A2)$$

Introducing the integration variable $u=v_z-v_z'$, (A1) reduces to

$$\tilde{f}=\frac{m^*\nu\tilde{\rho}}{q^2E_0n_0}F_0\int_0^\infty\exp[-B(u)]\,du-i\frac{\tilde{\varphi}}{E_0v_T^2}f_0\int_0^\infty[\zeta v_y-\beta u+\beta(v_z-v_0)]\exp[-C(u)]\,du \qquad (A3)$$

where

$$\left.\begin{aligned}B(u)&=\frac{u}{2v_T^2}(u-2v_z)-i\frac{m^*}{2qE_0}u\times[\beta u+2(\omega+i\nu-\zeta v_y-\beta v_z)]\\ C(u)&=B(u)+\frac{v_0u}{v_T^2}\end{aligned}\right\}\qquad (A4)$$

Substituting (A3) in (18), the integrations over v_x, v_y, and v_z can be evaluated shortly. We thus obtain

$$\tilde{\rho}=\frac{m^*\nu}{qE_0}\tilde{\rho}e^{-\alpha_1^2}\int_0^\infty\exp\Big[-\Big(\frac{m^*\Omega}{\sqrt{2}qE_0}u-i\alpha_1\Big)^2\Big]du-\Big(\frac{m^*}{qE_0}\Big)^2\varepsilon_s\omega_P^2(\zeta^2+\beta^2)\times\tilde{\varphi}e^{-\alpha_2^2}\int_0^\infty u\exp\Big[-\Big(\frac{m^*\Omega}{\sqrt{2}qE_0}u-i\alpha_2\Big)^2\Big]du \qquad (A5)$$

Introducing the transformations of integration variable defined by

$$\left.\begin{aligned}\theta_1&=-\frac{m^*\Omega}{\sqrt{2}qE_0}u+i\alpha_1,\\ \theta_2&=-\frac{m^*\Omega}{\sqrt{2}qE_0}u+i\alpha_2\end{aligned}\right\}\qquad (A6)$$

and taking into account the fact that $-\pi/4<\arg\Omega<\pi/4$, (A6) may be rewritten in the following form:

$$\tilde{\rho}=-\frac{\sqrt{2}\nu}{\Omega}\tilde{\rho}e^{-\alpha_1^2}\int_{-\infty}^{i\alpha_1}e^{-\theta_1^2}d\theta_1+\frac{2\varepsilon_s\omega_P^2}{\Omega^2}(\zeta^2+\beta^2)\tilde{\varphi}\times e^{-\alpha_2^2}\int_{-\infty}^{i\alpha_2}(\theta_2-i\alpha_2)e^{-\theta_2^2}d\theta_2. \qquad (A7)$$

Furthermore, introducing the plasma dispersion function defined by (20) into (A7), after some rearrangement of terms, we finally obtain (19).

For $qE_0/m^*<0$ we can also obtain (19) by proceeding in a manner similar to that for the case of $qE_0/m^*>0$.

Appendix B

Substituting the collision term (6) in (1) and following analytical methods similar to those presented in Section 2, Section 3, and Appendix A, for Pines' model we obtain the following dispersion equation:

$$\frac{\varepsilon_s}{\varepsilon}+(\pm)\frac{\beta}{\pi}\int_{-\infty}^{\infty}\frac{d\zeta}{(\zeta^2+\beta^2)D_P(\zeta)}=0 \qquad (B1)$$

where

$$D_P(\zeta)=1-\frac{\omega_P^2}{2\Omega^2}Z'(\alpha_2). \qquad (B2)$$

Similarly, using the collision term (7), for Robinson's model we obtain the following dispersion equation:

$$\frac{\varepsilon_s}{\varepsilon}+(\pm)\frac{\beta}{\pi}\int_{-\infty}^{\infty}\frac{1+i\dfrac{\nu Z(\alpha_2)}{\sqrt{2}\Omega}}{(\zeta^2+\beta^2)D_R(\zeta)}d\zeta=0 \qquad (B3)$$

where

$$D_R(\zeta)=1+i\frac{\nu Z(\alpha_2)}{\sqrt{2}\,\Omega}-\frac{\omega_P^2}{2\Omega^2}Z'(\alpha_2). \qquad \text{(B4)}$$

It should be noted that in the absence of collision, that is, in the case of $\nu=0$, (28), (B1), and (B3) reduce to the same dispersion equation. Using the approximation presented in Section 4, from (B1) and (B3) we are able to derive the approximate dispersion equation for each collision model.

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