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Convergence of Approximate Potential Functions for Vector Field in Electromagnetic Waveguides

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The convergence of an approximate electric and an approximte magnetic potential function representing vector field in electromagnetic waveguides is discussed. The two potential functions are expressed in the form of integral of Green's functions and the boundary values of the vector field. Based on these expressions, it is proved that two approximate potential functions converge uniformly to their true potential functions, respectively, when the approximate field satisfies the boundary conditions in vector field in the sense of mean square. Then, the inequalities about convergence of the approximate potential functions on a boundary are deduced. Using the inequalities, it is also shown that the boundary conditions in vector field can be satisfied when the approximate potential functions are expressed by the finite series of modal functions.

1. Introduction

Various numerical methods have been applied to the analysis of dielectric waveguides, which support vector field modes, in order to calculate the propagation characteristics precisely [1),2)]. It should be noted, however, that the solutions obtained by these methods do not necessarily converge to the true values. For the precise analysis of wave fields, Yasuura has developed a numerical method, i.e., the mode-matching method[3)]. This method has been applied successfylly not only to scalar field problems, in which the field can be expressed by one wave function, but also to vector field problems, in which the field can be expressed by two wave functions[4), 5)]. Its algorithm is straightforward. The approximate field is expressed by one or two truncated series of the modal functions and the expansion coeffcients are determined by matching the approximate field to the boundary conditions in the sense of mean squares. The uniform convergence of the solutions for scalar field problems is ensured theoretically by Yasuura's theorem[3)]. It may be expected, on the analogy of this theorem, that the solutions for vector field problems also converge to the true fields. However, it is important to clarify the theoretical basis for application of the mode-matching method to vector field problems.

In this paper, on the basis of Yasuura's theorem for scalar field, we shall derive the expressions for convergence of two approximate potential functions representing vector field. In section 2, we deduce the integral representations of an electric and a magnetic potential function in terms of the boundary values of the vector field. We introduce the approximate electric and the approximate magnetic potential function which satisfy the same equations as the true potential functions. Based on their integral representations, it is

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proved the two approximate potential functions converge uniformly to the true potential functions when the approximate vector field satisfies the boundary conditions in the sense of mean square. In section 3, we show that the approximate vector field satisfies the boundary conditions in the sense of mean square when the approximate potential functions are expressed by the finite series of modal functions. Using these results, Yasuura's theorem rewritten for vector field is deduced. The time factor is assumed to be of the form exp $(-iwt)$.

2. Uniform Convergence of Approximate Potential Functions

2.1 Integral representation of potential function

The geometry considered here is the dielectric waveguide as shown in **Fig. 1,** which is composed of the core region S_1 and the clad region S_2, and is uniform in the z direction.

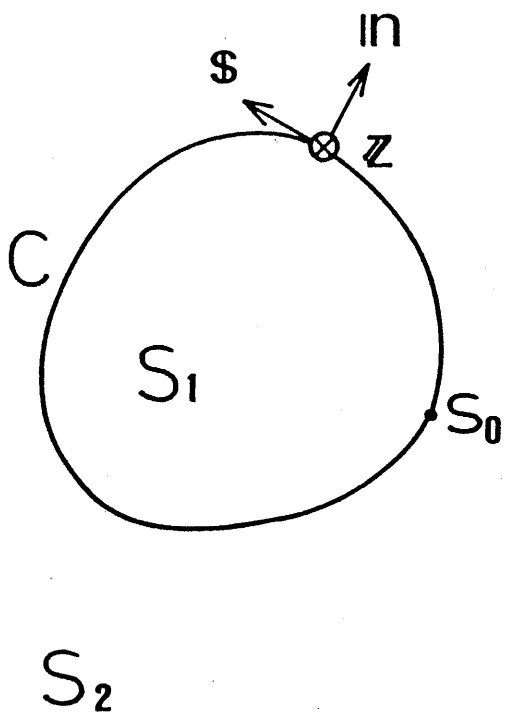

Fig. 1 Geometry of dielectric waveguide.

The refractive indices of two regions S_1 and S_2 are n_1 and n_2 $(n_1 \geq n_2)$, respectively. The boundary of the cross section is denoted by C, and **s** and **n** are the unit vectors in the tangential direction and the normal direcion pointing to outer region of C, respectively. **s**, **n**, and the unit vector **z** in the z direction satisfy the relation

$$\mathbf{s} \times \boldsymbol{n} = \mathbf{z}. \qquad (1)$$

The elecric and magnetic potential functions Π_j^e and Π_j^m, which vary in the form exp $(i\beta z)$, satisfy the Helmholtz equations

$$\left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + k^2 n_j^2\right) \Pi_j^p = 0, \qquad p = e,m, \qquad j = 1,2, \tag{2}$$

where the subscript j indicates the two regions S_1 and S_2, $\nabla_\perp$ is the transverse part of the vector operator Δ, and k is the wavenumber in free space. Using these two potential functions, the electromagnetic fields varying in the form exp $(i\beta z)$ can be represented as follows:

$$\mathbf{E}_j = \mathbf{z}\left(n_j^2 k^2 + \frac{\partial^2}{\partial z^2}\right) \Pi_j^e + \frac{\partial}{\partial z}\nabla_\perp \Pi_j^e - i\omega\mu_j \mathbf{z}\times \nabla_\perp \Pi_j^m,$$
$$\mathbf{E}_j = \mathbf{z}\left(n_j^2 k^2 + \frac{\partial^2}{\partial z^2}\right) \Pi_j^m + \frac{\partial}{\partial z}\nabla_\perp \Pi_j^m - i\omega\varepsilon_j \mathbf{z}\times \nabla_\perp \Pi_j^e, \tag{3}$$

where μ_j and ε_j are the permeability and the permittivity of the medium in the region S_j.

The electric and magnetic potential functions $\tilde{\Pi}_j^e$ and $\tilde{\Pi}_j^m$ which vary in the form exp $(-i\beta z)$, also satisfy the Helmholtz equations (2). Substituting $\tilde{\Pi}_j^e$ and $\tilde{\Pi}_j^m$ into Eqs. (3), the electromagnetic fields $\tilde{\mathbf{E}}_j$ and $\tilde{\mathbf{H}}_j$ can be expressed by these potential functions. The tilde on potential functions and fields means that the indicated physical quantity varies in the form $\exp(-i\beta z)$. Using these equations for $\mathbf{E}_j$, $\mathbf{H}_j$, $\tilde{\mathbf{E}}_j$, and $\tilde{\mathbf{H}}_j$, we obtain the following equation in region S_1:

$$\int_C (\mathbf{E}_1 \times \tilde{\mathbf{H}}_1 - \tilde{\mathbf{E}}_1 \times \mathbf{H}_1)\cdot \mathbf{n}ds$$
$$= -i\omega\varepsilon_1\tau_1^2 \int_C \left(\Pi_1^e \frac{\partial}{\partial n}\tilde{\Pi}_1^e - \tilde{\Pi}_1^e \frac{\partial}{\partial n}\Pi_1^e\right) ds$$
$$+ i\omega\mu_1\tau_1^2 \int_C \left(\Pi_1^m \frac{\partial}{\partial n}\tilde{\Pi}_1^m - \tilde{\Pi}_1^m \frac{\partial}{\partial n}\Pi_1^m\right) ds \quad , \tag{4}$$

with

$$\tau_1^2 = n_1^2 k^2 - \beta^2, \tag{5}$$

where s denotes arclength on C measured from a fixed point s_o. Applying Green's theorem to Π_1^p and $\tilde{\Pi}_1^p$ $(p = e, m)$, we have

$$\int_{S_1} (\Pi_1^p \nabla_\perp^2 \tilde{\Pi}_1^p - \tilde{\Pi}_1^p \nabla_\perp^2 \Pi_1^p) ds = \int_C \left(\Pi_1^p \frac{\partial}{\partial n}\tilde{\Pi}_1^p - \tilde{\Pi}_1^p \frac{\partial}{\partial n}\Pi_1^p\right) ds, \tag{6}$$

where dS is the areal element of region S_j. Using Eqs. (6), Eq. (4) can be rewritten into the following equation which expresses the relation between the electromagnetic fields on C and the potential functions in S_1:

$$-i\omega\varepsilon_1\tau_1^2 \int_{S_1} (\Pi_1^e \nabla_\perp^2 \Pi_1^e - \Pi_1^e \nabla_\perp^2 \Pi_1^e) dS$$
$$+ i\omega\mu_1\tau_1^2 \int_{S_1} (\Pi_1^m \nabla_\perp^2 \tilde{\Pi}_1^m - \tilde{\Pi}_1^m \nabla_\perp^2 \Pi_1^m) dS$$
$$= \int_C (\mathbf{E}_1 \times \tilde{\mathbf{H}}_1 - \tilde{\mathbf{E}}_1 \times \mathbf{H}_1)\cdot \mathbf{n}ds. \tag{7}$$

In a similar way, we can obtain the following equation in region S_2:

$$-i\omega\varepsilon_2\tau_2^2\int_{S_2}(\Pi_2^e\nabla_\perp^2\tilde{\Pi}_2^e-\tilde{\Pi}_2^e\nabla_\perp^2\Pi_2^e)dS$$
$$+i\omega\mu_2\tau_2^2\int_{S_2}(\Pi_2^m\nabla_\perp^2\tilde{\Pi}_2^m-\tilde{\Pi}_2^m\nabla_\perp^2\Pi_j^m)dS$$
$$=\int_C(\mathbf{E}_2\times\tilde{\mathbf{H}}_2-\tilde{\mathbf{E}}_2\times\mathbf{H}_2)\cdot nds. \tag{8}$$

With

$$\tau_2^2=n_2^2k^2-\beta^2, \tag{9}$$

Let us now consider four Green's functions which satisfy the following equations:

wave equations

$$(\nabla_{\perp Q_1}^2+\tau_1^2)\tilde{G}_{11}^{ee}(P_1,Q_1)=-\delta(\overline{P_1Q_1})\quad P_1,Q_1\in S_1, \tag{10}$$

$$(\nabla_{\perp Q_1}^2+\tau_1^2)\tilde{G}_{11}^{ee}(P_1,Q_1)=\ 0\qquad P_1,Q_1\in S_1, \tag{11}$$

$$(\nabla_{\perp Q_2}^2+\tau_2^2)\tilde{G}_{12}^{ee}(P_1,Q_2)=\ 0\qquad P_1\in S_1,\ Q_2\in S_2, \tag{12}$$

$$(\nabla_{\perp Q_2}^2+\tau_2^2)\tilde{G}_{12}^{ee}(P_1,Q_2)=\ 0\qquad P_1\in S_1,\ Q_2\in S_2, \tag{13}$$

boundary conditions on C

$$\mathbf{n}\times\tilde{\mathbf{E}}_{11,G}^e=\mathbf{n}\times[\mathbf{z}\tau_1^2\tilde{G}_{11}^{ee}-i\beta\nabla_\perp\tilde{G}_{11}^{ee}-i\omega\mu_1\mathbf{z}\times\nabla_\perp\tilde{G}_{11}^{em}]$$
$$=\mathbf{n}\times[\mathbf{z}\tau_2^2\tilde{G}_{12}^{ee}-i\beta\nabla_\perp\tilde{G}_{12}^{ee}-i\omega\mu_2\mathbf{z}\times\nabla_\perp\tilde{G}_{12}^{em}]$$
$$=\mathbf{n}\times\tilde{\mathbf{E}}_{12,G}^e, \tag{14}$$

$$\mathbf{n}\times\tilde{\mathbf{H}}_{11,G}^e=\mathbf{n}\times[\mathbf{z}\tau_1^2\tilde{G}_{11}^{em}-i\beta\nabla_\perp\tilde{G}_{11}^{em}+i\omega\epsilon_1\mathbf{z}\times\nabla_\perp\tilde{G}_{11}^{ee}]$$
$$=\mathbf{n}\times[\mathbf{z}\tau_2^2\tilde{G}_{12}^{ee}-i\beta\nabla_\perp\tilde{G}_{12}^{em}+i\omega\varepsilon_2\mathbf{z}\times\nabla_\perp\tilde{G}_{12}^{ee}]$$
$$=\mathbf{n}\times\tilde{\mathbf{H}}_{12,G}^e, \tag{15}$$

where $\overline{P_1Q_1}$ is the distance from the source point P_1 to the field point Q_1, $\delta(\overline{P_1Q_1})$ denotes a delta function, and $\nabla_{\perp Q_1}^2$ is the differential operator with respect to the coordinate of Q_1, and $\tilde{\mathbf{E}}_{1j,G}^e$ and $\tilde{\mathbf{H}}_{1j,G}^e$ denote the electric and magnetic fields deduced by $\tilde{G}_{1j}^{ee}$ and $\tilde{G}_{1j}^{em}$. $\tilde{G}_{1j}^{ee}$ and $\tilde{G}_{1j}^{em}$ are considered to be electric and magnetic potentials in S_j excited from the unit source of TM type located in region S_1. Using the potential functions $\tilde{G}_{11}^{ee}$ and $\tilde{G}_{11}^{em}$ as $\tilde{\Pi}_1^e$ and $\tilde{\Pi}_1^m$ in Eq. (7) and applying Eqs. (10) and (11) to Eq. (7), we have

$$i\omega\varepsilon_1\tau_1^2\Pi_1^e(P_1) = \int_C (\mathbf{E}_1 \times \tilde{\mathbf{H}}_{11,G}^e - \tilde{\mathbf{E}}_{11,G}^e \times \mathbf{H}_1)\cdot \mathbf{n}ds. \tag{16}$$

Using the potential functions $\tilde{G}_{12}^{ee}$ and $\tilde{G}_{12}^{em}$ as $\tilde{\Pi}_2^e$ and $\tilde{\Pi}_2^m$ in Eq. (8) and applying Eqs. (12) and (13) to Eq. (8), we have

$$0 = \int_C (\mathbf{E}_2 \times \tilde{\mathbf{H}}_{12,G}^e - \tilde{\mathbf{E}}_{12,G}^e \times \mathbf{H}_2)\cdot \mathbf{n}ds. \tag{17}$$

Subtracting Eq. (17) from Eq. (16) and taking account of the boundary condition (14) and (15), we obtain

$$i\omega\varepsilon_1\tau_1^2\Pi_1^e(P_1) = \int_C [\mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2)]\cdot \tilde{\mathbf{H}}_{11,G}^e\,ds + \int_C [\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2)]\cdot \tilde{\mathbf{E}}_{11,G}^e\,ds\,. \tag{18}$$

Using the tangential components of electromagnetic fields on C and the Green's functions, we obtain the similar integral representations for Π_1^m, Π_2^e, and Π_2^m to Eq. (18). Let us assume that the electrimagnetic fields satisfy the following relations on C:

$$\mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = \mathbf{f}(s),\ \mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{g}(s) \tag{19}$$

where $\mathbf{f}(s)$ and $\mathbf{g}(s)$ are the vectors related to the arbitrary incident wave of the problem.

2.2 Uniform convergence of approximate potential functions

Referring to the mode-matching method for scalar field, we introduce the complete set of wavefunctions $\{\phi_{j,l}(P_j)\}$ in S_j, which satisfy the radiation condition and the Helmholtz equations[6)]

$$(\nabla_\perp^2 + \tau_j^2)\,\phi_{j,l}(P_j) = 0, \qquad l = 0,\ \pm 1,\ \pm 2, \cdots, \tag{20}$$

and approximate the electric and magnetic potential functions $\Pi_j^e(P_j)$ and $\Pi_j^m(P_j)$ in terms of finite series of wavefunctions as follows:

$$\Pi_{j,N}^p(P_j) = \sum_{l=-N}^{N} A_{j,l}^p(N)\,\phi_{j,l}(P_j), \quad p = e, m \tag{21}$$

where $A_{j,l}^p(N)$ are the expansion coefficients, and N is a truncated number. Since $\Pi_{1,N}^e(P_1)$ satisfies the same Helmholtz equation as $\Pi_1^e(P_1)$, based on the discussion in sec. 2.1, we obtain the following equation:

$$i\omega\varepsilon_1\tau_1^2\Pi_{1,N}^e(P_1) = \int_C [\mathbf{n} \times (\mathbf{E}_{1,N} - \mathbf{E}_{2,N})]\cdot \tilde{\mathbf{H}}_{11,G}^e\,ds + \int_C [\mathbf{n} \times (\mathbf{H}_{1,N} - \mathbf{H}_{2,N})]\cdot \tilde{\mathbf{E}}_{11,G}^e\,ds\,, \tag{22}$$

where $\mathbf{E}_{j,N}$ and $\mathbf{H}_{j,N}$ are the approximate fields given by Eqs. (3) and (21). Subtracting Eq. (22) from Eq. (18) and using Eqs. (19), we have

$$i\omega\varepsilon_1\tau_1^2\ \{\Pi_1^e\ (P_1)-\Pi_{1,N}^e\ (P_1)\}$$
$$=\int_C[\mathbf{f}(s)\ -\mathbf{n}\times\ (\mathbf{E}_{1,N}-\mathbf{E}_{2,N})]\cdot\tilde{\mathbf{H}}_{11,G}^e\,\mathrm{ds}$$
$$+\int_C[\mathbf{g}(s)\ -\mathbf{n}\times\ (\mathbf{H}_{1,N}-\mathbf{H}_{2,N})]\cdot\tilde{\mathbf{E}}_{11,G}^e\,\mathrm{ds}\ . \tag{23}$$

On the condition that P_1 is not on C, $\tilde{\mathbf{E}}_{11,g}^e$ and $\tilde{\mathbf{E}}_{11,g}^e$ are square integrable over the integral path of Eq. (23). Applying the Schwarz inequality to Eq. (23), we obtain

$$|\Pi_1^e(P_1)\ -\ \Pi_{1,N}^e\ (P_1)|\leq\frac{A\ (H_1^e)}{|\omega\ \varepsilon_1\tau_1^2\ |}\ ||\mathbf{f}\ (s\ )\ -\mathbf{n}\times\ (\mathbf{E}_{1,N}-\ \mathbf{E}_{2,N})||$$
$$+\frac{A\ (E_1^e)}{|\omega\ \varepsilon_1\tau_1^2|}\ ||\ \mathbf{g}\ (s\ )\ -\mathbf{n}\times\ (\mathbf{H}_{1,N}-\mathbf{H}_{2,N})\ ||\ , \tag{24}$$

where $A\ (E_1^e)$ and $A\ (H_1^e)$ are positive constants independent of P_1 and $||\ \ ||$ denotes the norm of function defined by

$$||\ \mathbf{E}_{1,N}-\mathbf{E}_{2,N}\ ||\ =\ \left(\int_C|\mathbf{E}_{1,N}-\mathbf{E}_{2,N}\ |^2ds\right)^{\frac{1}{2}} \tag{25}$$

Through the same process, we obtain the similar inequalities for $\Pi_2^e(P_2)$, $\Pi_1^m(P_1)$, and Π_2^m (P_2) to Eq. (24). From these inequalities, we have the following lemma:

Lemma-1 For increasing N, the approximate potential functions $\Pi_{j,N}^e\ (P_j)$ and $\Pi_{j,N}^m$ (P_j) converge uniformly to the true potential functions $\Pi_j^e\ (P_j)$ and $\Pi_j^m(P_j)$ in any closed subset of S_j, respectively, if the relations

$$\lim_{N\to\infty}||\ \mathbf{f}(s)\ -\mathbf{n}\times\ (\mathbf{E}_{1,N}-\mathbf{E}_{2,N})\ ||\ =0,$$
$$\lim_{N\to\infty}||\ \mathbf{g}(s)\ -\mathbf{n}\times\ (\mathbf{H}_{1,N}-\mathbf{H}_{2,N})\ ||\ =0, \tag{26}$$

hold true.

3. Convergence of Approximate Fields to Boundary Values

In this section we prove that the approximate fields vonverge to the boundary value in the sense of mean square. The electric and magnetic potential functions $\Pi_1^e\ (P_1)$ and Π_1^m (P_1) in region S_1 can be represented as follows [3] :

$$\Pi_1^p\ (P_1)\ =\int_C\nu^p(s)\ \zeta\ (kr_1)\,ds,\qquad P_1\in S_1,\ p=e,m, \tag{27}$$

with

$$\zeta\ (kr_1)\ =\frac{1}{4i}H_0^{(1)}\ (kr_1),\quad r_1=\overline{P_1Q},\qquad Q\in C, \tag{28}$$

where $\nu^{p}(s)$ are the single-layer source distributions on boundary C, and $H_0^{(1)}$ is the zero order Hankel function of the first kind. $\Pi_1^{p}(P_1)$ can be approximated by the finite series of modal functions $\phi_{1,l}(P_1)$ as follows:

$$\Pi_{1,N}^{p}(P_1)=\sum_{l=-N}^{N}A_{1,l}^{p}(N)\,\phi_{1,l}(P_1),\qquad P_1\in S_1, \tag{29}$$

where $A_{1,l}^{p}(N)$ are expansion cofficients dependent on the value of N. Using single-layer source distributions $\{\nu_l(s)\}$, the modal functions also can be represented by

$$\phi_{1,l}(P_1)=\int_C\nu_l(s)\,\zeta\ (kr_1)\,ds,\qquad l=0,\ \pm 1,\ \pm 2,\cdots \tag{30}$$

The substitution of Eqs. (30) into Eqs. (29) yields

$$\Pi_{1,N}^{p}(P_1)=\sum_{l=-N}^{N}A_{1,l}^{p}(N)\int_C\nu_l(s)\,\zeta\,(kr_1)\,ds. \tag{31}$$

The subtraction of Eqs. (27) from Eqs. (31) yields

$$\Pi_{1,N}^{p}(P_1)-\Pi_1^{p}(P_1)=\int_C\delta M_N^{p}(S)\,\zeta\ (kr_1)\,ds, \tag{32}$$

with

$$\delta M_N^{p}(s)=\sum_{l=-N}^{N}A_{1,l}^{p}(N)\,\nu_l(s)-\nu^{p}(s)\,. \tag{33}$$

In the limit when the point P_1 in region S_1 approaches the point P on the boundary C, Eqs. (32) bexomes

$$\Pi_{1,N}^{p}(P)-\Pi_1^{p}(P)=\int_C\delta M_N^{p}(s)\,\zeta\ (kr)\ ds,\qquad P\in C, \tag{34}$$

where r is the distance from point P to point Q. Since $\zeta\ (kr)$ is square integrable over the integral path of Eqs. (34), the application of Schwarz inequality to these equations yields

$$||\Pi_{1,N}^{p}(s)-\Pi_1^{p}(s)||\leq A\cdot||\delta M_N^{p}(s)||\,, \tag{35}$$

where A is a positive constant independent of N. The gradient of Eqs. (32) is given by

$$\nabla_{\perp P_1}\{\Pi_{1,N}^{p}(P_1)-\Pi_1^{p}(P_1)\}=\int_C\delta M_N^{p}(s)\,\nabla_{\perp P_1}\zeta\ (kr_1)\ ds,\ P_1\in S_1. \tag{36}$$

In the limit when P_1 approaches P, the inner product of Eqs. (36) and the unit vector **s** in the tangential direction at P becomes

$$\frac{\partial}{\partial s_P}\{\Pi^p_{1,N}(P) - \Pi^p_1(P)\} = \int_C \delta M^p_N(s) \frac{\partial}{\partial s_P} \zeta(kr)\, ds,\ P \in C. \tag{37}$$

Eqiations (37) can be derived by using the concept of the principal value of a Cauchy integral[7]. Since $\partial \zeta(kr)/\partial s_P$ is square integrable over the integral path of Eqs. (37), the application of Schwarz inequality to these equations yields

$$\left\| \frac{\partial}{\partial s_P} \Pi^p_{1,N}(s) - \frac{\partial}{\partial s_P} \Pi_1(s)^p \right\| \leq A_s \cdot \| \delta M^p_N(s) \|, \tag{38}$$

where A_s is a positive constant independent of N. In the limit when P_1 approaches P, the inner product of Eqs. (36) and the unit vector $\boldsymbol{n}$ in the normal direction at P becomes[3]

$$\frac{\partial}{\partial n_P} \Pi^p_{1,N}(P) - \frac{\partial}{\partial n_P} \Pi^p_1(P) = \frac{1}{2} \delta M^p_N(P) + \int_C \delta M^p_N(s) \frac{\partial}{\partial n_P} \zeta(kr)\, ds,$$

$$P \in C. \tag{39}$$

From Eqs. (39), the following inequalities can be derived [8] :

$$\| \delta M^p_N(s) \| \leq A_n \cdot \left\| \frac{\partial}{\partial n_P} \Pi^p_{1,N}(s) - \frac{\partial}{\partial n_P} \Pi^p_1(s) \right\|, \tag{40}$$

where A_n is a positive constant independent of N.

For a complete set of wave functions $\{\phi_{1,l}(P_1)\}$, $\{\partial \phi_{1,l}(s)/\partial n\}$ is complete in the space of L^2 function on the boundary C[3]. Then we have the convergence relations

$$\lim_{N\to\infty} \left\| \frac{\partial}{\partial n_P} \Pi^p_{1,N}(s) - \frac{\partial}{\partial n_P} \Pi^p_1(s) \right\| = 0, \tag{41}$$

for proper expansion coefficients $\{A^p_{1,l}(N)\}$. According to Eqs. (35), (38) and (40), the convergence relations

$$\lim_{N\to\infty} \left\| \frac{\partial}{\partial s_P} \Pi^p_{1,N}(s) - \frac{\partial}{\partial s_P} \Pi^p_1(s) \right\| = 0, \tag{42}$$

$$\lim_{N\to\infty} \| \Pi^p_{1,N}(s) - \Pi^p_1(s) \| = 0, \tag{43}$$

can be satisfied by the expansion coefficients. Also for the potential functions $\Pi^p_2(P_2)$ in region S_2, we have the same relations.

The transformations of $\mathbf{f}(s) - \mathbf{n} \times (\mathbf{E}_{1,N} - \mathbf{E}_{2,N})$ and $\mathbf{g}(s) - \mathbf{n} \times (\mathbf{H}_{1,N} - \mathbf{H}_{2,N})$ by using Eqs. (3) and (19) and application of the Schwarz inequality yield

$$\| \mathbf{f}(s) - \mathbf{n} \times (\mathbf{E}_{1,N} - \mathbf{E}_{2,N}) \|$$

$$\leq \beta \left\| \frac{\partial}{\partial s} \Pi^e_{1,N}(s) - \frac{\partial}{\partial s} \Pi^e_1(s) \right\| + \omega\mu_1 \left\| \frac{\partial}{\partial n} \Pi^m_{1,N}(s) - \frac{\partial}{\partial n} \Pi^m_1(s) \right\|$$

$$+ (n_1^2 k^2 - \beta^2) || \Pi_{1,N}^m (s) - \Pi_1^m(s) ||$$

$$+ \beta || \frac{\partial}{\partial s} \Pi_{2,N}^e (s) - \frac{\partial}{\partial s} \Pi_2^e (s) || + \omega \mu_2 || \frac{\partial}{\partial n} \Pi_{2,N}^m (s) - \frac{\partial}{\partial n} \Pi_2^m (s) ||$$

$$+ (n_2^2 k^2 - \beta^2) || \Pi_{2,N}^m (s) - \Pi_2^m(s) ||, \tag{44}$$

$$|| \mathbf{g} (s) - \mathbf{n} \times (\mathbf{H}_{1,N} - \mathbf{H}_{2,N}) ||$$

$$\leq \beta || \frac{\partial}{\partial s} \Pi_{1,N}^m (s) - \frac{\partial}{\partial s} \Pi_1^m (s) || + \omega \varepsilon_1 || \frac{\partial}{\partial n} \Pi_{1,N}^e (s) - \frac{\partial}{\partial n} \Pi_1^e(s) ||$$

$$+ (n_1^2 k^2 - \beta^2) || \Pi_{1,N}^m (s) - \Pi_1^m (s) ||$$

$$+ \beta || \frac{\partial}{\partial s} \Pi_{2,N}^m (s) - \frac{\partial}{\partial s} \Pi_2^m(s) || + \omega \varepsilon_2 || \frac{\partial}{\partial n} \Pi_{2,N}^e(s) - \frac{\partial}{\partial n} \Pi_2^e (s) ||$$

$$+ (n_2^2 k^2 - \beta^2) || \Pi_{2,N}^m (s) - \Pi_2^m (s) ||, \tag{45}$$

Taking account of Eqs. (41) - (43) and the equations for $\Pi_{2,N}^p (P_2)$ in correspondence to these equations, we obtain the following lemma:

Lemmaw-2 Let $\Pi_{j,N}^e (P_j)$ and $\Pi_{j,N}^m (P_j)$ be approximate electric and magnetic potential functions given by

$$\Pi_{j,N}^p (P_j) = \sum_{l=-N}^{N} A_{j,l}^p(N) \phi_{j,l} (P_j), \quad p = e, m \tag{46}$$

where $\{\phi_{j,l} (P_j)\}$ are complete sets of wavefunctions in S_j. Then there exist infinite sequences of the approximate potential functions $\Pi_{j,N}^e (P_j)$ and $\Pi_{j,N}^m (P_j)$ which satisfy the relations

$$\lim_{N \to \infty} || \mathbf{f} (s) - \mathbf{n} \times (\mathbf{E}_{1,N} - \mathbf{E}_{2,N}) || = 0,$$

$$\lim_{N \to \infty} || \mathbf{g} (s) - \mathbf{n} \times (\mathbf{H}_{1,N} - \mathbf{H}_{2,N}) || = 0, \tag{47}$$

From lemma-1 and lemma-2, we have the Yasuura's theorem rewritten for the vector field in dielectric waveguides as follows:

Yassura's theorem for the vector field in dielectric waveguides :

Let $\Pi_{j,N}^e (P_j)$ and $\Pi_{j,N}^m (P_j)$ be approximate electric and magnetic potential functions given by

$$\Pi_{j,N}^p (P_j) = \sum_{l=-N}^{N} A_{j,l}^p (N) \phi_{j,l} (P_j), \quad p = e, m \tag{48}$$

where $\{\phi_{j,l} (P_j)\}$ are complete sets of wavefunctions in S_j, $A_{j,l}^p (N)$ are the expansion coefficients, and N is a truncated number. Then there exist infinite sequences of

$$\{\Pi^{e}_{j,N}(P_j) : N = 1,2,3,\cdots\},\quad \{\Pi^{m}_{j,N}(P_j) : N = 1,2,3,\cdots\} \tag{49}$$

which satisfy the relations

$$\begin{aligned} \lim_{N\to\infty} \| \mathbf{f}(s) - \mathbf{n} \times (\mathbf{E}_{1,N} - \mathbf{E}_{2,N}) \| &= 0, \\ \lim_{N\to\infty} \| \mathbf{g}(s) - \mathbf{n} \times (\mathbf{H}_{1,N} - \mathbf{H}_{2,N}) \| &= 0, \end{aligned} \tag{50}$$

and converge uniformly to the true potential functions in respective region S_j. Here $\mathbf{E}_{j,N}$ and $\mathbf{H}_{j,N}$ are the approximate electromagnetic fields derived from $\Pi^{e}_{j,N}$ and $\Pi^{m}_{j,N}$.

When $\Pi^{e}_{j,N}(P_j)$ and $\Pi^{m}_{j,N}(P_j)$ vonverge uniformly to $\Pi^{e}_{j}(P_j)$ and $\Pi^{m}_{j}(P_j)$, $\mathbf{E}_{j,N}(P_j)$ and $\mathbf{H}_{j,N}(P_j)$ derived through the differential operation on $\Pi^{e}_{j,N}(P_j)$ and $\Pi^{m}_{j,N}(P_j)$ also converge uniformly to $\mathbf{E}_j(P_j)$ and $\mathbf{H}_j(P_j)$, respectively.

4. Conclusion

In this paper, we have proved the convergence of approximate electric and magnetic potential functions representing vector fields in dielectric waveguides, and showed the Yasuura's theorem rewritten for vector field. This theorem shows that there exist infinite sequences of approximate electric and magnetic potential functions which converge uniformly to the true potential functions and whose approximate field converges to the boundary value in the sense of mean square. By this theorem, the uniform convergence of solutions of the mode-matching method for the vector field problems has been ensured theoretically.

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