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Vortex Rings in Rotating and Stratified Fluids

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The traveling distances of a single vortex ring in rotating and stably stratified fluids have been measured by means of the method of flow visualization. In rotating fluids, the normalized maximum traveling distance along the axis of fluid rotation is inversely proportional to the dimensionless angular velocity (T) at $T > 140$, below which the normalized traveling distance remains to be unity. In stably stratified fluids, the maximum traveling distance in the vertical direction decreases with increasing dimensionless buoyancy frequency (K) over the examined range of K; the rate of its decrease varies appreciably at $K \approx 180$.

1. Introduction

Vortex rings form commonly in diverse types of flows, and play an important role in transporting and mixing homogeneous or inhomogeneous fluids[1,2]. For this reason, many theoretical and experimental investigations concerning a single vortex ring or an assembly of multiple vortex rings have been carried out[3-5].

No vortex rings, however, in rotating or stratified fluids have been investigated extensively. In rotating fluids, the Coriolis force dominates the fluid motion and the flow may be entirely two-dimensional in planes perpendicular to the axis of rotation; the effect is due to the so called Taylor-Proudman theorem[2]. In stratified fluids, buoyancy forces arise as a result of density variations and act in the vertical direction. The motion of vortex rings in rotating or stratified fluids may be necessarily affected by these respective forces[2,6,7].

The purpose of this paper is to report some laboratory results concerning the difference in behavior between the vortex rings in homogeneous fluid at rest and those in either rotating or stratified fluids.

2. Experimental Setup

A side view of experimental apparatus used in the rotating fluid experiment is illustrated in **Fig. 1**; u and Ω indicate the mean speed of a piston stroke and the angular velocity of rotation of the system, respectively. A water tank made of acrylic resin was placed on a rotating table. A thin brass plate with a circular hole for producing vortex rings was set in the tank horizontally as in **Fig. 1**. The thickness of the plate was 1.0mm, and the hole diameter (d) ranged from 2.0 to 3.0 cm. The ratio of the maximum value of d to the dimension of the tank (40.0cm) is 0.075. No effects of the side walls of the tank on the

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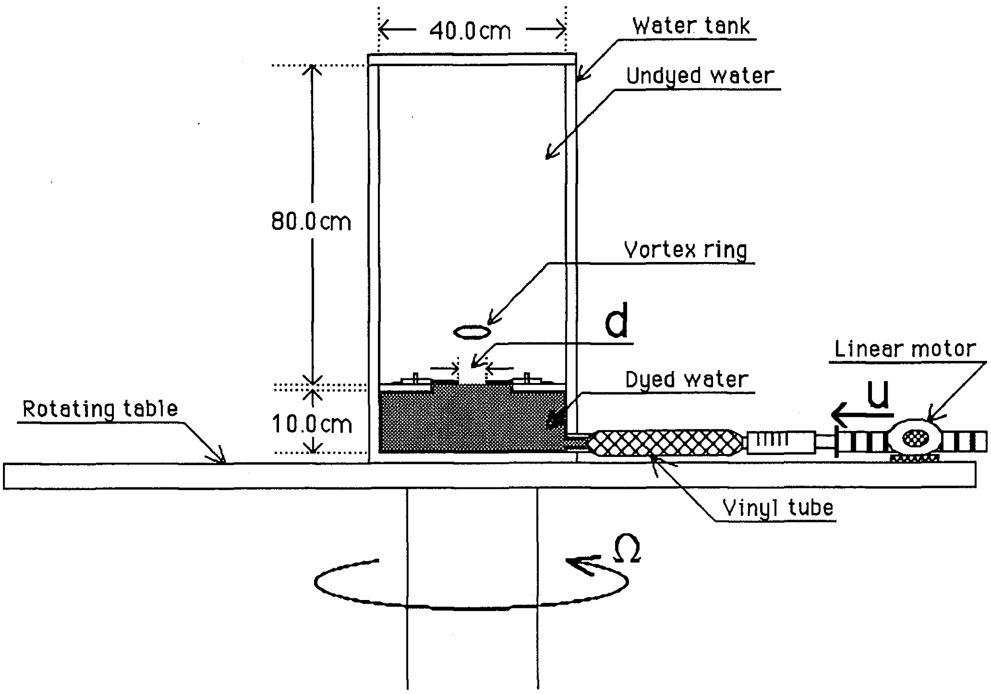


Fig. 1 Experimental apparatus on rotating table.

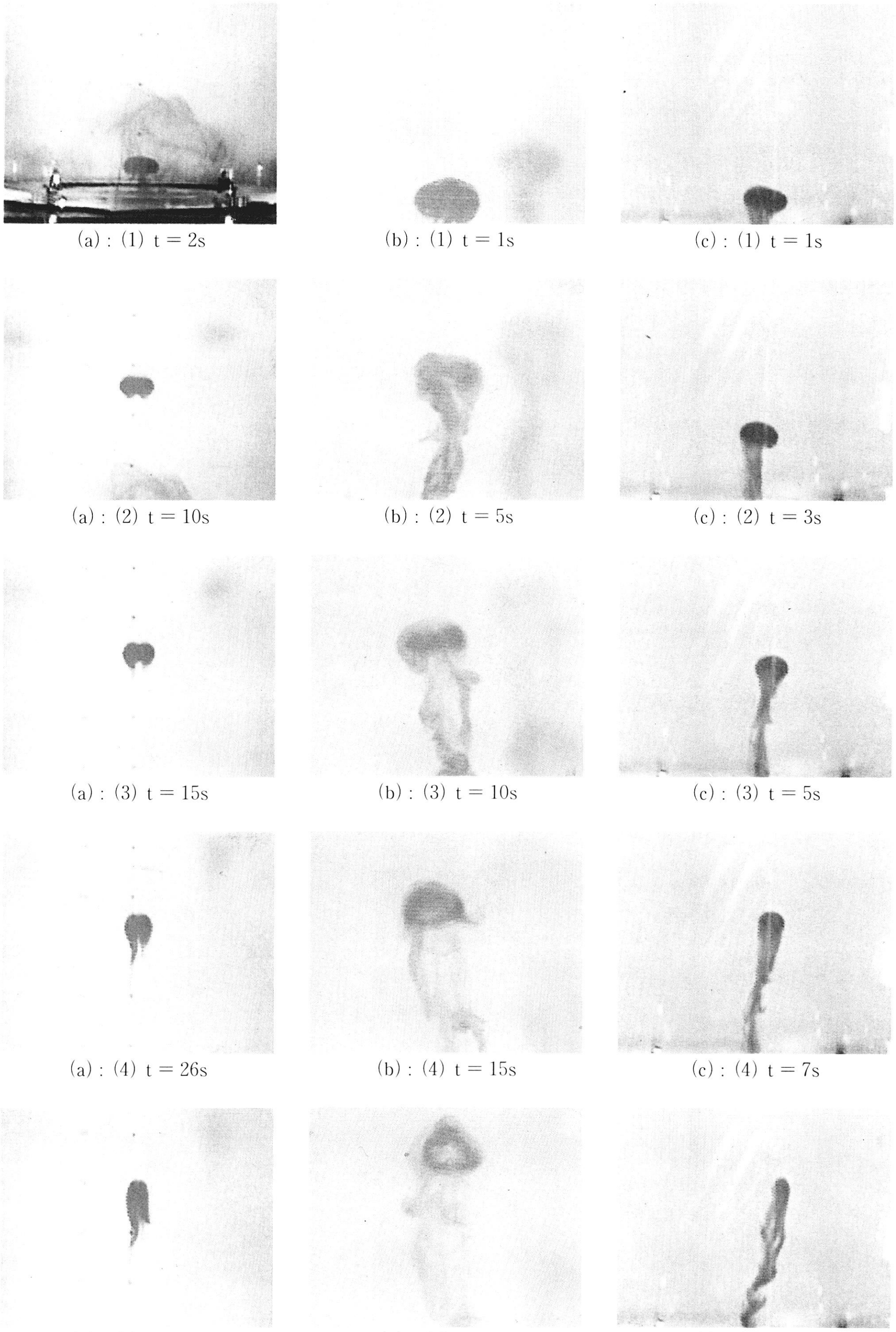
motion of vortex rings have been examined in the present study.

The lower portion of the tank was equipped with a piston system including a linear motor; the speed and amplitude of a piston stroke were made adjustable. Vortex ring motions were visualized using a water-soluble dye. Dyed water was set only in the lower portion of the tank initially, and the upper layer of water was left transparent. As the piston set in motion, a dyed water mass was ejected upward to produce a vortex ring into the upper undyed water. The density difference between the dyed and undyed waters was small and no effect on the vortex ring motion was discernible. The vortex ring motions were video-taped using a camera placed in front of the water tank.

The whole apparatus including the tank and the piston system was placed on a non-rotating table when the stratified-fluid experiment was concerned. The stratification has been set up by means of the two-tank method, and only a linear density profile examined.

3. Results and Discussion

Time-wise developments of the three types of vortex rings are displayed in **Fig. 2**, where (a), (b), and (c) show those in a still, rotating, and stratified fluid, respectively. In each case, the photographs are arranged in the order of increasing time from top to bottom. In a homogeneous fluid at rest (a), a vortex ring initially moves upward rather stably. After a while, however, the ring begins to be distorted and eventually stops to rise up before breaking.



(a) : (1) t = 2s
(b) : (1) t = 1s
(c) : (1) t = 1s

(a) : (2) t = 10s
(b) : (2) t = 5s
(c) : (2) t = 3s

(a) : (3) t = 15s
(b) : (3) t = 10s
(c) : (3) t = 5s

(a) : (4) t = 26s
(b) : (4) t = 15s
(c) : (4) t = 7s

(a) : (5) t = 30s
(b) : (5) t = 20s
(c) : (5) t = 9s

Fig. 2 Side views of vortex rings: (a) Homogeneous fluid at rest, (b) Rotating fluid, (c) Stratified fluid.

In a rotating fluid (b), a vortex ring initially moves upward similarly as in the case of the motion in still water. As time elapses, however, the outer edge of the ring is elongated downward, and the breaking up of the ring occurs at an earlier stage than in a fluid at rest.

In a stably stratified fluid (c), a vortex ring is elongated as a whole in the vertical direction as it moves upward. Owing to the effect of buoyant forces, the upward movement of the ring ceases earlier than in a homogeneous fluid at rest. Then the ring collapses to form a mass of dyed water which falls back downward.

Based on the flow visualization as above described, the traveling distance of each type of a vortex ring has been measured photographically. Notation is given in **Table. 1**. The traveling distance in the vertical direction h is defined as the full distance between the hole and the position of breaking of a vortex ring. Thus, h is the maximum distance traveled by the ring keeping its original shape. The value of h for the case of homogeneous fluids at rest is denoted as h_0. The nondimensional parameters R, T, and K are defined ad ud/ν, $\Omega d^2/\nu$, and $[(g/\rho_0)\ (d\rho/dz)]^{1/2}\ d^2/\nu$, respectively. These are the ratios of the effect of inertia, rotation, and stratification to that of viscosity, respectively. The parameter R is the Reynolds number, T the Ekman number, and K the dimensionless buoyancy frequency. The values of h_0/d plotted against R are shown in **Fig. 3**, in which h_0/d increases monotonically with increasing R. This shows that the traveling distance of a vortex ring increases as R increases; the increase of R means that of u. However, the rate of increase of h_0/d is reduced at about $R \approx 1200$. This seems to take place because the ring is disturbed considerably from the beginning of its formation when R is large enough.

Table. 1 Notation

d : Hole diameter
u : Stroke speed of piston
l : Distance traveled by piston
Ω : Angular velocity
ρ : Fluid density
ν : Kinematic viscosity of fluid
h : Full distance traveled by vortex ring
h_0 : Parameter h for homogeneous fluids at rest
g : Acceleration due to gravity
R : Reynolds number
T : Dimensionless angular velocity
K : Dimensionless buoyancy frequency

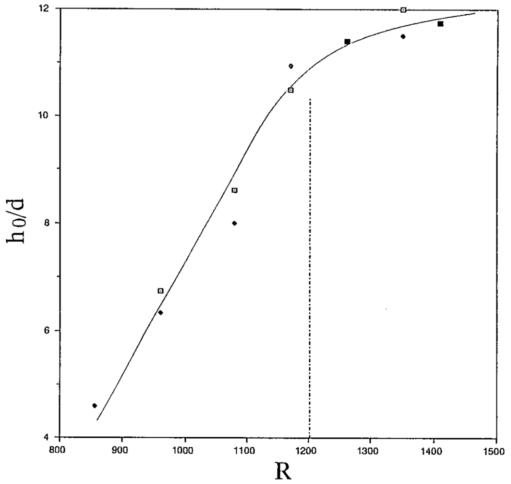


Fig. 3 Dependence of h_0/d on R.

The dependence of h/h_0 on T is shown in **Fig. 4**, in which h/h_0 decreases monotonically with increasing T. In the range $50 < T < 140$, h/h_0 is nearly unity. This means that the traveling distance of a vortex ring does not depend on the rotation rate when T is small. In the range $140 < T < 500$, h/h_0 is inversely propotional to T as satisfying the equation $h/h_0 = 130/T$. The elongation of the outer edge of a vortex ring occurs at an earlier stage, and it is more conspicuous than in the range $T < 140$. Above this critical value of $T_c \approx 140$, the effects of rotation of the system on the motion of vortex rings is remarkable. In **Fig. 5** are plotted h/h_0 against R, and h/h_0 are almost constant over the range $1080 < R < 1410$. This means that the traveling distance normalized by h_0 is independent of u.

The dependence of h/h_0 on K is shown in **Fig. 6**, in which h/h_0 decreases monotonically

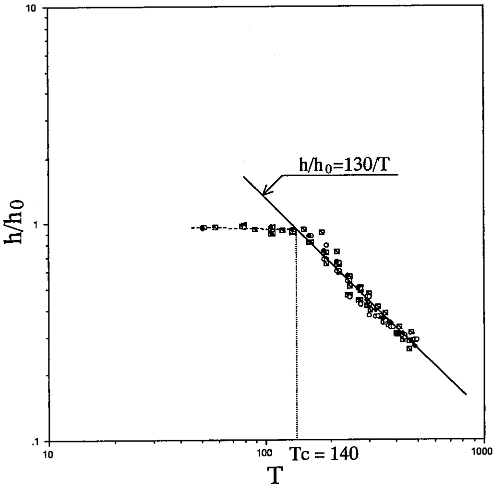


Fig. 4 Dependence of h_0/d on T:R=1080 (□), 1260 (○), and 1410 (◆).

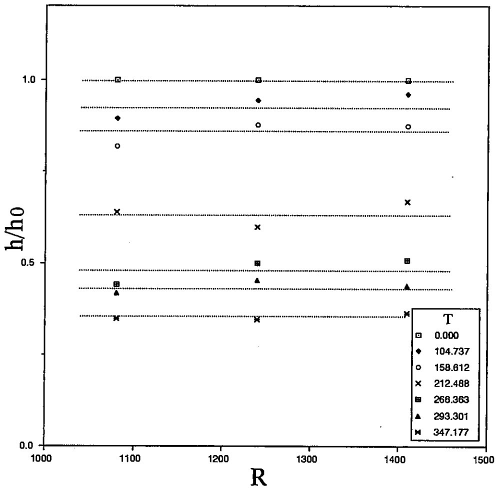


Fig. 5 Dependence of h/h_0 on R.

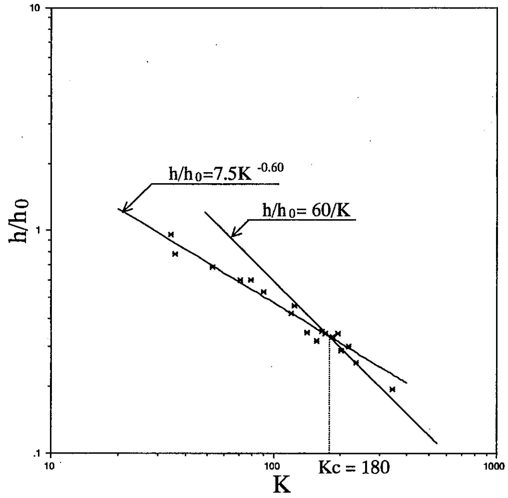


Fig. 6 Dependence of h/h_0 on K

with increasing K. The rate of the decrease of h/h_0 seems to vary at $K=K_c\approx 180$, which is indicated by a dotted line in the figure. The data lie on a line $h/h_0 = 7.5K^{-0.60}$ approximately at $30<K<180$, and on another line $h/h_0 = 60/T$ approximately at $180<K<400$.

In both cases h/h_0 do not exceed unity, i. e. the effect of either rotation or stratification of fluids is to reduce the maximum traveling distance of a single vortex ring.

Conclusions

The main results so far described are summarized as follows.

(1) In rotating fluids, the vertical traveling distance of vortex rings depends on T as $h/h_0 = 130/T$ at $T>T_c\approx 140$. The vortex rings in rotating and non-rotating fluids can travel almost the same distance when $T<140$. Above $T>140$, however, the vortex rings collapse at smaller traveling distances as T increases. In this range, the traveling distance decreases monotonically with T.

(2) In stratified fluids, the traveling distance of vortex rings depends on K as $h/h_0 = 7.5K^{-0.60}$ at $K<K_c\approx 180$, and $h/h_0 = 60/K$ at $K>K_c$. The distance traveled by the vortex rings in stably stratified fluids does not exceed that traveled by the rings in non-stratified homogeneous fluids. The traveling distance decreases monotonically with K over the whole range of K examined.

References

1) Tatsumi, T.: *Fluid Dynamics* (Baifukan, Tokyo, 1982: in Japanese)

2) Batchelor, G. K.: *An introduction to fluid dynamics* (Cambridge University Press, Cambridge, 1967)

3) Fraenkel, L. E.: On steady vortex rings of small cross-section in an ideal fluid, Proc. Roy. Soc. Lond. **A. 316** (1970) 29-62.

4) Norbury, J.: A family of steady vortex rings, J Fluid Mech. **57** (1973) 417-431.

5) Widnall, S. E. & Sullivan, J. P.: On the stability of vortex rings, Proc, Roy. Soc. Lond. **A. 332** (1973) 335-353.

6) Turner, J. S.: Buoyant vortex rings, Proc. Roy. Soc. Lond. **A. 239** (1957) 61-75.

7) Honji, H. & Tatsuno, M.: Vortex rings in a stratified fluid, J. Phys. Soc. Jpn. **41** (1976) 2121-2125.