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Interaction of Ion Acoustic Surface Waves with a Low-Density Ion Beam

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The interaction of ion acoustic surface waves propagating along a plane surface of density discontinuity in a warm, isotropic plasma with a low-density ion beam is investigated. The dispersion relation of the ion acoustic surface waves and the growth rates after the resonant and nonresonant interactions are obtained. The description is done on the basis of two-fluid hydrodynamics for warm electrons and cold ions.

1. Introduction

The investigation of the propagation of surface waves on bounded plasma systems is of significant importance in connection with widespread laboratory experiments with plasmas. Considerable attention has been currently given to ion acoustic surface waves which are low-frequency potential surface modes supported by a nonisothermal bounded plasma. The existence of such modes has been pointed out by Mcbride and Kaw[1] in the study of the surface waves on a homogeneous, isotropic, and semi-infinite plasma bounded by vacuum. Shoucri[2] has investigated the ion acoustic surface waves along a plasma cylinder, either with or without a strong longitudinal external magnetic field, and the excitation of these waves by a low-density electron beam. The dispersion equation for the ion acoustic surface waves and the growth rates of the oscillations resulting from a resonant interaction with the beam have been obtained. It must be stressed that in this work, the thermal motion of electrons normal to the boundary surface is disregarded. Such an anisotropic velocity distribution of electrons seems to be rather unrealistic for the plasma without an external magnetic field. Besides, the possibility of instability through a nonresonant interaction has not been considered, although the growth rates may be small in this case. Zhelyazkov and Nenovski[3] have presented a detailed analysis of the spectrum of ion acoustic surface waves along an isotropic plasma layer bounded by dielectrics, or along a dielectric layer bounded by isotropic plasmas. On the basis of kinetic plasma description, Atanassov et al.[4] have discussed the spectrum and Landau damping of ion acoustic surface waves propagating on the interface between an isotropic plasma and a dielectric.

In the present paper, we shall investigate the propagation of ion acoustic surface waves along a plane surface of density discontinuity in a nonisothermal isotropic plasma and their excitation by a low-density ion beam. By assuming a sharp boundary, in Section 2, the dispe-

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rsion equation for the ion acoustic surface waves is derived. In Section 3, the interaction of the waves with the ion beam is discussed. The growth rates of the oscillations are obtained both for the resonant and nonresonant instabilities.

2. Dispersion equation

We investigate first the dispersion equation for ion acoustic surface waves that propagate along a interface separating two regions of nonisothermal ($T_e \gg T_i$, where T_e and T_i are the electron and ion temperatures), homogeneous, and isotropic plasma with the different electron densities. We assume that the plane $y=0$ is the interface. The upper ($y>0$) region 1 and the lower ($y>0$) region 2 are characterized by the equilibrium electron (or ion) densities n_{01} and n_{02}, respectively, where $n_{01}<n_{02}$.

We describe this system using Poisson's equation and the two-fluid hydrodynamics for electrons and ions. Since we are concerned with the low-frequency surface waves with phase velocity that is much larger than the ion thermal velocity but much smaller than the electron thermal velocity v_{Te}, we can neglect the ion temperature and the inertia of electron. Then the set of basic equations is given as follows:

$$\left.\begin{array}{l} \dfrac{\partial}{\partial t}n_{i\nu}+n_{0\nu}\nabla\cdot\bar{v}_{i\nu}=0,\ \dfrac{\partial}{\partial t}\bar{v}_{i\nu}=-\dfrac{e}{m_i}\nabla\varphi_\nu \\ 0=\dfrac{e}{m_e}\nabla\varphi_\nu-\dfrac{v_{Te}^2}{n_{0\nu}}\nabla n_{e\nu},\ \varepsilon_0\nabla^2\varphi_\nu=e(n_{e\nu}-n_{i\nu}) \end{array}\right\} \tag{1}$$

where $-e$ is the electron charge, ε_0 is the permittivity in free space, φ is the perturbation of potential field, m_μ, n_μ, and $\bar{v}_\mu$ are the mass, the perturbed density, and the perturbed velocity for the μth ($\mu=e$, i) species, respectively, and the subscript ν ($\nu=1, 2$) refers to the plasma regions 1 and 2. Assuming that the perturbation is of the form $exp\,[-i(\omega t-kz)]$ and is independent of x, we obtain the following equation for the potential:

$$\frac{\partial^2}{\partial y^2}\varphi_\nu-\gamma_\nu^2\varphi_\nu=0 \tag{2}$$

where

$$\gamma_\nu=\left(k^2+\frac{\omega_{P\nu}^2}{\varepsilon_\nu c_s^2}\right)^{1/2},\quad \varepsilon_\nu=1-\frac{\omega_{P\nu}^2}{\omega^2}, \tag{3}$$

$\omega_{P\nu}=(e^2n_{0\nu}/\varepsilon_0 m_i)^{1/2}$ is the ion plasma frequency, and $c_s=v_{Te}(m_e/m_i)^{1/2}$ is the ion acoustic speed. Allowing for the fact that the surface-wave potentials decay exponentially from the interface both inside two plasma regions, from (2) one sees that γ_1 and γ_2 are required to be both positive real. Then the surface-wave solutions of (2) in two regions are presented as

$$\varphi_1=Ae^{-\gamma_1 y}(y>0),\quad \varphi_2=Be^{\gamma_2 y}(y<0) \tag{4}$$

where A and B are arbitrary constants.

The boundary conditions which must be satisfied by the solutions (4) are

$$(\varphi_1-\varphi_2)_{y=0}=0,$$
$$\varepsilon_0\left(\frac{\partial}{\partial y}\varphi_1-\frac{\partial}{\partial y}\varphi_2\right)_{y=0}=-\rho_s \tag{5}$$

where ρ_s represents the surface charge density, which may be readily obtained by calculating the normal displacement of ions at the interface and is given as follows:

$$\rho_s=-\varepsilon_0\left(\frac{\omega_{P1}^2}{\omega^2}\frac{\partial}{\partial y}\varphi_1-\frac{\omega_{P2}^2}{\omega^2}\frac{\partial}{\partial y}\varphi_2\right)_{y=0} \tag{6}$$

The condition for the existence of solutions for the set of equations (5) and (6), after the substitution of φ with the expressions (4), gives the following dispersion equation:

$$\varepsilon_1 \gamma_1 + \varepsilon_2 \gamma_2 = 0. \tag{7}$$

Making use of the expressions (3) for γ_ν and ε_ν and taking account of the fact that $\omega_{P1} < \omega_{P2}$, it is found that the solutions of (7) giving real values of ω and k are possible only if

$$\omega_{P1} < \omega < \frac{\Omega_+}{\sqrt{2}} \tag{8}$$

where $\Omega_+ = (\omega_{P1}^2 + \omega_{P2}^2)^{1/2}$. From (7) we thus obtain the ion acoustic surface waves which may propagate along the interface between two regions of plasma as follows:

$$k^2 = \frac{\omega^2(\Omega_+^2 - \omega^2)}{c_s^2(\Omega_+^2 - 2\omega^2)}, \tag{9}$$

The inequality (8) determines the range of the frequency spectrum of the ion acoustic surface waves. From (8) and (9) it is found that the wave-number spectrum is restricted in the following range:

$$\frac{\omega_{P1}^2 \omega_{P2}^2}{c_s^2 \Omega_-^2} < k^2 \tag{10}$$

where $\Omega_- = (\omega_{P2}^2 - \omega_{P1}^2)^{1/2}$. When $\omega_{P1} = 0$, (9) gives the dispersion equation of the ion acoustic surface waves along the vaccum-plasma interface. We note that the range of the frequency spectrum is narrowed and the phase velocity is reduced by the presence of the low-density plasma region. In the another limit $\omega_{P1} \to \omega_{P2}$, all of k, γ_1, and γ_2 tend to infinity and the ion acoustic surface waves disappear.

3. Interaction with a ion beam

We investigate an interaction of a low-density, cold ion beam with the ion acoustic surface waves which have been examined in Section 2. We assume that the ion beam of plasma frequency ω_{Pb} ($\ll \omega_{P2}$) fills the lower region 2 of high-density plasma and propagates in the z direction with the velocity v_0 (>0). In this case, Poisson's equation in region 2 and the expression (6) for the surface charge density are slightly changed by the presence of the ion beam. Bearing this in mind, we resolve the set of equations (1), (5), and (6) together with the fluid equations for the ion beam and thus obtain the following dispersion equation:

$$\varepsilon_1 \gamma_1 + \varepsilon_{2b} \gamma_{2b} = 0 \tag{11}$$

where

$$\left.\begin{aligned} \gamma_{2b} &= \left[k^2 + \frac{\omega_{P2}^2}{\varepsilon_{2b} c_s^2} \right]^{1/2} \\ \varepsilon_{2b} &= 1 - \frac{\omega_{P2}^2}{\omega^2} - \frac{\omega_{Pb}^2}{(\omega - kv_0)^2}. \end{aligned}\right\} \tag{12}$$

Since we restrict our study to the case of a low-density ion beam, we may solve (11) by a perturbation method. We neglect the ion contribution as a first approximation and thus obtain from (11) the dispersion equation (9) for the ion acoustic surface waves. We next consider a small perturbation by the passage of the ion beam. One sees that an effective interaction takes place when the dispersion curve of (9) intersects that of the synchronous wave $\omega = kv_0$ associated with the ion beam. Since $(\omega/k)^2 < c_s^2(1 - \omega_{P1}^2/\omega_{P2}^2)$ as is seen from (9), this interaction is possible if

$$\frac{v_0^2}{c_s^2} < 1 - \frac{\omega_{P1}^2}{\omega_{P2}^2}. \tag{13}$$

We consider first the resonant interaction. This occurs in the neighborhood of the synchronous point (ω_0, k_0) which satisfies $\omega_0 = k_0 v_0$ and $D(\omega_0, k_0) = 0$, where $D(\omega, k) = 0$ is the dispersion equation given by (9). The synchronous point is determined as follows:

$$\omega_0^2=\frac{1-v_0^2/c_s^2}{2-v_0^2/c_s^2}\Omega_+^2, \quad k_0=\frac{\omega_0}{v_0}. \tag{14}$$

In this case one can obtain the solution of (11) by assuming a small frequency shift $\Delta\omega=\omega-\omega_0$ and wave-number shift $\Delta k=k-k_0$. Keeping only the first-order terms with respect to $\Delta\omega$, Δk, and $\omega_{P1}^2/\omega_{P2}^2$, a straightforward Taylor expansion of (11) gives the following equation:

$$(\Delta\omega-v_g\Delta k)(\Delta\omega-v_0\Delta k)^2=\frac{\alpha\omega_{P2}^2\omega_0^3}{2\Omega_-^2}\cdot\frac{(2-v_0^2/c_s^2)\omega_{P2}^2-2\omega_0^2}{(2-v_0^2/c_s^2)\Omega_+^2-2\omega_0^2} \tag{15}$$

where $\alpha=\omega_{Pb}^2/\omega_{P2}^2(\ll1)$ is a small parameter and v_g is the group velocity of the ion acoustic surface waves at the synchronous point, which is given by

$$v_g=\frac{v_0^3}{c_s^2}\left[1+\left(1-\frac{v_0^2}{c_s^2}\right)^2\right]^{-1}. \tag{16}$$

Equation (15) can be solved for ω (k real) or for k (ω real). It has been shown by Kotsarenko and Federchenko[5] that for an equation of the form (15) with $v_gv_0>0$, any solution with Δk complex denotes a convective instability. Since in our case, $v_gv_0>0$ by the assumption $v_0>0$, a solution of (15) with Δk complex indicates a convective instability. The maximum growth rate is obtained at $\Delta\omega=0$ and given by

$$\mathrm{Im}k=-\frac{\sqrt{3}}{2^{4/3}}\alpha^{1/3}\frac{\omega_0}{v_0}\left[\frac{v_0}{v_g}\frac{\omega_{P2}^2}{\Omega_-^2}\cdot\frac{(2-v_0^2/c_s^2)\omega_{P2}^2-2\omega_0^2}{(2-v_0^2/c_s^2)\Omega_+^2-2\omega_0^2}\right]^{1/3} \tag{17}$$

where Imk denotes the imaginary part of k. When $\omega_{P1}=0$, (17) is simplified as follows:

$$\mathrm{Im}k=-\frac{\sqrt{3}}{2^{4/3}}\alpha^{1/3}\frac{\omega_0}{v_0}\left(\frac{c_s}{v_0}\right)^{2/3}\left[1+\left(1-\frac{v_0^2}{c_s^2}\right)^2\right]^{1/3} \tag{18}$$

where we have used (16) for v_g. This expression can be compared directly with that of Shoucri[2], which has been derived by assuming an anisotropic temperature such that the thermal velocity of electrons normal to the interface is identically zero. We notice that our growth rate (18) differs from the latter by amount of the factor $[1+(1-v_0^2/c_s^2)^2]^{1/3}$.

It sould be noted that in the limiting case $\omega_{P1}\to\omega_{P2}$, the dispersion equation (11) is still valid. Since (11) does not give any surface-wave solutions when $\omega_{P1}=\omega_{P2}$ and $\omega_{Pb}=0$ ($\alpha=0$), however, the presence of the ion beam is essential to determine the interface along which the surface waves are excited. In this case, an exact solution of (11) is necessary. It is found that the surface waves exist only if

$$0<1-\frac{\omega_{P2}^2}{\omega^2}<\frac{\alpha\omega_{P2}^2}{(\omega-kv_0)^2}. \tag{19}$$

When this inequality is satisfied, (11) can be rewritten in the form

$$k^2\left[2\left(1-\frac{\omega_{P2}^2}{\omega^2}\right)-\frac{\alpha\omega_{P2}^2}{(\omega-kv_0)^2}\right]+\frac{\omega_{P2}^2}{c_2^s}=0. \tag{20}$$

We note that for a low-density ion beam ($\alpha\ll1$), the solutions of (20) do not indicate any instabilities. Equations (19) and (20) imply, however, that it is possible to excite an ion acoustic surface mode by a relatively dense ion beam ($\alpha\gtrsim1$) penetrating in a homogeneous, warm plasma.

We next consider the nonresonant interaction. This takes place in the nonsynchronous region where the solution of $\omega-kv_0=0$ is not too near that of $D(\omega, k)=0$. In this case we redefine a small frequency shift $\Delta\omega=\omega-kv_0$ and expand (11) around $\omega-kv_0=0$. After rationalizing the result, we obtain

$$a(\Delta\omega)^4 - b\alpha\omega_{P2}^2(\Delta\omega)^2 + \alpha^2\omega_{P2}^4 k^2 = 0 \qquad (21)$$

where

$$\left.\begin{aligned} a &= \left(2 - \frac{v_0^2}{c_s^2}\right)\frac{\Omega_-^2}{v_0^2}\left[\left(\frac{\omega_0}{kv_0}\right)^2 - 1\right] \\ b &= 2k^2\left[1 - \left(\frac{\omega_{P2}}{kv_0}\right)^2\right] + \frac{\omega_{P2}^2}{c_s^2}. \end{aligned}\right\} \qquad (22)$$

Equation (21) gives two solutions for $(\Delta\omega)^2$. It is readily shown that one solution does not satisfy (11) but is introduced in process of rationalization. The proper solution is given as follows:

$$(\Delta\omega)^2 = \frac{\alpha\omega_{P2}^2}{2a}\left[b - \sqrt{b - 4ak^2}\right]. \qquad (23)$$

Assuming $kv_0 \simeq \omega > \omega_{P1}$, which is the condition (8) for the existence of the ion acoustic surface waves, and using the inequality (13), it is verified that $(\Delta\omega)^2$ becomes negative if $kv_0 < \omega_0$. The temporal growth rate is given by

$$\mathrm{Im}\omega = \frac{\alpha^{1/2}\omega_{P2}}{\sqrt{2a}}\left|b - \sqrt{b^2 - 4ak^2}\right|^{1/2} \qquad (24)$$

and the condition for the nonresonant instability is

$$\omega_{P1} < kv_0 < \omega_0. \qquad (25)$$

From the expression (23) for $(\Delta\omega)^2$ it is obvious that the instability is convective. The spacial growth rate $\mathrm{Im}k$ can be obtained from (24) by introducing the following transformation of variables:

$$\mathrm{Im}k = -\mathrm{Im}\omega/v_0 (\Delta k = -\Delta\omega/v_0), \quad k = \omega/v_0. \qquad (26)$$

For the sake of simplicity, we consider the situation such that $\omega_{P1} = 0$ and $\omega^2 \ll \omega_{P2}^2 v_0^2/c_s^2$ or $\omega^2 \gg \omega_{P2}^2 v_0^2/c_s^2$. For each case, the growth rate is given as follows:

$$\mathrm{Im}k = -\frac{\alpha^{1/2}\omega_{P2}/v_0}{\left[(2 - v_0^2/c_s^2)(\omega_0^2/\omega^2 - 1)\right]^{1/2}} \quad (\omega^2 \ll \omega_{P2}^2 v_0^2/c_s^2) \qquad (27)$$

$$\mathrm{Im}k = -\frac{\alpha^{1/2}\omega_{P2}/\sqrt{2}\,v_0}{(\omega_0^2/\omega^2 - 1)^{1/2}} \quad (\omega^2 \gg \omega_{P2}^2 v_0^2/c_s^2). \qquad (28)$$

It should be noted that the order of the growth rate for the nonresonant interaction is characterized by $\alpha^{1/2}$, while that for the resonant interaction is by $\alpha^{1/3}$.

We have restricted our attention only to the case where the ion beam occupies the high-density plasma region. It is also of interest, however, to consider another situation where the ion beam occupies the low-density plasma region. Although the detailed calculation is not presented here, we can analyze the resonant and the nonresonant interactions by following the approximation method presented in this Section. For instance, we obtain the dispersion equation for the resonant interaction as follows:

$$(\Delta\omega - v_g\Delta k)(\Delta\omega - v_0\Delta k)^2 = -\frac{\alpha\omega_{P2}^2\omega_0^3}{2\Omega_-^2} \cdot \frac{(2 - v_0^2/c_s^2)\omega_{P1}^2 - 2\omega_0^2}{(2 - v_0^2/c_s^2)\Omega_+^2 - 2\omega_0^2}. \qquad (29)$$

When $\omega_{P1} = 0$, the maximum growth rate is given by

$$\mathrm{Im}k = -\frac{\sqrt{3}}{2}\alpha^{1/3}\frac{\omega_0}{v_0}\left(\frac{c_s}{v_0}\right)^{2/3}\left(1 - \frac{v_0^2}{c_s^2}\right)^{1/3} \qquad (30)$$

It follows from (18) and (30) that the growth rate in the present configuration is smaller than in the preceding case.

4. Concluding remarks

The dispersion equation of ion acoustic surface waves supported by a plane surface of density discontinuity in a warm, isotropic plasma has been derived. The growth rates of these waves by the resonant and the nonresonant interactions with a low-density ion beam have been obtained.

Although we have assumed the plane surface to simplify a theoretical treatment, the present analysis can be also carried out for ion acoustic surface waves along a cylindrical surface of density discontinuity in a plasma-filled cylindrical wave-guide and their excitation by a low-density ion beam. It has been shown[6] in that case that when the radii of the wave-guide and the cylindrical surface are about ten times as long as the electron Debye length, the dispersion equation for the axially symmetric mode and the dependence on the beam velocity of the maximum growth rate agree approximately with (9) and (17), respectively.

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