Formation of Periodic Vortex Streets Driven by the Lorents Force

Ikehata, Yoshito
Department of Earth System Science and Technology, Interdisciplinary Graduate School of Engineering Sciences, Kyushu University

Honji, Hiroyuki
Department of Earth System Science and Technology, Interdisciplinary Graduate School of Engineering Sciences, Kyushu University

Sugihara, Yuji
Department of Earth System Science and Technology, Interdisciplinary Graduate School of Engineering Sciences, Kyushu University

https://doi.org/10.15017/17395

出版情報：九州大学大学院総合理工学報告. 17 (4), pp.437-442, 1996-03-01. 九州大学大学院総合理工学研究科

九州大学大学院総合理工学研究科報告
第17巻 第4号 437-442頁 平成8年3月

Engineering Sciences Reports, Kyushu University
(KYUSHU DAIGAKU SOGORIKOGAKU KENKYUKA HOKOKU)
Vol. 17, No. 4, pp. 437 - 442, March 1996

Formation of Periodic Vortex Streets Driven by the Lorentz Force

Yoshito IKEHATA*, Hiroyuki HONJI** and Yuji SUGIHARA**

(Received November 20, 1995)

Quasi-2D periodic vortex streets, driven by the Lorentz force due to the interaction of a localized magnetic field with an electrolytic current, have been investigated experimentally using a shallow water tank with a movable bottom floor. The vortex street formation has also been investigated numerically and some simulated flow patterns are presented.

1. Introduction

When a cylindrical body moves in fluid a boundary layer forms on the surface of the body. It is well-known that at moderately high Reynolds numbers the boundary layer separates to form Kármán's periodic vortex street behind the body. Such a vortex street formation, however, is not necessarily connected with boundary layer separation. Honji[1] and Honji & Haraguchi[2] investigated the formation of quasi-2D vortex wakes behind a localized magnetic field, which interacts with an electrolytic direct current to produce the Lorentz force acting on an aqueous electrolyte solution as a working fluid. In their experimental setup no rigid bodies which give rise to flow separation are present in the fluid.

The purpose of this work is to extend the preceding investigations and shed some new light on the mechanism of vortex street formation. In section 2, a belt-driven water tank developed specifically for the present investigation is described. In section 3, a numerical model for quasi-2D flows induced by the Lorentz force is outlined together with the method of numerical calculation. Section 4 is devoted to results and discussion.

2. Experimental apparatus and methods

A schematic diagram of the newly developed water tank used for visual observations of flow is illustrated in **Fig. 1**. The tank is equipped with a 25cm-wide endless rubber belt, which is moved in a fluid at a constant speed with a motor, thereby producing a nearly uniform stream above the belt. The fluid is a 5.0% $NaHCO_3$ aqueous solution. The tank is also equipped with a pump which serves to circulate the fluid. The thickness of a thin layer of the fluid above the belt surface was 1.0 cm, which defined the depth of the uniform stream.

Vortex wakes were generated by electromagnetically disturbing an upstream localized region of the above uniform stream. A flat circular-cylindrical permanent magnet with the magnetic flux density $\boldsymbol{B}$ (0.2T) was placed under the belt as shown in **Fig. 1**. The fluid was electrolyzed by applying a d.c. voltage between an anode and a cathode placed on both ends of the fluid layer, thereby giving rise to an electrolytic current. The Lorentz

*Graduate Student, Department of Earth System Science and Technology
**Department of Earth System Science and Technology

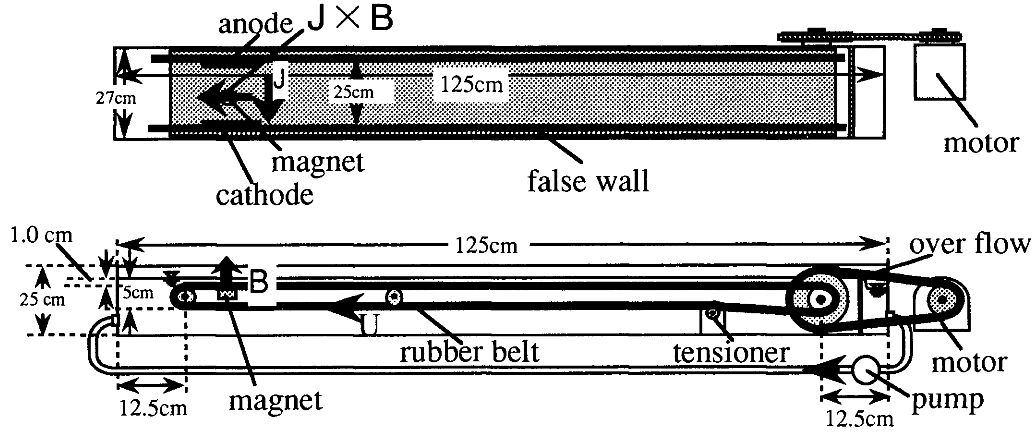


Fig. 1　Schematic diagram of experimental apparatus

force was selected always to direct upstream, i.e. opposite to the direction of the belt motion. The fluid acted on by the Lorentz force was thus moved upstream. As shown in a preceding paper[2)***)], the balance between the uniform downstream and induced upstream flows governs the three types of flows, open-streamline, vortex pair, and periodic ones, depending on Reynolds number and reduced Lorentz force. The magnitude of the Lorentz force was controlled by varying the electric current from a d.c. power supply. For visualizing the surface flows polystyrene beads were scattered on the surface of the fluid and illuminated with a slide projector. Flow patterns were photographed from above with a 35 mm or video camera.

3. Numerical model and methods

A quasi-2D flow of a thin layer of fluid driven by the Lorentz force acting only horizontally is considered ; the vertical fluid motion motion may be neglected to be small as compared with the horizontal one. Approximating the vertical profile of a horizontal flow velocity with a quadratic function, we have the incompressible flow equations[3)]

$$\frac{\partial \boldsymbol{u}_s}{\partial t}+(\boldsymbol{u}_s\cdot\nabla_h)\boldsymbol{u}_s=\nu\nabla_h^2\boldsymbol{u}_s-\kappa\frac{2\nu}{h^2}\boldsymbol{u}_s+\frac{1}{\rho}(\boldsymbol{J}\times\boldsymbol{B}) \tag{1}$$

and

$$\nabla_h\cdot\boldsymbol{u}_s=0 \tag{2}$$

***) In Ref.2 the following corrections should be made. On 9th line from top on p.2275 (left) for,"...is B_0 = 0.48T..." read"...is 0.48 T...". B_0 is 0.29 T at z =7mm. On 8th line from bottom on p.2277 (left), for "...downstream," read "...upstream,".

where $\boldsymbol{u}_s$ (u, v) is the surface velocity vector, ρ the fluid density, h the depth, $\boldsymbol{J}$ the electric current density, and $\boldsymbol{B}$ the magnetic flux density. The term $\boldsymbol{J}\times\boldsymbol{B}$ describes the Lorentz force. The uniform flow is in the x direction to which the horizontal y direction is perpendicular ; u and v are the x-and y-components of $\boldsymbol{u}_s$, respectively. Letting $\boldsymbol{i}$ and $\boldsymbol{j}$ be the unit vectors in the x and y directions respectively, we have $\nabla_h=(\partial/\partial x)\boldsymbol{i}+(\partial/\partial y)\boldsymbol{j}$.

Equation (1) does not includes the pressure term for only the surface flows are considered. The second term $-(2\kappa\nu/h^2)\boldsymbol{u}_s$ on r.h.s. of the equation describes the viscous effect at the bottom floor, where κ is an adjustable parameter depending on the deviation of the velocity profile form the quadratic form. We write

$$\boldsymbol{J}=-J_0\boldsymbol{j}$$

and

$$\boldsymbol{B}=B_0\, exp\left[-\left\{\frac{(x-a)^2+(y-(b/2))^2}{d^2}\right\}\right]\boldsymbol{k}, \tag{3}$$

where J_0 is the applied current density, B_0 the maximum value of $|\boldsymbol{B}|$ on the axis of a magnet, a the stream-wise distance between the upstream edge of a computational domain and the magnet axis, b the y-directional width of the region, d the magnet diameter, and $\boldsymbol{k}$ the unit vector in the z-direction. The Lorentz force $\boldsymbol{f}$ is

$$\boldsymbol{f}=\boldsymbol{J}\times\boldsymbol{B}=-J_0B_0\, exp\left[-\left\{\frac{(x-a)^2+(y-(b/2))^2}{d^2}\right\}\right]\boldsymbol{i}. \tag{4}$$

Substituting eq. (4) into eq. (1), taking $\boldsymbol{rot}$ of the resulting equation, and non-dimensionalizing it with J_0 and B_0, we have a nonlinear evolution equation

$$\frac{\partial\omega'}{\partial t}+\frac{\partial\psi'}{\partial y'}\frac{\partial\omega'}{\partial x'}-\frac{\partial\psi'}{\partial x'}\frac{\partial\omega'}{\partial y'}$$
$$=\frac{\nabla'^2\omega'}{Re}-\frac{\omega'}{Rh}-2Q\left(y'-\frac{b}{2d}\right)exp\left[-\left\{\left(x'-\frac{a}{d}\right)^2+\left(y'-\frac{b}{2d}\right)^2\right\}\right], \tag{5}$$

where the primes indicate non-dimensional quantities including the vorticity ω' is the z-components of $\nabla\times\boldsymbol{u}_s'$, and the streamfunction ψ' defining $\boldsymbol{u}_s'$ as $\boldsymbol{u}_s'=(\partial\psi'/\partial y', -\partial\psi'/\partial x')$ with

$$Re=\frac{Ud}{\nu},\ Rh=\frac{Uh^2}{2\kappa\nu d},\ \text{and}\ Q=\frac{J_0B_0d}{\rho U^2} \tag{6}$$

This stream function ψ' can be evaluated using

$$\nabla^2\psi'=-\omega' \tag{7}$$

The convection terms in eq. (5) are approximated using a first order upwind scheme and

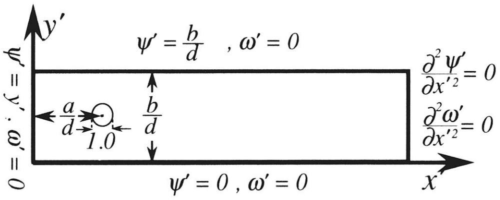


Fig. 2 Boundary conditions

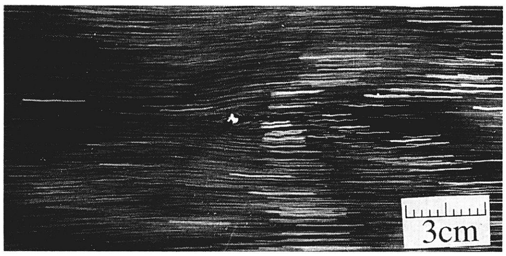


Fig. 3 Open-streamline flow ($Q=1.54$, $Re=375$, $Rh=20.8$)

Fig. 4 Vortex pair flow ($Q=2.46$, $Re=375$, $Rh=20.8$)

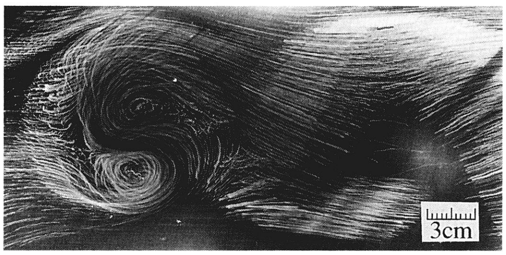


Fig. 5 Periodic flow ($Q=7.29$, $Re=375$, $Rh=20.8$)

other terms by means of a central difference scheme. This evolution equation has been numerically integrated under the boundary conditions as depicted in **Fig. 2** with κ the coefficient of Rh being kept as unity.

4. Results and discussion

Three different types of visualized flow patterns are shown in **Figs. 3** to **5** at $Re=375$ and $Rh=20.8$. **Figure 3** shows a flow having no closed streamlines at a relatively small value of $Q=1.54$. It will be seen that the induced flow is blown off in the flow direction. When Q is increased to be 2.64, a steady vortex pair is formed as shown in **Fig. 4**. With the further increase of Q, the vortex pair becomes unstable and oscillatory. **Figure 5** shows an oscillatory flow at a large value of $Q=7.29$. A streakline pattern of the downstream wake is displayed in **Fig. 6** at the same value of Q. The formation of a periodic unsteady far wake will be seen clearly. The wake looks similar to Kármán's vortex street, although this wake formation is free from any boundary layer separation from rigid bodies. According to Honji and Haraguchi[2], the Strouhal number for this type of periodic wake formation is about half the value of 0.2 for the common vortex streets behind circular-cylindrical rigid bodies. When Q and Re are increased further, the vortex streets become irregular and turbulent.

Some numerical results will follow here the above experimental results. **Figures 7** and **8** show an open-streamline flow and a vortex pair, respectively. The former figure corresponds to the flow shown in **Fig. 3** and the latter to the one shown in **Fig. 4**. Both corresponding flow patterns look quite similar to each other, and the developed numerical model seems to work well for the quasi-2D steady flows. **Figure 9** shows the time evolutions of streamlines and vorticity lines for an oscillatory unsteady wake. The vortex

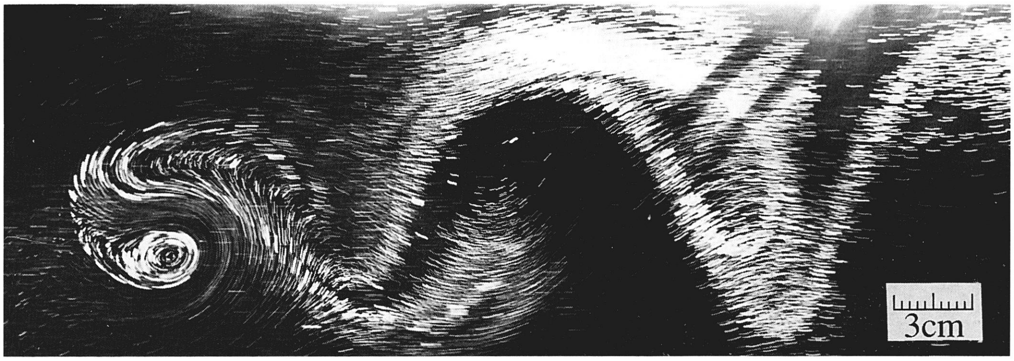


Fig. 6 Streakline pattern of periodic flow (Q=7.29, Re=375, Rh=20.8)

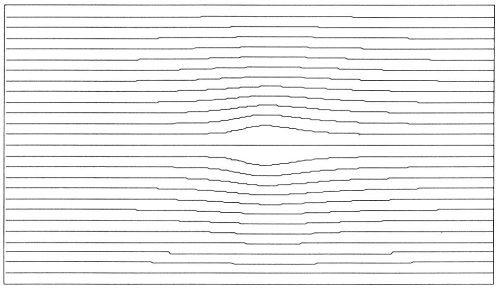


Fig. 7 Streamline pattern of computed open-streamline flow (Q=1.0, Re=150, Rh=2.1, a/d=6.5, b/d=13)

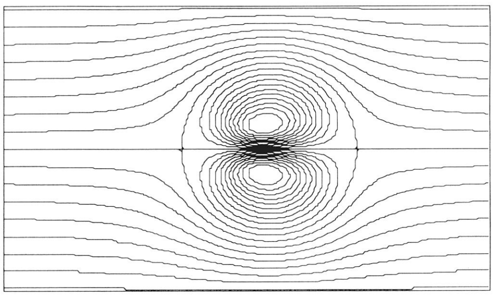
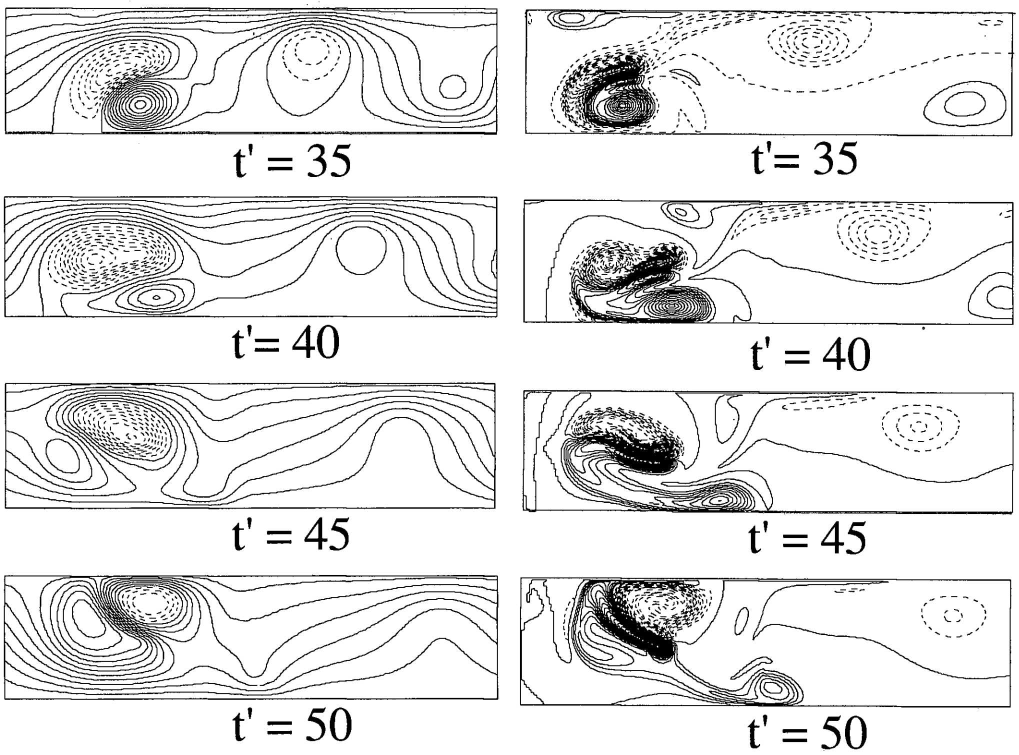


Fig. 8 Streamline pattern of computed vortex pair flow (Q=125, Re=30, Rh=0.42, a/d=6.5, b/d=13)

pair will be seen to oscillate up and down, thereby each vortex being subject to alternative expansion and reduction in its size. A periodic release of vorticity occurs behind the oscillatory vortex pair to form a periodic vortex street. A resulting similarity of the flow patterns between those in **Figs. 5** & **6** and **9** seems to support the validity of the present numerical model. The calculated value of the Strouhal number is about 0.1, which agrees well with the previously referred experimental value. It should be noted here, however, that the outer boundary of the computational region is reached fully by the oscillatory vortex pair ; a much larger computational domain should be used in further studies.

The computed formation regions for the three types of flows are compared with the experimental results of Honji and Haraguchi[2] in **Fig. 10**, where the computed flow types are shown with the symbols ○, △, and ×. The agreement with the experimental result is satisfactory.

In conclusion, the flow patterns and formation regions have been investigated experimentally and numerically, and the numerical model so far developed has proved to be promising. One of the authors (HH) thanks Prof. Y. Nakamura for pointing out the passages that needed correction in Ref. 2. The authors thank Dr. N. Matsunaga for helpful discussions.



(a) Streamline patterns (b) Vorticity patterns

Fig. 9 Time evolution of computed periodic flow (Q=60, Re=600, Rh=8.4, a/d=15, b/d=13)

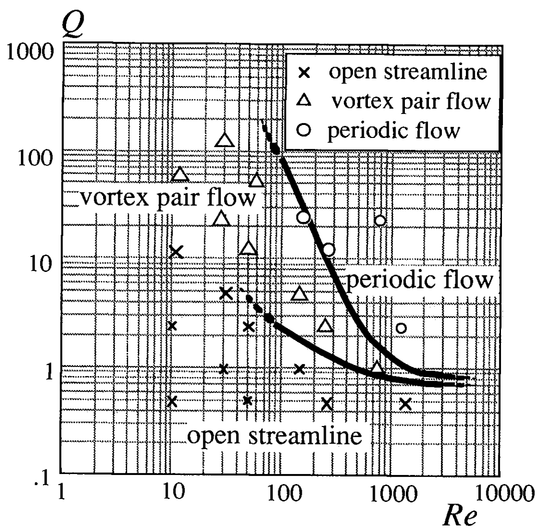


Fig. 10 Formation region diagram

References

1) Honji, H., Wavy wake formation in the absence of submerged bodies in electrolyzed water, J. Phys. Soc. Jpn., **Vol. 60**, No. 4 (1991) p. 1161.

2) Honji, H. and Haraguchi, Y., Electrolytically-induced quasi-two-dimensional vortex pairs, J. Phys. Soc. Jpn., **Vol. 64**, No. 7 (1995) p. 2274.

3) Murakami, Y , Stability of a quasi-two-dimensional flow, Nagare, **Vol. 13** (1994) p. 115.