# Effects of Sloping Sea Bottom on Wave Forces Acting on a Large Floating Platform

Ohkusu, Makoto
Research Institute for Applied Mechanics, Department of Earth System and Technology

Imai, Yasutaka
Department of Earth System and Technology, Interdisciplinary Graduate School of Engineering Sciences, Kyushu University

https://doi.org/10.15017/17360

出版情報：九州大学大学院総合理工学報告. 16 (4), pp.433-443, 1995-03-01. 九州大学大学院総合理工学研究科

九州大学大学院総合理工学研究科報告
第16巻 第4号 433-443頁 平成7年3月

Engineering Sciences Reports, Kyushu University
(KYUSHU DAIGAKU SOGORIKOGAKU KENKYUKA HOKOKU)
Vol. 16, No. 4, pp. 433-443, March. 1995

# Effects of Sloping Sea Bottom on Wave Forces Acting on a Large Floating Platform

Makoto OHKUSU* and Yasutaka IMAI**

(Received November 30, 1994)

**ABSTRACT**

A method is presented of estimating wave forces on a large floating platform whose dimension is comparable with the topographical scale at the site. We assume the structure is supported by numerous floats very small in size relatively to the structure. This method accounts for the effects of the topography such as bottom slope and wave breaking at the shore. Some examples of the predicted wave forces on a floating structure of extremely long size are presented and compared with those for no topographical effects.

**KEYWORDS**

Floating airport; wave force; sloping bottom; wave breaking

## 1. INTRODUCTION

Floating offshore structures of very large size, such as floating-airport which we expect to be realized in near future, will be of the configuration of platform supported on numerous floats of simple geometry. One example is a conceptual design proposed more than ten years ago in Japan of an international airport[1]. It is 5,000 *m* long and supported on more than 25,000 floats.

Such structures will be deployed not far offshore but rather close to the land where water depth is relatively shallow and waves are affected by the bottom topography. The structures' size will be so large as comparable to the scale of the bottom topography and the effect of the bottom topography must be accounted for when predicting wave forces on the structures.

Objective of this study is to develop a theoretical method to predict wave forces on a large floating structure supported by numerous floats and located on the sea of sloping bottom. This method must model the effect of steepening and breaking of waves on reasonable basis.

We assume that the structure's horizontal dimensions are of the same order of the topographical scale, while each float element is relatively very small. Waves propagating distance long enough relatively to the float size will be refracted under the effect of the bottom topography. They will be described by the mild shope equation. But within the distance comparable to the float size the wave motion will be considered at that propagating on the flat sea bottom of waves valid close to the float must be metched with the wave

*Research Institute for Applied Mechanics, Department of Earth System and Technology
**Department of Earth System and Technology

motions away from the float satisfying the mild slope equation. Wave breaking at the seashore must be incorporated in the mild slope equation governing the refraction of waves.

Some examples of the predicted wave forces computed with this method will be presented on a floating structure extremely long in the direction parallel to the shore line and composed of a lot of floats of of cylindrical shape.

## 2. MATHEMATICAL MODELLING

We assume a structure supported by numerous floats of vertical cylindrical shape. The structure in floating on the free surface with its longerside parallel to the shoreline as shown in **Fig. 1**. The sea bottom is sloping from deep water to the shore. The bottom contour is parallel to the $y$ axis.

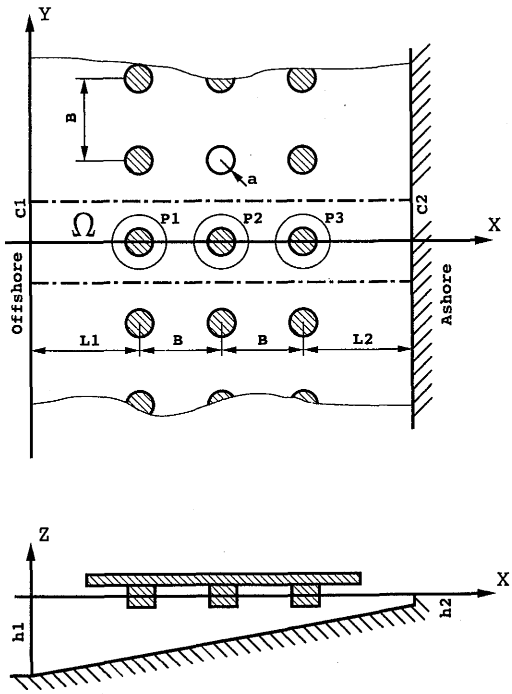



**Fig. 1**　Coordinate System

When waves with the crest line parallel to the $y$ axis are incident on, the followings will occur.

1. Waves will be refracted under the influence of the sloping bottom and become steeper and shorter as it approaches the shore.
2. Waves are diffracted by the floats and the interaction occurs between them.
3. If waves will be sufficiently steep, they will break at some distance from the shoreline.

We need a mathematical model accounting for these phenomena.

### 2.1 Refraction and diffraction

Refraction of waves on the bottom whose slope is much smaller than the slope is described by the mild slope equation[2)]

$$\nabla\left(\frac{CC_g}{g}\nabla\phi\right)+\frac{\omega^2}{g}\frac{C_g}{C}\phi=0 \quad (1)$$

where we express the velocity potential as follows.

$$\Phi(x,y,z,t)=\phi(x,y)\frac{\cosh k(z+h)}{\cosh kh}e^{-i\omega t} \quad (2)$$

$\omega$ is the wave frequency and $h$ is the water depth varying slowly. The wave number $k$ at the reference depth $h$ satisfies the relation

$$\omega^2=gk\tanh kh \quad (3)$$

where $C$ and $C_g$ are the local phase and group velocity of the waves.

The mild slope equation does not correctly describe the localized effect of scattering and radiation of the waves by floating bodies. We use the following approach to incorporate the scattering effect into the solution. We assume that the bottom slope varies in the scale of the whole structure. Therefore the scale of the bottom slope is very large relatively to the size of each floating body composing the structure. We can consider the water depth is constant when analyzing the scattering wave field close to the floating body. Those diffraction waves close to the body will be matched with the solution of the mild slope equation valid away from the body to provide a complete wave field.

Assuming simple geometry of vertical circular cylinder of radius $a$ and draft $d$ for the floating body, the wave motion close to it will be given by

$$\phi(r, \theta, z)=\sum_{m=0}^{\infty} F_m\Big[J_m(kr)\frac{Z_0(kz)}{Z_0(kh)}+B_{m0}\frac{H_m^{(1)}(kr)}{H_m^{(1)}(ka)}Z_0(kz)$$

$$+\sum_{p=1}^{\infty} B_{mp}\frac{K_m(\alpha_p r)}{K_m(\alpha_p a)}Z_p(\alpha_p z)\Big]\cos m\theta \qquad (4)$$

where $(r, \theta)$ is the polar coordinate and its origin is on the free surface and at the center of the cylinder. The wave number $k$ and the water depth $h$ refer to the values at the cylinder location. $J_m$ is $m$th order of Bessel function of first kind, $K_m$ is $m$th order of modified Bessel function of second kind, and $H_m^{(1)}$ is $m$th order of Hankel function of first kind. $\alpha_p$ is the $p$th solution of the equation

$$\alpha_p \tan \alpha_p h+\frac{\omega^2}{g}=0$$

Other notations in (4) are

$$Z_0(kz)=\frac{\cosh k(z+h)}{\sqrt{N_0}}, \qquad N_0=\frac{1}{2}\left(1+\frac{\sinh 2kh}{2kh}\right)$$

$$Z_p(\alpha_p z)=\frac{\cos \alpha_p(z+h)}{\sqrt{N_p}}, \qquad N_p=\frac{1}{2}\left(1+\frac{\sin 2\alpha_p h}{2\alpha_p h}\right)$$

$B_{mp}$ in this expression is determined so that

$$\frac{\partial \phi}{\partial n}=0$$

is satisfied on the cylinder.

$F_m$ are the unknowns representing the incoming waves on the body which were already refreacted on the sloping bottom on their way. The origins of those waves are the scattered waves from the other bodies and the incident waves comming from offshore. $F_m$ are to be determined later by the matching condition to the outer solution satisfying the mild slope equation.

When we are concerned with the radiation waves generated by the motions of the structure, a particular solution satisfying the inhomogeneous condition on the body boundary,

$$\frac{\partial \phi}{\partial n} = V_n$$

will be added to equation (4). $V_n$ is the normal velocity of the body surface caused by its motion.

Upon letting $r \rightarrow \infty$, (4) becomes

$$\phi\,(r,\ \theta\,,z) = \sum_{m=0}^{\infty} F_m^* \left[ J_m(kr) + \gamma_m H_m^{(1)}(kr) \right] \frac{\cosh k(z+h)}{\cosh kh} \cos m\theta \qquad (5)$$

in which $\gamma_m$ is determined from $B_{m0}$. Equation (5) is used as the boundary condition at an appropriate $r$ ($>$ a) for the outer solution of the mild slope equation. $F_m^*$ will be determined together with the outer solution. For the radiation problem this boundary condition for the outer solution must include the particular solution mentioned before.

### 2.2 Effect of wave breaking

Waves become steeper as they approach the region of shallower depth and break if their height is over a critical value. This effect must be taken into account in the computation of wave forces on the structure.

Wave diffraction by the numerous bodies as well as the wave refraction on the sloping bottom leads to very complicated wave pattern. No universal way of modelling the effect of breaking of such complicated waves seems to be possible. We introduce a model of wave breaking for the simpler case when the crest lines of waves are always parallel to the shoreline, that is, no diffraction by the bodies exist. We use this model even for the case when the waves are diffracted on the bodies and the wave pattern is not simple.

We suppose that the structure is not so close to the shore as it is in the midst of surf zone. This means we do not need to model the pressure field of breaking waves when we compute the wave forces on the structure. The effect we have to account for is wave energy dissipation of the incident waves at the breaking zone and reduction of the amplitude of the reflected waves from the beach.

The following is semi-empirically established knowledge of wave breaking at the sloping beach[3]. Waves approaching to the shore will break when the wave height becomes as large as 0.7 to 1.2 of the local water depth. Reflection coefficient of a beach affected by the breaking will be inversely proportional to a parameter $\varepsilon_0$ defined by

$$\varepsilon_0 = \sqrt{2\pi}\, A_0 \omega^2 / g\alpha^{2.5} \qquad (6)$$

where $A_0$ is the amplitude of the incident waves in deep water and $\alpha$ the beach slope.

In our problem the beach slope may not be so steep because we assume the horizontal scale of the slope is of the order of a large structure's size and $\alpha$ will be less than, say, 0.1. In practical context we are concerned with the case of relatively steep incident waves, say, steeper than 0.01. Then very small almost zero, reflection coefficient of the beach is predicted with equation (6). Generally we are not uninterested in the wave forces when very low waves are incident and refracted. But in this preliminary study we assume a mathematical model in which all the energy of the waves dissipates by the breaking and no waves are reflected back from the beach.

Boudet *et al.* proposed a mathematical model of wave absorbing for achieving numerical water tank[4]. They introduced some damping terms in both the kinematical and dynamical conditions on the free surface. We employed their idea to realize no reflection from the beach. We selected magnitude of the damping terms and the distance from the beach within which the free surface conditions are to be damped so that the incident waves break at the location where their height becomes as large as the local water depth and no reflection comes from the beach.

The mild slope equation modified with these damping effects will be written as

$$\nabla\left(\frac{CC_g}{g}\nabla\phi\right)+\frac{\omega^2}{g}\frac{C_g}{C}\phi-\frac{1}{g}(2vi\omega+v^2)\phi=0 \tag{7}$$

where $v$ is given by

$$v=\begin{cases}\dfrac{\omega(x-x_0)^2}{\lambda^2} & \text{damping zone}\\ 0 & \text{otherwise}\end{cases}$$

$x_0$ in this equation is the coordinate where the breaking will occur and $\lambda$ the local wave length.

With simple ray theory we computed variation in wave height when a plane incident wave approaches on the sloping bottom to the shore to find the critical position $x_0$ beyond which the wave height becomes larger than the local water depth. Numerical solution of the equation (7) obtained upon using this value for $x_0$ gives the relation between the local wave height $H$ and the depth $h$. One example is shown in **Fig. 2**.

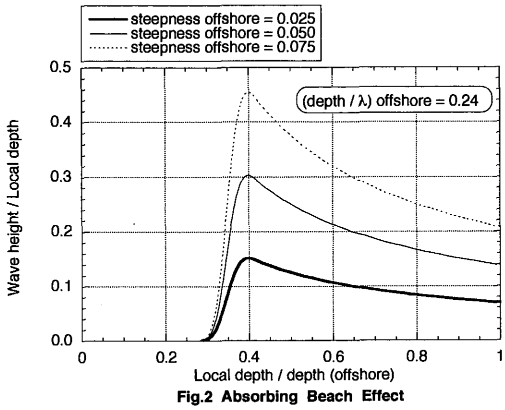



**Fig. 2** Aborbing Beach Effect

## 3. NUMERICAL COMPUTATION

Let a plane incident wave, whose velocity potential given by

$$\phi_I=e^{i(kx-\omega t)} \tag{8}$$

approach to the shore from offshore. We assume a large floating structure composed of three rows of cylindrical floats. Each row is parallel to the shore line. Number of the floating bodies in one row is very large and the flow will be periodical in the $y$ direction. We need to analyze the flow only within $-B/2 < y < B/2$ as seen in **Fig. 1**. The flow will be symmetrical with respect to $y = 0$ because we assume the crest lines of the plane incident waves parallel to the shoreline.

Let the fluid region within the boundaries $C_1$, $C_2$ and $P_i$ $(i = 1, 2, \ldots\ldots, N)$ be denoted by $\Omega$. $C_1$ is the boundary of the sloping bottom region and the offshore sea of the constant depth. $C_2$ is the shoreline and $P_i$ is the closures around the body $i$ on which the inner solutions will be matched to the solutions in $\Omega$.

In the domain $\Omega$ the equation (7) will be the governing equation. Then the boundary conditions for $\phi$ are:

$$\phi = \bar{\phi}, \qquad \frac{\partial \phi}{\partial n} = \frac{\partial \bar{\phi}}{\partial n} \quad \text{on} \quad C_1 \quad \text{and} \quad P_i$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on} \quad y = \pm B/2$$

where $\bar{\phi}$ denotes the velocity potential outside the region $\Omega$.

By virtue of the periodicity of the flow in $y$ direction, the scattering wave on $C_1$ defined as $\bar{\phi} - \phi_I$ is expressed as

$$\bar{\phi}_s(x, y) = \sum_{p=1}^{pmax} A_p e^{-ik_\infty x \sqrt{1-\left(\frac{2\pi(p-1)}{k_\infty B}\right)^2}} \cos\left(k_\infty y \frac{2\pi(p-1)}{k_\infty B}\right) \tag{9}$$

where $k_\infty$ is the wave number off-shore of constant depth and $A_p$ is the unknown constants to be determined by solving the equation (7). $p_{max}$ is given by the formula

$$p_{max} = \left[\frac{k_\infty B}{2\pi}\right] + 1$$

Define the following functional $J$, the solution of our boundary value problem will be $\phi$ giving the stationary value of $J$.

$$J(\phi, \bar{\phi}) = \iint_\Omega \frac{1}{2}\left[\frac{CC_g}{g}(\nabla\phi)^2 - \left\{\frac{\omega^2}{g}\frac{C_g}{C} - \frac{1}{g}(2vi\omega + v^2)\right\}\phi^2\right] dA$$

$$+ \int_{C1} \frac{CC_g}{g}\left[\left(\frac{1}{2}\bar{\phi}_s - \bar{\phi}_s\right)\frac{\partial\bar{\phi}}{\partial n} - \frac{1}{2}\bar{\phi}_s\frac{\partial\bar{\phi}_I}{\partial n}\right] dS$$

$$+ \sum_{i=1}^{N}\int_{Pi}\frac{CC_g}{g}\left(\frac{1}{2}\bar{\phi} - \phi\right)\frac{\partial\bar{\phi}}{\partial n} dS \tag{10}$$

We substitute $\bar{\phi}_s$ given by equation (9) into the second integral on $C_1$ and the matching condition (5) for $\bar{\phi}$ in the third integral on $P_i$.

Numerical implementation of the solution is to subdivide the region $\Omega$ into a number of traingular panels, represent $\phi$ and $\nabla\phi$ by local basic functions and solve resulting linear simultaneous equations.

## 4. WAVE FORCES

The solution $\phi$ in the region $\Omega$ determines $F_m^*$ in the equation (5) through the matching condition. Then the velocity potential[5] defined in the region $r < a$ and $-h < z < -d$ under the cylindrial body will be given by

$$\phi(r,\theta,z) = \sum_{m=0}^{\infty} F_m^* \left[ A_{m0} \left(\frac{r}{a}\right)^m + 2\sum_{n=0}^{\infty} A_{mn} \frac{I_m\left(\frac{n\pi r}{d_1}\right)}{I_m\left(\frac{n\pi a}{d_1}\right)} \cos\frac{n\pi(z+h)}{d_1} \right] \cos m\theta \quad (11)$$

Where $A_{mn}$ are the coefficients determined by the body boundary condition and the matching condition of the solution (4) and (11) on $r = a$, $I_m$ the first kind modified Bessel function of $m$th order and $d_1 = h - d$.

It is straightforward to derive the formulas for wave exciting forces on each cylindrical body from equations (4) and (11). The $x$ component of the force normalized is given by

$$\frac{F_x}{\pi a^2 \rho g \zeta} = 2iF_1^* \left[ \left( \frac{J_1(ka)}{Z_0(kh)} + B_{10} \right) \frac{\sinh kh - \sinh kd_1}{ka\sqrt{N_0}} + \sum_{p=1}^{\infty} B_{1p} \frac{\sin \alpha_p h - \sin \alpha_p d_1}{\alpha_p a \sqrt{N_p}} \right] \quad (12)$$

where $\rho$ is the fluid density and $\zeta$ the amplitude of the incident waves in deep water.

The $z$ component of the force is

$$\frac{F_z}{\pi a^2 \rho g \zeta} = 2F_0^* \left[ \frac{A_{00}}{2} + 2\sum_{n=0}^{\infty} A_{0n} \frac{(-1)^n I_1\left(\frac{n\pi a}{d_1}\right)}{\left(\frac{n\pi a}{d_1}\right) I_0\left(\frac{n\pi a}{d_1}\right)} \right] \quad (13)$$

The wave forces and moment on the whole structure will be given by summing these components on all the cylinders. Hereafter, however, the wave forces denote the sum of the forces on one cylinder of all the rows. If the number of the cylindrical bodies in one row is $M$, then the real forces on the structure are $M\times$ wave forces.

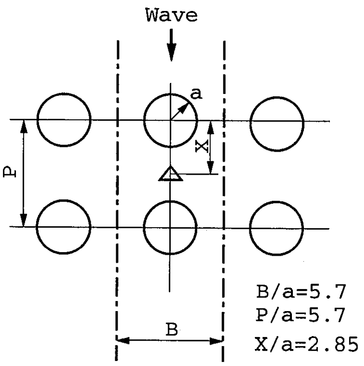
## 5. NUMERICAL RESULTS

In order to confirm the validy of the present method we applied it to a case of two rows of vertical cylinders standing on the flat water bottom with the wave incident head on the rows. For this problem actually we do not need the matching between the solutions in the outer and inner domain of the cylinders. But we used matching technique on purpose for the confirmation.

The predicted wave amplitude normalized by the amplitude of the incident waves is compared with the measured[6] *in* **Fig. 3** The amplitude is at the middle of two adjacent cylinders. Complete agreement between them confirms the validity of our method except at $ka=1.1$. The wave length at $ka=1.1$ is equal to the distance $B$ in this figure and the linear solution is impossible.

Our model of large floating platform for numerical computation is, as shown in **Fig. 1,** three rows of float elements whose geometry is vertical circular cylinder. Diameter of each float is $10m$, the distance between the centers of two adjacent cylinders is $34m$ in the $x$ direction and $32m$ in the $y$ direction. Distance from the shoreline $C_2$ to the deep water boundary $C_1$ is assumed to be $300m$. Water depth is $15m$ at $C_1$. The bottom slope is 0.05.

These parameters are apparently too small as the real situation. Reason is the limited capacity of available computers. Yet our model will suffice for our purpose to develop a methodology to predict wave forces.



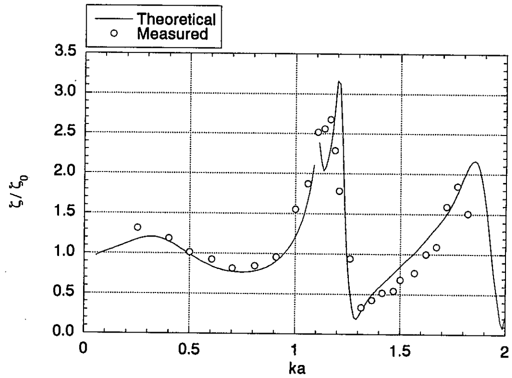




Fig.3 Wave Amplitude at X ( Flat Bottom )

**Fig. 3** Wave Amplitude at X (Flat Bottom)

Half of the region $\Omega$ is subdivided into around 2,000 triangular panels. The size of the panel must be small enough relatively to the wave length we are concerned. Some trial computations revealed that their size must be much smaller in particular close to the shoreline. It is as small as one twenty-fourth of the local wave length in our computation within $15m$ to shoreline. Actually the $x$ coordinate of the onset location of wave breaking $x_0$ given in the section 2.2 is within this distance.

The amplitudes of wave forces on three rows are shown in **Figs. 4, 5** and **6**. Rolling moment is defined about the middle row of the cylinders on the free surface. All the results shown hereafter are for the steepness 0.05 of the incident waves offshore. Comparison with the thin lines depicting the wave forces when water depth is constant and no shore exists, apparently suggests that the wave forces are larger in magnitude under the influence of the sloping bottom due to the steepening of the waves.

This is more significant on the internal forces caused between two adjacent cylinders on the shore side. They are illustrated in **Figs. 7, 8** and **9**.

Computed wave patterns around the structure are depicted in **Fig. 10**. There are almost no waves on the shore as expected. As the length of the incident waves becomes shorter, the reflection from the structure is considerable. This will be the case for constant depth too but certainly the steepening of the waves intensifies it.

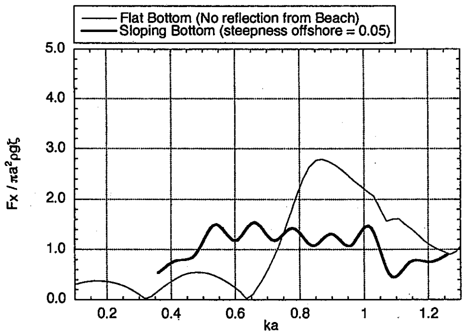



**Fig. 4**　Wave Force (Horizontal)

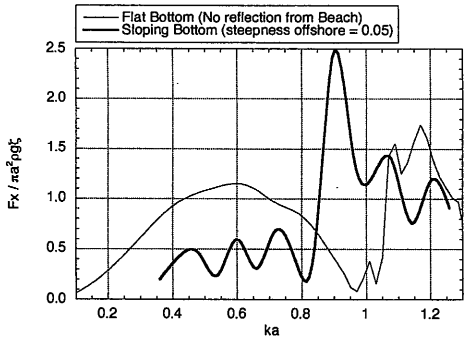



**Fig. 7**　Internal Force (Between 2 Shoreside Cylinders)

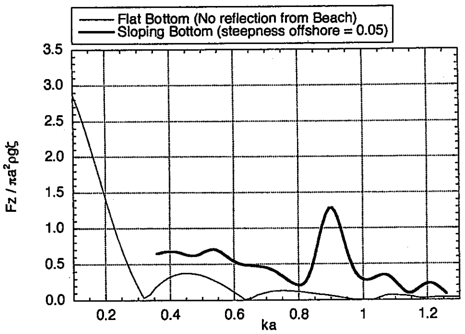



**Fig. 5**　Wave Force (Vertical)

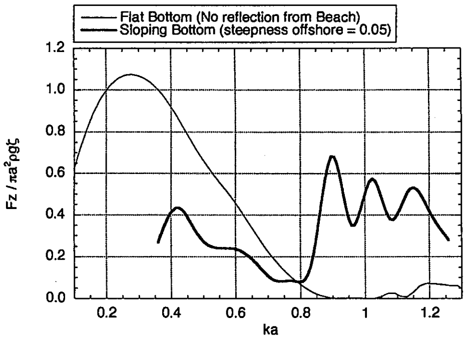



**Fig. 8**　Internal Force (Between 2 Shoreside Cylinders)

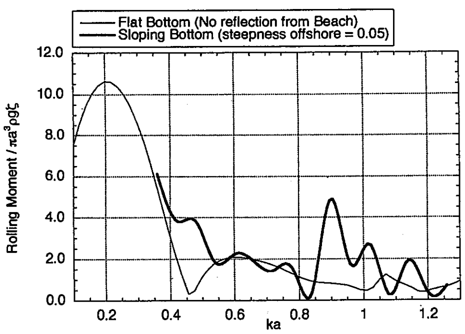



**Fig. 6**　Rolling Moment

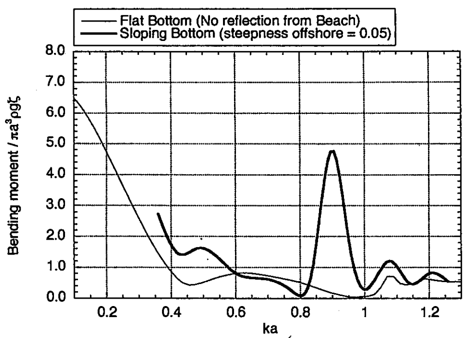



**Fig. 9**　Bending Moment (Between 2 Shoreside Cylinders)

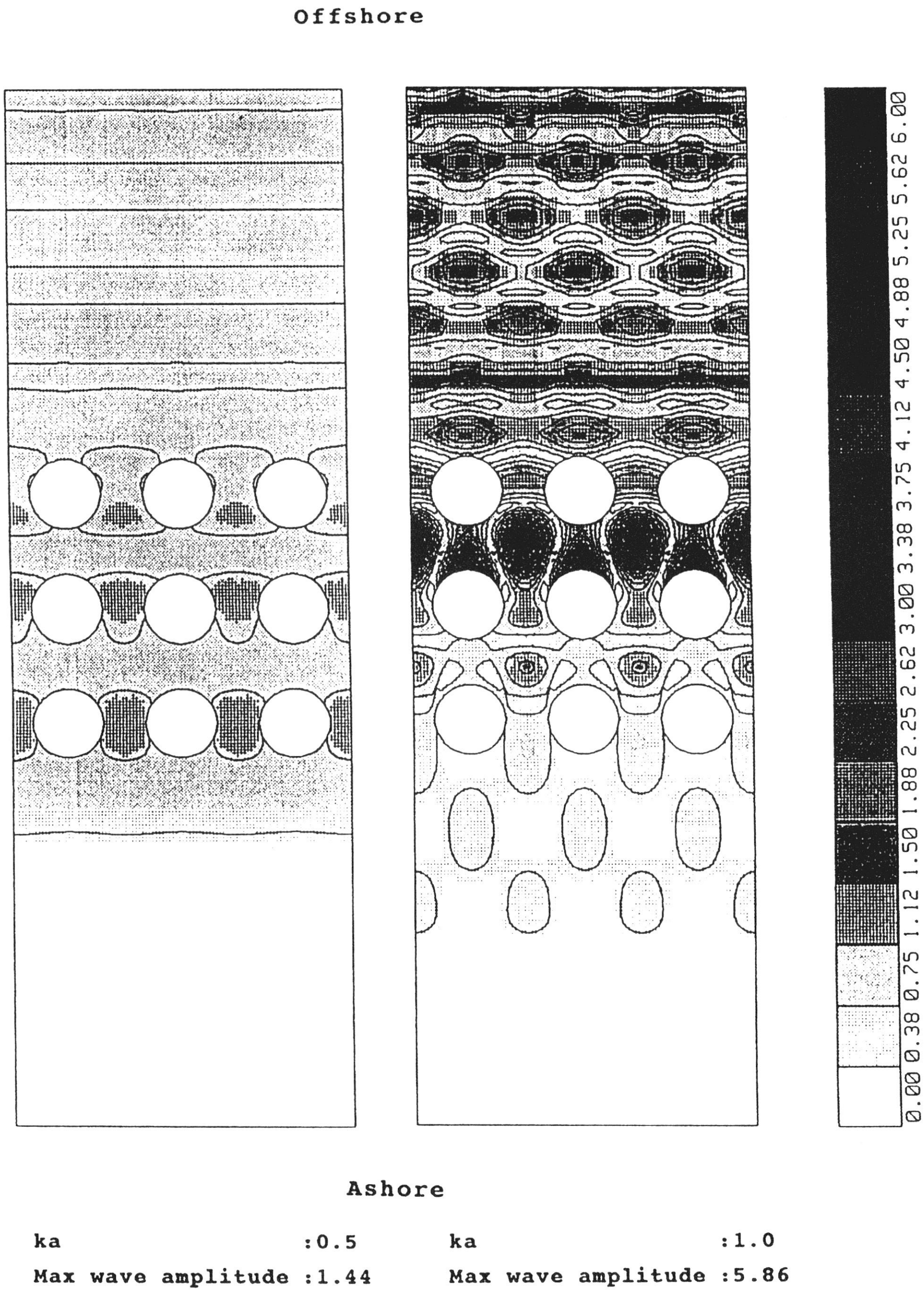



**Fig. 10** Wave Amplitude around Multiple Floating Structures

## 6. CONCLUDING REMARKS

Wave forces on very large floating structure will be affected by the sea bottom topography because such structure will be located close to the land and its scale is large enough to be compared with the topographical scale. We developed a method to predict the wave forces on the structures floating on the sloping sea bottom. In this method we considered the effects of the refraction and diffraction of waves and the breaking on the shore rationally. Results of numerical computation suggest that this method predicts physically realistic wave forces. After experimental confirmation for simple geometry, this is to be applied to more complicated and larger scale structures and topography.

## REFERENCES


1) Japan Ship Industries Association, Report of feasibility study of the floating platform concept for New Kansai International Airport (1979).

2) Mei, Chian C., *The Applied Dynamics of Ocean Surface Waves*, John Wiley & Sons. (1989)

3) Battjes, J. A., Surf zone dynamics, Annual Review of Fluid Mechanics Vol. 20 (1988).

4) Boudet, L., Cointe, R. and Molin, B., Standing waves in numerical tanks, 7th International Workshop on Water Waves and Floating Bodies (1992).

5) Garret, C. J. R., Wave forces on a circular dock. J. Fluid Mechanics, vol 46, part-1 (1971).

6) Ohkusu, M., Reflection and transmission of waves by a group of vertical cylinders. Bulletin of Research Institute for Applied Mechanics, Kyushu Univ. (in Japanese) No. 57. (1982)