Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field

Takaesu, Toshimitsu
Faculty of Mathematics, Kyushu University

<http://hdl.handle.net/2324/16962>

出版情報 : MI Preprint Series. MI 2010-18, 2010-04-16. 九州大学大学院数理学研究院
バージョン :
権利関係 :

MI Preprint Series

**Kyushu University
The Global COE Program
Math-for-Industry Education & Research Hub**

Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field

Toshimitsu Takaesu

MI 2010-18

(Received April 16, 2010)

Faculty of Mathematics
Kyushu University
Fukuoka, JAPAN

Scaling Limits for the System of Semi-Relativistic Particles Coupled to a Scalar Bose Field

Toshimitsu TAKAESU

*Faculty of Mathematics, Kyushu University,
Fukuoka, 812-8581, Japan*

Abstract. In this paper the Hamiltonian for the system of semi-relativistic particles interacting with a scalar bose field is investigated. A scaled total Hamiltonian of the system is defined and its scaling limit is considered. Then the semi-relativistic Schrödinger operator with an effective potential is derived.

Mathematics Subject Classification 2010 : 81Q10, 62M15.

key words : Spectral analysis, Relativistic Schrödinger operator, Quantum field theory.

1 Introduction

In this paper we consider the Hamiltonian of the system of N particles linearly coupled to a scalar bose field. We assume that particles obey the semi-relativistic Schrödinger operator

$$H_p = \sum_{j=1}^N \sqrt{-\Delta_j + M^2},$$

where $M > 0$ is a rest mass. There has been many results on the spectral properties of H_p . Refer to e.g. [21, 6, 3, 4, 5, 20]. The free Hamiltonian H_b of the scalar bose field is defined by the second quantization of the multiplication operator ω , which is formally expressed by

$$H_b = \int_{\mathbf{R}^d} \omega(\mathbf{k}) a^*(\mathbf{k}) a(\mathbf{k}) d\mathbf{k}$$

The state space of the interacting system is defined by $\mathcal{H} = L^2(\mathbf{R}_x^{dN}) \otimes \mathcal{F}_b(L^2(\mathbf{R}_k^d))$ where $\mathcal{F}_b(L^2(\mathbf{R}_k^d))$ is the boson Fock space on $L^2(\mathbf{R}_k^d)$. The total Hamiltonian is given by

$$H = H_p \otimes I + I \otimes H_b + \kappa H_I, \quad \kappa \in \mathbf{R}. \quad (1)$$

Here the interaction H_I is denoted by formally

$$H_I = \sum_{j=1}^N \frac{1}{\sqrt{2}} \int_{\mathbf{R}^d} \left(\overline{f_{x_j}(\mathbf{k})} \otimes a(\mathbf{k}) + f_{x_j}(\mathbf{k}) \otimes a^*(\mathbf{k}) \right),$$

where f_x is an multiplication operator on $L^2(\mathbf{R}_x^d)$.

We consider the scaled Hamiltonian

$$H(\Lambda) = H_p \otimes I + \Lambda^2 I \otimes H_b + \kappa \Lambda H_I, \quad \Lambda > 0. \quad (2)$$

We investigate the asymptotic behavior of $H(\Lambda)$ as $\Lambda \rightarrow \infty$. The unitary evolution $e^{-itH(\Lambda)}$ generated by $H(\Lambda)$ is given by

$$e^{-itH(\Lambda)} = e^{-i\Lambda^2 t \left(\sum_{j=1}^N \sqrt{\left(\frac{\hat{\mathbf{p}}_j}{\Lambda^2}\right)^2 + \left(\frac{M}{\Lambda^2}\right)^2} + H_b + \left(\frac{\kappa}{\Lambda}\right) H_I \right)}.$$

Here $\Lambda^2 t$ denotes the scaled time, $\Lambda^{-2} \hat{\mathbf{p}}$ the scaled momentum for $\hat{\mathbf{p}} = -i\nabla$, $\Lambda^{-2} M$ the scaled mass, and $\Lambda^{-1} \kappa$ the scaled coupling constant. As far as we know scaling limits of the Hamiltonians of the form (2) is initiated by E. B. Davies [2], where $H(\Lambda)$ with semi-relativistic Schrödinger operator replaced by a standard Schrödinger operator is considered and a Schrödinger operator with an effective potential is derived as $\Lambda \rightarrow \infty$. This model is called the Nelson model, and our result can be regarded as a semi-relativistic version of [2]. In [1], a general theory of scaling limits is established and it is applied to scaling limits of a spin-boson model and non-relativistic QED models. In [10], by removing ultraviolet cutoffs and taking a scaling limit of the Nelson model *simultaneously*, a Schrödinger operator with the Yukawa potential or the Coulomb potential is derived. Refer to see also [8, 17, 18, 16, 19].

In the main theorem, it is shown that for $z \in \mathbf{Z} \setminus \mathbf{R}$,

$$s - \lim_{\Lambda \rightarrow \infty} (H(\Lambda) - z)^{-1} = \left(\sum_{j=1}^N \sqrt{-\Delta_j + M^2} + V_{\text{eff}}(\mathbf{x}_1, \dots, \mathbf{x}_N) - z \right)^{-1} P_{\Omega_b}, \quad (3)$$

where

$$V_{\text{eff}}(\mathbf{x}_1, \dots, \mathbf{x}_N) = -\frac{\kappa^2}{4} \sum_{j,l} \int_{\mathbf{R}^d} \frac{\overline{f_{\mathbf{x}_j}(\mathbf{k})} f_{\mathbf{x}_l}(\mathbf{k}) + \overline{f_{\mathbf{x}_l}(\mathbf{k})} f_{\mathbf{x}_j}(\mathbf{k})}{\omega(\mathbf{k})} d\mathbf{k},$$

and P_{Ω_b} is the projection onto the closed subspace spanned by the Fock vacuum Ω_b of the bose field.

For the strategy of the proof of the main theorem, we use a unitary transformation, called *the dressing transformation*. Then we apply the general theory investigated in [1] to the unitary transformed Hamiltonian $U(\Lambda)^{-1} H(\Lambda) U(\Lambda)$, and the consider the asymptotic behavior of $U(\Lambda)^{-1} H(\Lambda) U(\Lambda)$ as $\Lambda \rightarrow \infty$.

This paper is organized as follows. In Section 2, the theory of boson Fock space is described. Then the total state space and the total Hamiltonian is defined, and the main results are stated. In Section 3, the proof of the main theorem is given.

2 Main Results

2.1 Boson Fock Spaces

In this subsection we give the mathematically rigorous definition of the bose field. The state space of the bose field is given by the boson Fock space $\mathcal{F}_b(L^2(\mathbf{R}^d)) = \bigoplus_{n=0}^{\infty} (\bigotimes_s^n L^2(\mathbf{R}^d))$, where $\bigotimes_s^n L^2(\mathbf{R}^d)$ denotes the n -fold symmetric tensor product of $L^2(\mathbf{R}^d)$ with $\bigotimes_s^0 L^2(\mathbf{R}^d) := \mathbf{C}$. The Fock vacuum is defined by $\Omega_b = \{1, 0, 0, \dots\} \in \mathcal{F}_b(L^2(\mathbf{R}^d))$. The finite particle subspace $\mathcal{F}_b^{\text{fin}}(\mathcal{D})$ on the subspace $\mathcal{D} \subset L^2(\mathbf{R}^d)$ is defined by the set of $\Psi = \{\Psi^{(n)}\}_{n=0}^{\infty}$ satisfying that $\Psi^{(n)} \in \bigotimes_s^n \mathcal{D}$, $n \geq 0$, and $\Psi^{(n')} = 0$ for all $n' > N$ with some $N \geq 0$. Let $a(\xi)$, $\xi \in L^2(\mathbf{R}^3)$, and $a^*(\eta)$, $\eta \in L^2(\mathbf{R}^d)$, be the annihilation operator and the creation operator on $\mathcal{F}_b(L^2(\mathbf{R}^d))$, respectively. Then they satisfy the canonical commutation relations on $\mathcal{F}_b^{\text{fin}}(L^2(\mathbf{R}^d))$:

$$[a(\xi), a^*(\eta)] = (\xi, \eta), \quad [a(\xi), a(\eta)] = [a^*(\xi), a^*(\eta)] = 0.$$

Let S be a self-adjoint operator on $L^2(\mathbf{R}^d)$. The second quantization of S is defined by

$$d\Gamma(S) = \bigoplus_{n=0}^{\infty} \left(\sum_{j=1}^n (I \otimes \dots \otimes I \otimes \underbrace{S}_{j\text{th}} \otimes I \dots \otimes I) \right),$$

For $\eta \in \mathcal{D}(S^{-1/2})$, it is seen that $a(\eta)$ and $a^*(\eta)$ are relatively bounded with respect to $d\Gamma(S)^{1/2}$ with the bound

$$\|a(\eta)\Psi\| \leq \|S^{-1/2}\eta\| \|d\Gamma(S)^{1/2}\Psi\|, \quad \Psi \in \mathcal{D}(d\Gamma(S)^{1/2}), \quad (4)$$

$$\|a^*(\eta)\Psi\| \leq \|S^{-1/2}\eta\| \|d\Gamma(S)^{1/2}\Psi\| + \|\eta\| \|\Psi\|, \quad \Psi \in \mathcal{D}(d\Gamma(S)^{1/2}). \quad (5)$$

The field operator and its conjugate operator are defined by

$$\phi(\xi) = \frac{1}{\sqrt{2}} \left(a(\xi) + a^*(\xi) \right), \quad \Pi(\eta) = \frac{i}{\sqrt{2}} \left(-a(\eta) + a^*(\eta) \right).$$

2.2 Main Theorem

In this subsection we define the total Hamiltonian and state the main results. The state space of the system for the N -particles coupled to bose field is defined by

$$\mathcal{H} = L^2(\mathbf{R}_x^{dN}) \otimes \mathcal{F}_b(L^2(\mathbf{R}_k^d)).$$

The free Hamiltonian of particles and the bose field are defined by

$$H_p = \sum_{j=1}^N \sqrt{-\Delta_j + M^2}, \quad H_b = d\Gamma_b(\omega),$$

where $M > 0$ is a rest mass and ω denotes the multiplication operator by the function $\omega(\mathbf{k})$, which describes the energy of the boson with momentum $\mathbf{k}$. We assume the following condition :

(A.1) ω is non-negative.

The interaction H_I is defined by

$$H_I = \sum_{j=1}^N \phi(f_{\mathbf{x}_j}),$$

where $f_{\mathbf{x}}$ is the multiplication operator satisfying the following condition :

$$(A.2) \quad \sup_{\mathbf{x} \in \mathbf{R}^d} \int_{\mathbf{R}^d} |f_{\mathbf{x}}(\mathbf{k})|^2 d\mathbf{k} < \infty, \quad \text{and} \quad \sup_{\mathbf{x} \in \mathbf{R}^d} \int_{\mathbf{R}^d} \frac{|f_{\mathbf{x}}(\mathbf{k})|^2}{\omega(\mathbf{k})} d\mathbf{k} < \infty.$$

The total Hamiltonian of this system is given by

$$H = H_0 + \kappa H_I,$$

where $H_0 = H_p \otimes I + I \otimes H_b$. By (4), (5), and the assumption (A.2), it is seen that the H_I is relatively bounded with respect to $I \otimes H_b^{1/2}$. Hence H_I is relatively bounded with respect to H_0 with infinitely small bound. Then the Kato-Rellich theorem shows that H is self-adjoint and essentially self-adjoint any core of H_0 . Then in particular, H is essentially self-adjoint on $\mathcal{D}_0 = C_0^\infty(\mathbf{R}^{dN}) \hat{\otimes} \mathcal{F}_b^{\text{fin}}(\mathcal{D}(\omega))$, where $\hat{\otimes}$ denotes the algebraic tensor product.

Let us introduce the scaled total Hamiltonian

$$H(\Lambda) = H_0(\Lambda) + \kappa \Lambda H_I,$$

where $H_0(\Lambda) = H_p \otimes I + \Lambda^2 I \otimes H_b$. We introduce an additional assumption on the interaction.

$$(A.3) \quad \sup_{\mathbf{x} \in \mathbf{R}^d} \int_{\mathbf{R}^d} \frac{|f_{\mathbf{x}}(\mathbf{k})|^2}{\omega(\mathbf{k})^2} d\mathbf{k} < \infty, \quad \text{and} \quad \sup_{\mathbf{x} \in \mathbf{R}^d} \int_{\mathbf{R}^d} \frac{|f_{\mathbf{x}}(\mathbf{k})|^2}{\omega(\mathbf{k})^3} d\mathbf{k} < \infty.$$

$$(A.4) \quad \sup_{\mathbf{x} \in \mathbf{R}^d} \int_{\mathbf{R}^d} \frac{|\partial_{\mathbf{x}}^v f_{\mathbf{x}}(\mathbf{k})|^2}{\omega(\mathbf{k})^2} d\mathbf{k} < \infty, \quad \sup_{\mathbf{x} \in \mathbf{R}^d} \int_{\mathbf{R}^d} \frac{|\triangle f_{\mathbf{x}}(\mathbf{k})|^2}{\omega(\mathbf{k})^2} d\mathbf{k} < \infty \quad \text{and} \quad (\frac{\partial_{\mathbf{x}}^v f_{\mathbf{x}}}{\omega}, \frac{f_{\mathbf{y}}}{\omega}) \in \mathbf{R}, \quad \mathbf{x}, \mathbf{y} \in \mathbf{R}^d.$$

Under the condition $(\frac{\partial_{\mathbf{x}}^v f_{\mathbf{x}}}{\omega}, \frac{f_{\mathbf{y}}}{\omega}) \in \mathbf{R}$ in (A.4), it follows that $[\Pi(\frac{\partial_{\mathbf{x}}^v f_{\mathbf{x}}}{\omega}), \Pi(\frac{f_{\mathbf{y}}}{\omega})] = 0, \mathbf{x}, \mathbf{y} \in \mathbf{R}^d$.

Remark 2.1 Let us define that $f_{\mathbf{x}}(\mathbf{k}) = \frac{\chi_R(|\mathbf{k}|)}{\sqrt{\omega(\mathbf{k})}} e^{-i\mathbf{k} \cdot \mathbf{x}}$ with $\omega(\mathbf{k}) = \omega(-\mathbf{k})$. Here χ_R denotes the characteristic function on $[0, R)$. Then the conditions (A.1)-(A.4) are satisfied, and the interaction H_I is formally expressed by

$$H_I = \sum_{j=1}^N \int_{\mathbf{R}^d} \frac{\chi_R(|\mathbf{k}|)}{\sqrt{2\omega(\mathbf{k})}} \left(a(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}_j} + a^*(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{x}_j} \right) d\mathbf{k}.$$

The main theorem in this paper is as follows

Theorem 2.1 Assume (A.1)-(A.4). Then for $z \in \mathbb{C} \setminus \mathbb{R}$ it follows that

$$s - \lim_{\Lambda \rightarrow \infty} (H(\Lambda) - z)^{-1} = (H_p + V_{\text{eff}}(\mathbf{x}_1, \dots, \mathbf{x}_n) - z) \otimes P_{\Omega_b},$$

where

$$V_{\text{eff}}(\mathbf{x}_1, \dots, \mathbf{x}_n) = -\frac{\kappa^2}{4} \sum_{j,l} \int_{\mathbb{R}^d} \frac{\overline{f_{\mathbf{x}_j}(\mathbf{k})} f_{\mathbf{x}_l}(\mathbf{k}) + \overline{f_{\mathbf{x}_l}(\mathbf{k})} f_{\mathbf{x}_j}(\mathbf{k})}{\omega(\mathbf{k})} d\mathbf{k},$$

and P_{Ω_b} is the projection onto the closed subspace spanned by the Fock vacuum Ω_b .

Remark 2.2 When $f_{\mathbf{x}}(\mathbf{k}) = \frac{\chi_R(|\mathbf{k}|)}{\sqrt{\omega(\mathbf{k})}} e^{-i\mathbf{k} \cdot \mathbf{x}}$ with $\omega(\mathbf{k}) = \omega(-\mathbf{k})$, the effective potential is given by

$$V_{\text{eff}}(\mathbf{x}_1, \dots, \mathbf{x}_n) = -\frac{\kappa^2}{2} \sum_{j,l} \int_{\mathbb{R}^d} \frac{|\chi_R(\mathbf{k})|^2}{\omega(\mathbf{k})^2} e^{-i\mathbf{k} \cdot (\mathbf{x}_j - \mathbf{x}_l)} d\mathbf{k},$$

By using the norm convergence theorem considered in ([18] ; Lemma 2.7), the the next corollary follows.

Corollary 2.2 Assume (A.1)-(A.4). Then for $z \in \mathbb{C} \setminus \mathbb{R}$ it follows that

$$s - \lim_{\Lambda \rightarrow \infty} e^{-itH(\Lambda)} (I \otimes P_{\Omega_b}) = e^{-it(H_p + V_{\text{eff}}(\mathbf{x}_1, \dots, \mathbf{x}_n))} \otimes P_{\Omega_b}.$$

3 Proof of Main Theorem

The outline of the proof of Theorem 2.1 is as follows. A unitary transformation $U(\Lambda)$, called the dressing transformation, is defined and we consider the unitarily transformed Hamiltonian $U(\Lambda)^{-1}H(\Lambda)U(\Lambda)$. Then we apply the general theory on scaling limits in [1] to $U(\Lambda)^{-1}H(\Lambda)U(\Lambda)$.

Under the condition (A.3), the following unitary operator can be defined :

$$U(\Lambda) = e^{i\left(\frac{\kappa}{\Lambda}\right) \sum_{j=1}^N \Pi\left(\frac{f_{\mathbf{x}_j}}{\omega}\right)}.$$

It is seen that on the finite particle subspace

$$[\Pi(\xi), H_b] = -i\phi(\omega\xi), \quad \xi \in \mathcal{D}(\omega), \quad (6)$$

$$[\Pi(\xi), \phi(\eta)] = \frac{-i}{2} \left((\xi, \eta) + (\eta, \xi) \right), \quad \xi, \eta \in L^2(\mathbb{R}^3). \quad (7)$$

By (6) and (7), we have

$$U(\Lambda)^{-1}H(\Lambda)U(\Lambda) = H_0(\Lambda) + K(\Lambda) \quad (8)$$

where

$$K(\Lambda) = U(\Lambda)^{-1} (H_p \otimes I) U(\Lambda) - H_p \otimes I + V_{\text{eff}}(\mathbf{x}_1, \dots, \mathbf{x}_N). \quad (9)$$

Now we apply the general theory on scaling limits investigated in [1]. Let us set the total Hilbert space by $\mathcal{Z} = \mathcal{X} \otimes \mathcal{Y}$. Let A and B be non-negative self-adjoint operators on $\mathcal{X}$ and $\mathcal{Y}$, respectively. Here we assume that $\ker B \neq \{0\}$. We consider a family of symmetric operators $\{C(\Lambda)\}_{\Lambda>0}$ satisfying the conditions :

(S.1) For all $\varepsilon > 0$ there exists a constant $\Lambda(\varepsilon) > 0$ such that for all $\Lambda > \Lambda(\varepsilon)$, $\mathcal{D}(A \otimes I) \cap \mathcal{D}(I \otimes B) \subset \mathcal{D}(C(\Lambda))$, and there exists $b(\varepsilon) \geq 0$ such that

$$\|C(\Lambda)\Phi\| \leq \varepsilon\|(A \otimes I + \Lambda I \otimes B)\Phi\| + b(\varepsilon)\|\Phi\|.$$

(S.2) There exists a symmetric operator C on $\mathcal{Z}$ such that $\mathcal{D} \otimes \ker B \subset \mathcal{D}(C)$ and for all $z \in \mathbf{C} \setminus \mathbf{R}$,

$$s - \lim_{\Lambda \rightarrow \infty} C(\Lambda)(A \otimes I + \Lambda I \otimes B - z) = C(A - z)^{-1} \otimes P_B,$$

where P_B is the orthogonal projection from $\mathcal{Y}$ onto $\ker B$.

Proposition A ([1] ; Theorem 2.1) Assume (S.1) and (S.2). Then (i)-(iii) follows.

(i) There exists $\Lambda_0 \geq 0$ such that for all $\Lambda > \Lambda_0$,

$$X(\Lambda) = A \otimes I + \Lambda I \otimes B + C(\Lambda)$$

is self-adjoint on $\mathcal{D}(A \otimes I) \cap \mathcal{D}(I \otimes B)$ and uniformly bounded from below for Λ , furthermore $X(\Lambda)$ is essentially self-adjoint on any core of $A \otimes I + I \otimes B$.

(ii) Let $X = A \otimes I + (I \otimes P_B)C(I \otimes P_B)$. Then X is self-adjoint on $\mathcal{D}(A \otimes I)$ and bounded from below, and essentially self-adjoint on any core of $A \otimes I$.

(iii) Let $z \in \bigcap_{\Lambda \geq \Lambda_0} \rho(X(\Lambda)) \cap \rho(X)$, where $\rho(\mathcal{O})$ denotes the resolvent set of an operator $\mathcal{O}$. Then

$$s - \lim_{\Lambda \rightarrow \infty} (X(\Lambda) - z)^{-1} = (X - z)^{-1}(I \otimes P_B).$$

Now we consider $H(\Lambda)$ again. What we have to prove is that $H(\Lambda)$ satisfies the condition (S.1) and (S.2) by applying $H_0(\Lambda)$ to $A \otimes I + \Lambda I \otimes B$ and $K(\Lambda)$ to $C(\Lambda)$. First let us consider the term $U(\Lambda)^{-1}(H_p \otimes I)U(\Lambda)$ in (9). Let us set $\hat{\mathbf{p}} = (\hat{p}^1, \dots, \hat{p}^d) = (-i \frac{\partial}{\partial x^1}, \dots, -i \frac{\partial}{\partial x^d})$. Then by the spectral decomposition theorem,

$$U(\Lambda)^{-1}(H_p \otimes I)U(\Lambda) = \sum_{j=1}^N \sqrt{\left(U(\Lambda)^{-1}(\mathbf{p}_j \otimes I)U(\Lambda)\right)^2 + M^2}, \quad (10)$$

follows. We see that

$$[\Pi(f_{\mathbf{x}}), \hat{p}^v] = i\Pi(\partial_{x^v} f_{\mathbf{x}}). \quad (11)$$

Then by (A.4), it follows that for $\Psi \in \mathcal{D}_0$,

$$\left(U(\Lambda)^{-1}(\mathbf{p}_j \otimes I)U(\Lambda)\right)^2 \Psi = \left(\sum_{v=1}^d \left(\hat{p}_j^v \otimes I + \left(\frac{\kappa}{\Lambda}\right) \Pi\left(\frac{\partial_{x_j^v} f_{\mathbf{x}_j}}{\omega}\right)\right)^2 + M^2\right) \Psi.$$

Then we have

$$\left(U(\Lambda)^{-1}(\mathbf{p}_j \otimes I)U(\Lambda)\right)^2 = \overline{\left(\sum_{v=1}^d \left(\hat{p}_j^v \otimes I + \left(\frac{\kappa}{\Lambda}\right) \Pi\left(\frac{\partial_{x_j^v} f_{\mathbf{x}_j}}{\omega}\right)\right)^2 + M^2\right)}_{\upharpoonright \mathcal{D}_0}, \quad (12)$$

where $\bar{Y}$ denotes the closure of the operator Y . Here we abbreviate as

$$\begin{aligned}\Pi\left(\frac{\nabla f_{\mathbf{x}}}{\omega}\right) \cdot (\hat{\mathbf{p}} \otimes I) &= \sum_{v=1}^d \Pi\left(\frac{\partial_{x^v} f_{\mathbf{x}}}{\omega}\right) (\hat{p}^v \otimes I), \\ \Pi\left(\frac{\nabla f_{\mathbf{x}}}{\omega}\right) \cdot \Pi\left(\frac{\nabla f_{\mathbf{x}}}{\omega}\right) &= \sum_{v=1}^d \Pi\left(\frac{\partial_{x^v} f_{\mathbf{x}}}{\omega}\right) \phi\left(\frac{\partial_{x^v} f_{\mathbf{x}}}{\omega}\right).\end{aligned}$$

Then we see that

$$U(\Lambda)^{-1} (H_p \otimes I) U(\Lambda) = \sum_{j=1}^N \left(\overline{\left(-\Delta_j \otimes I + Q_j(\Lambda) + M^2 \right)}_{\upharpoonright \mathcal{D}_0} \right)^{1/2}, \quad (13)$$

where

$$Q_j(\Lambda) = \left(\frac{\kappa}{\Lambda} \right) \left(2\Pi\left(\frac{\nabla f_{\mathbf{x}_j}}{\omega}\right) \cdot (\hat{\mathbf{p}}_j \otimes I) - i\Pi\left(\frac{\Delta f_{\mathbf{x}}}{\omega}\right) \right) + \left(\frac{\kappa}{\Lambda} \right)^2 \Pi\left(\frac{\nabla f_{\mathbf{x}_j}}{\omega}\right) \cdot \Pi\left(\frac{\nabla f_{\mathbf{x}_j}}{\omega}\right).$$

Proposition 3.1 *Assume (A.1)-(A.4). Then for $\varepsilon > 0$, there exists $\Lambda(\varepsilon) \geq 0$ such that for all $\Lambda > \Lambda(\varepsilon)$,*

$$\|U(\Lambda)^{-1} (H_p \otimes I) U(\Lambda) \Psi - (H_p \otimes I) \Psi\| \leq \varepsilon \|H_0(\Lambda) \Psi\| + b(\varepsilon) \|\Psi\| \quad (14)$$

where $b(\varepsilon)$ is a constant independent of $\Lambda \geq \Lambda(\varepsilon)$.

Before proving Proposition 3.1, we show the following lemma.

Lemma 3.2 *For $\lambda > 0$ and $\delta \in (0, \frac{1}{10})$, there exists $M_v(\delta)$, $v = 1, \dots, d$, such that*

$$\|\hat{p}^v (-\Delta + M^2 + \lambda)^{-1} (\sqrt{-\Delta + M^2 + 1})^{-1/2}\| \leq \frac{1}{\lambda^{\frac{1}{2}+\delta}} M_v(\delta). \quad (15)$$

(Proof) For $\mathbf{p} = (p^1, \dots, p^d) \in \mathbf{R}^d$, $v = 1, \dots, d$, we see that

$$|p^v (\mathbf{p}^2 + M^2 + \lambda)^{-1} (\sqrt{\mathbf{p}^2 + M^2 + 1})^{-1/2}| = \frac{1}{\lambda^{\frac{1}{2}+\delta}} |\lambda^{\frac{1}{2}+\delta} p^v| (\mathbf{p}^2 + M^2 + \lambda)^{-1} (\sqrt{\mathbf{p}^2 + M^2 + 1})^{-1/2}.$$

We shall show that

$$\sup_{\lambda > 0, \mathbf{p} \in \mathbf{R}^d} |\lambda^{\frac{1}{2}+\delta} p^v| (\mathbf{p}^2 + M^2 + \lambda)^{-1} (\sqrt{\mathbf{p}^2 + M^2 + 1})^{-1/2} < \infty, \quad (16)$$

and hence (15) follows from the spectral decomposition theorem. The Young's inequality shows that for $q > 1$ and $\tilde{q} > 1$ satisfying $\frac{1}{q} + \frac{1}{\tilde{q}} = 1$,

$$\lambda^{\frac{1}{2}+\delta} |p^v| \leq \frac{1}{q} \lambda^{(\frac{1}{2}+\delta)q} + \frac{1}{\tilde{q}} |p^v|^{\tilde{q}} \quad (17)$$

follows. Let us take $q = (\frac{1}{2} + \delta)^{-1}$ for $\delta \in (0, \frac{1}{10})$, and hence $\tilde{q} = (\frac{1}{2} - \delta)^{-1}$. Then we have

$$\lambda^{\frac{1}{2}+\delta} |p^v| \leq \left(\frac{1}{2} + \delta\right) \lambda + \left(\frac{1}{2} - \delta\right) |p^v|^{(\frac{1}{2}-\delta)^{-1}}. \quad (18)$$

Note that

$$\sup_{\lambda > 0, \mathbf{p} \in \mathbf{R}^d} \lambda (\mathbf{p}^2 + M^2 + \lambda)^{-1} (\sqrt{\mathbf{p}^2 + M^2} + 1)^{-1/2} < \infty. \quad (19)$$

Since $0 < \delta < \frac{1}{10}$, we see that $(\frac{1}{2} - \delta)^{-1} < \frac{5}{2}$, and hence

$$\sup_{\mathbf{p} \in \mathbf{R}^d} |p^\nu|^{(\frac{1}{2} - \delta)^{-1}} (\mathbf{p}^2 + M^2)^{-1} (\sqrt{\mathbf{p}^2 + M^2} + 1)^{-1/2} < \infty. \quad (20)$$

Then we have

$$\begin{aligned} \sup_{\lambda > 0, \mathbf{p} \in \mathbf{R}^d} |p^\nu|^{(\frac{1}{2} - \delta)^{-1}} (\mathbf{p}^2 + M^2 + \lambda)^{-1} (\sqrt{\mathbf{p}^2 + M^2} + 1)^{-1/2} \\ \leq \sup_{\mathbf{p} \in \mathbf{R}^d} |p^\nu|^{(\frac{1}{2} - \delta)^{-1}} (\mathbf{p}^2 + M^2)^{-1} (\sqrt{\mathbf{p}^2 + M^2} + 1)^{-1/2} < \infty. \end{aligned} \quad (21)$$

By (18), (19) and (21), we obtain (16). ■

(Proof of Proposition 3.1)

It follows that for a nonnegative self-adjoint operator S ,

$$\sqrt{S}\Phi = \frac{1}{\pi} \int_0^\infty \frac{1}{\sqrt{\lambda}} (S + \lambda)^{-1} S \Phi d\lambda, \quad \Phi \in \mathcal{D}(S). \quad (22)$$

Let

$$\begin{aligned} A_j(\Lambda) &= \overline{\left(-\Delta_j \otimes I + Q_j(\Lambda) + M^2 \right)}_{\upharpoonright \mathcal{D}_0}, \\ B_j &= -\Delta_j \otimes I + M^2. \end{aligned}$$

Then we have for $\Psi \in \mathcal{D}_0$,

$$\begin{aligned} \left(U(\Lambda)^{-1} (H_p \otimes I) U(\Lambda) - H_p \otimes I \right) \Psi &= \sum_{j=1}^N \frac{1}{\pi} \int_0^\infty \frac{1}{\sqrt{\lambda}} \{ (A_j(\Lambda) + \lambda)^{-1} A_j(\Lambda) - (B_j + \lambda)^{-1} B_j \} \Psi d\lambda \\ &= \sum_{j=1}^N \frac{1}{\pi} \int_0^\infty \sqrt{\lambda} (A_j(\Lambda) + \lambda)^{-1} (A_j(\Lambda) - B_j) (B_j + \lambda)^{-1} \Psi d\lambda \\ &= \sum_{j=1}^N \frac{1}{\pi} \int_0^\infty \sqrt{\lambda} (A_j(\Lambda) + \lambda)^{-1} Q_j(\Lambda) (B_j + \lambda)^{-1} \Psi d\lambda. \end{aligned} \quad (23)$$

By (13) and the spectral decomposition theorem, $\|(A_j(\Lambda) + \lambda)^{-1}\| \leq \frac{1}{\lambda + M^2}$, $\lambda > 0$ follows, and then we have

$$\left\| \left(U(\Lambda)^{-1} (H_p \otimes I) U(\Lambda) - H_p \otimes I \right) \Psi \right\| \leq \sum_{j=1}^N \frac{1}{\pi} \int_0^\infty \frac{\sqrt{\lambda}}{\lambda + M^2} \|Q_j(\Lambda) (B_j + \lambda)^{-1} \Psi\| d\lambda. \quad (24)$$

We see that

$$\begin{aligned} \|Q_j(\Lambda) (B_j + \lambda)^{-1} \Psi\| &\leq \left(\frac{\kappa}{\Lambda} \right) \left(\left\| \Pi \left(\frac{\nabla f_{\mathbf{x}_j}}{\omega} \right) \cdot (\mathbf{p}_j \otimes I) (B_j + \lambda)^{-1} \Psi \right\| + \left\| \Pi \left(\frac{\Delta f_{\mathbf{x}_j}}{\omega} \right) (B_j + \lambda)^{-1} \Psi \right\| \right) \\ &\quad + \left(\frac{\kappa}{\Lambda} \right)^2 \left\| \Pi \left(\frac{\nabla f_{\mathbf{x}_j}}{\omega} \right) \cdot \Pi \left(\frac{\nabla f_{\mathbf{x}_j}}{\omega} \right) (B_j + \lambda)^{-1} \Psi \right\|. \end{aligned} \quad (25)$$

Note that

$$\begin{aligned}
& \|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot (\hat{\mathbf{p}}_j \otimes I)(B_j + \lambda)^{-1} \Psi\| \\
& \leq \sum_v \|\Pi(\frac{\partial_{x_j^v} f_{\mathbf{x}_j}}{\omega})(I \otimes H_b + 1)^{-1/2}\| \|(\hat{\mathbf{p}}_j^v \otimes I)(B_j + \lambda)^{-1}(H_p \otimes I + 1)^{-1/2}\| \|(H_p \otimes I + 1)^{1/2}(I \otimes H_b + 1)^{1/2} \Psi\|
\end{aligned} \tag{26}$$

Here we used the boundness (4) and (5). Applying the Lemma 3.2 to $\|(\hat{\mathbf{p}}_j^v \otimes I)(B_j + \lambda)^{-1}(H_p \otimes I + 1)^{-1/2}\|$ in (26), it is seen that for $\delta \in (0, \frac{1}{10})$, there exist $\alpha_j(\delta) \geq 0$ such that

$$\|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot (\hat{\mathbf{p}}_j \otimes I)(B_j + \lambda)^{-1} \Psi\| \leq \frac{\alpha_j(\delta)}{\lambda^{\frac{1}{2}+\delta}} \|(H_p \otimes I + 1)^{1/2}(I \otimes H_b + 1)^{1/2} \Psi\|,$$

and hence we have

$$\|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot (\hat{\mathbf{p}}_j \otimes I)(B_j + \lambda)^{-1} \Psi\| \leq \frac{\alpha_j(\delta)}{\lambda^{\frac{1}{2}+\delta}} \left(\|H_p \Psi\| + \|H_b \Psi\| + \|\Psi\| \right). \tag{27}$$

Since $(\|H_p \otimes I \Psi\| + \|I \otimes H_b \Psi\|)^2 \leq 2\|H_0(\Lambda)\|^2$, we have

$$\int_0^\infty \frac{\sqrt{\lambda}}{\lambda + M^2} \|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot (\hat{\mathbf{p}}_j \otimes I)(B_j + \lambda)^{-1} \Psi\| d\lambda \leq \alpha_j(\delta) \left(\int_0^\infty \frac{1}{(\lambda + M^2)\lambda^\delta} d\lambda \right) \left(\sqrt{2}\|H_0(\Lambda)\Psi\| + \|\Psi\| \right). \tag{28}$$

By $\|\Pi(\frac{\Delta f_{\mathbf{x}_j}}{\omega})(I \otimes H_b + 1)^{-1/2}\| < \infty$ and $\|(B_j + \lambda)^{-1}\| \leq \frac{1}{\lambda + M^2}$, we have

$$\begin{aligned}
\|\Pi(\frac{\Delta f_{\mathbf{x}_j}}{\omega})(B_j + \lambda)^{-1} \Psi\| & \leq \|\Pi(\frac{\Delta f_{\mathbf{x}_j}}{\omega})(I \otimes H_b + 1)^{-1/2}\| \|(B_j + \lambda)^{-1}\| \|(I \otimes H_b + 1)^{1/2} \Psi\| \\
& \leq \frac{1}{\lambda + M^2} \|\Pi(\frac{\Delta f_{\mathbf{x}_j}}{\omega})(I \otimes H_b + 1)^{-1/2}\| \|(I \otimes H_b + 1)^{1/2} \Psi\|.
\end{aligned} \tag{29}$$

Then by $\|(I \otimes H_b + 1)^{1/2} \Psi\| \leq \|H_0(\Lambda)\Psi\| + \|\Psi\|$, we have

$$\begin{aligned}
& \int_0^\infty \frac{\sqrt{\lambda}}{\lambda + M^2} \|\Pi(\frac{\Delta f_{\mathbf{x}_j}}{\omega})(B_j + \lambda)^{-1} \Psi\| d\lambda \\
& \leq \|\Pi(\frac{\Delta f_{\mathbf{x}_j}}{\omega})(I \otimes H_b + 1)^{-1/2}\| \left(\int_0^\infty \frac{\sqrt{\lambda}}{(\lambda + M^2)^2} d\lambda \right) \left(\|H_0(\Lambda)\Psi\| + \|\Psi\| \right).
\end{aligned} \tag{30}$$

In addition we also see that

$$\begin{aligned}
\|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot \Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega})(B_j + \lambda)^{-1} \Psi\| & \leq \|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot \Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega})(I \otimes H_b + 1)^{-1}\| \|(B_j + \lambda)^{-1}\| \|(I \otimes H_b + 1) \Psi\| \\
& \leq \frac{1}{\lambda + M^2} \|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot \Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega})(I \otimes H_b + 1)^{-1}\| \|(I \otimes H_b + 1) \Psi\|.
\end{aligned} \tag{31}$$

Since $\|\Pi(\xi)\Pi(\eta)(H_b + 1)^{-1}\| < \infty$ for $\xi, \eta \in \mathcal{D}(\omega)$, we obtain

$$\begin{aligned} & \int_0^\infty \frac{\sqrt{\lambda}}{\lambda + M^2} \|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot \Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega})(B_j + \lambda)^{-1} \Psi\| d\lambda \\ & \leq \|\Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega}) \cdot \Pi(\frac{\nabla f_{\mathbf{x}_j}}{\omega})(H_b + 1)^{-1}\| \left(\int_0^\infty \frac{\sqrt{\lambda}}{(\lambda + M^2)^2} d\lambda \right) (\|H_0(\Lambda)\Psi\| + \|\Psi\|). \end{aligned} \quad (32)$$

Then from (28), (30), (32) and (25), the proposition follows. ■

Proposition 3.3 Assume (A.1) - (A.4).

(1) For $\varepsilon > 0$, there exists $\Lambda(\varepsilon) \geq 0$ such that for all $\Lambda > \Lambda(\varepsilon)$,

$$\|K(\Lambda)\Psi\| \leq \varepsilon \|H_0(\Lambda)\Psi\| + v(\varepsilon)\|\Psi\|, \quad \Psi \in \mathcal{D}_0, \quad (33)$$

holds, where $v(\varepsilon)$ is a constant independent of $\Lambda \geq \Lambda(\varepsilon)$.

(2) Then for all $z \in \mathbf{C} \setminus \mathbf{R}$, it follows that

$$s - \lim_{\Lambda \rightarrow \infty} K(\Lambda)(H_0(\Lambda) - z)^{-1} = V_{\text{eff}}(H_p - z)^{-1} \otimes P_{\Omega_b}, \quad (34)$$

(Proof)

(1) By the condition (A.4), V_{eff} is bounded. Then (1) follows from Proposition 3.1.

(2) It is seen that

$$K(\Lambda)(H_0(\Lambda) - z)^{-1} = K(\Lambda)(H_p - z)^{-1} \otimes P_{\Omega_b} + K(\Lambda)(H_0(\Lambda) - z)^{-1} (I \otimes (1 - P_{\Omega_b})).$$

By Proposition 3.1, we have

$$s - \lim_{\Lambda \rightarrow \infty} K(\Lambda) \left((H_p - z)^{-1} \otimes P_{\Omega_b} \right) \Psi = V_{\text{eff}} \left((H_p - z)^{-1} \otimes P_{\Omega_b} \right) \Psi. \quad (35)$$

By (33), we see that for $\varepsilon > 0$ there exists $\Lambda(\varepsilon) \geq 0$ such that for all $\Lambda > \Lambda(\varepsilon)$

$$\|K(\Lambda)(H_0(\Lambda) - z)^{-1} \Phi\| \leq \varepsilon \|\Phi\| + (\varepsilon|z| + v(\varepsilon)) \|(H_0(\Lambda) - z)^{-1} \Phi\|, \quad \Phi \in \mathcal{H}.$$

Note that $\lim_{\Lambda \rightarrow \infty} \|(H_0(\Lambda) - z)^{-1} (I \otimes (1 - P_{\Omega_b})) \Psi\| = 0$, and hence we obtain

$$\lim_{\Lambda \rightarrow \infty} \left\| K(\Lambda)(H_0(\Lambda) - z)^{-1} (I \otimes (1 - P_{\Omega_b})) \Psi \right\| = 0. \quad (36)$$

By (35) and (36), we obtain (34). ■

(Proof of Theorem 2.1)

By Proposition 3.3, it is shown that $H(\Lambda)$ satisfies the condition (S.1) and (S.2) by applying $H_0(\Lambda)$ to $A \otimes I + \Lambda I \otimes B$ and $K(\Lambda)$ to $C(\Lambda)$. Hence by the Proposition A, we have for $z \in \mathbf{C} \setminus \mathbf{R}$,

$$s - \lim_{\Lambda \rightarrow \infty} (H(\Lambda) - z)^{-1} = \lim_{\Lambda \rightarrow \infty} U(\Lambda) (H_0(\Lambda) - z)^{-1} U(\Lambda)^{-1} = (H_p + V_{\text{eff}} - z)^{-1} \otimes P_{\Omega_b}.$$

Thus the proof is completed. ■

Acknowledgments

It is a pleasure to thank Professor Fumio Hiroshima for his advice and comments.

References

- [1] A. Arai, Asymptotic analysis and its application to the nonrelativistic limit of the Pauli-Fierz and a spin-boson model, *J. Math. Phys.* **32** (1990), 2653-2663.
- [2] E. B. Davies, Particle interactions and the weak coupling limit, *J. Math. Phys.* **20** (1979), 345-351.
- [3] I. Daubechies, One electron molecules with relativistic kinetic energy : Properties of the discrete spectrum, *Comm. Math. Phys.*, **94** (1984), 523-535.
- [4] J. Dereziński, The Mourre estimate for dispersive N -body Schrödinger operators, *Trans. Amer. Math. Soc.* **317** (1990), 773-798.
- [5] C. Gérard, The mourre estimate for regular dispersive systems, *Ann. Inst. H. Poincaré Phys. Théor.* **54** (1991), 59-88.
- [6] I. Herbst, Spectral theory of the operator $(p^2 + m^2)^{1/2} - Ze^2/r$, *Commun. Math. Phys.*, **53** (1977), 285-294.
- [7] F. Hiroshima, Analysis of ground states of atoms interacting with a quantized radiation field, *Topics in the theory of Schrödinger operators*, (H.Araki and H.Ezawa eds.) World Scientific, 2004, 145-273.
- [8] F. Hiroshima, Scaling limit of a model of quantum electrodynamics, *J. Math. Phys.* **34** (1993), 4478-4518.
- [9] F. Hiroshima, Scaling limit of a model of quantum electrodynamics with many nonrelativistic particles, *Rev. Math. Phys.* **9** (1997), 201-225.
- [10] F. Hiroshima, Weak coupling limit with a removal of an ultraviolet cutoff for a Hamiltonian of particles interacting with a massive scalar field, *Inf. Dim. Ana. Quantum Prob. Rel. Top.* **1** (1998), 407-423.
- [11] F. Hiroshima, Weak coupling limit and a removing ultraviolet cutoff for a Hamiltonian of particles interacting with a quantized scalar field, *J. Math. Phys.* **40** (1999), 1215-1236.
- [12] F. Hiroshima, Observable effects and parametrized scaling limits of a model in non-relativistic quantum electrodynamics, *J. Math. Phys.* **43** (2002), 1755-1795.
- [13] F. Hiroshima and H.Spohn, Enhanced binding through coupling to a quantum field, *Ann. Henri. Poincaré* **2** (2001), 1150-1187.
- [14] F. Hiroshima and I.Sasaki, On the ionization of the semi-relativistic Pauli-Fierz model for a single particle, (arxiv : 1003.1661v4).
- [15] E. Lieb and M. Loss, *Analysis* (second edition), Amer. Math. Soc. 2001.
- [16] A. Ohkubo, Scaling limit for the Dereziński-Gérard Model, to appear in Hokkaido Math. J.
- [17] A. Suzuki, Scaling limits for a general class of quantum field models and its applications to nuclear physics and condensed matter physics, *Inf. Dim. Ana. Quantum Prob. Rel. Top.* **10** (2007), 43-65.
- [18] A. Suzuki, Scaling limits for a generalization of the Nelson model and its application to nuclear physics, *Rev. Math. Phys.* **19** (2007), 131-155.
- [19] T. Takaesu, On the scaling limit of quantum electrodynamics with spatial cutoffs. (arxiv : 0908.2080v1)
- [20] T. Umeda, Radiation conditions and resolvent estimates for relativistic Schrödinger operators, *Ann. Inst. H. Poincaré Phys. Théor.* **63** (1995), 277-296.
- [21] R. Weder, Spectral properties of one-body relativistic spin-zero hamiltonians, *Ann. Inst. H. Poincaré, Sect. A*, **20** (1974), 211-220.

List of MI Preprint Series, Kyushu University

The Global COE Program
Math-for-Industry Education & Research Hub

MI

- MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata
- MI2008-2 Eiji ONODERA
The initial value problem for a third-order dispersive flow into compact almost Hermitian manifolds
- MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space
- MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme
- MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in infinite extensions over a p-adic field
- MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields
- MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI
The random walk model revisited
- MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition
- MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials

- MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE
- MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds
- MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the L^2 a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator
- MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials
- MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality
- MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions
- MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings
- MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors
- MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the L_1 regularization
- MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions
- MI2009-5 Toshiro HIRANOUCI & Yuichiro TAGUCHI
Flat modules and Groebner bases over truncated discrete valuation rings

- MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations
- MI2009-7 Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow
- MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization
- MI2009-9 Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity
- MI2009-10 Shingo SAITO
Generalisation of Mack's formula for claims reserving with arbitrary exponents for the variance assumption
- MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve
- MI2009-12 Tetsu MASUDA
Hypergeometric q -functions of the q -Painlevé system of type $E_8^{(1)}$
- MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination
- MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications
- MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on L^p spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain

- MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring
- MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions
- MI2009-18 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force
- MI2009-19 Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application
- MI2009-20 Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions
- MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations
- MI2009-22 Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces
- MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions
- MI2009-24 Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map
- MI2009-25 Takehiko KINOSHITA & Mitsuhiro T. NAKAO
On very accurate enclosure of the optimal constant in the a priori error estimates for H_0^2 -projection

- MI2009-26 Manabu YOSHIDA
Ramification of local fields and Fontaine's property (Pm)
- MI2009-27 Yu KAWAKAMI
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space
- MI2009-28 Masahisa TABATA
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme
- MI2009-29 Yoshiyuki KAGEI & Yasunori MAEKAWA
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance
- MI2009-30 Yoshiyuki KAGEI & Yasunori MAEKAWA
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis
- MI2009-31 Masato WAKAYAMA & Yoshinori YAMASAKI
Hecke's zeros and higher depth determinants
- MI2009-32 Olivier PIRONNEAU & Masahisa TABATA
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type
- MI2009-33 Chikashi ARITA
Queueing process with excluded-volume effect
- MI2009-34 Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA
Projective reduction of the discrete Painlevé system of type $(A_2 + A_1)^{(1)}$
- MI2009-35 Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI
Finite element computation for scattering problems of micro-hologram using DtN map

- MI2009-36 Reiichiro KAWAI & Hiroki MASUDA
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes
- MI2009-37 Hiroki MASUDA
On statistical aspects in calibrating a geometric skewed stable asset price model
- MI2010-1 Hiroki MASUDA
Approximate self-weighted LAD estimation of discretely observed ergodic Ornstein-Uhlenbeck processes
- MI2010-2 Reiichiro KAWAI & Hiroki MASUDA
Infinite variation tempered stable Ornstein-Uhlenbeck processes with discrete observations
- MI2010-3 Kei HIROSE, Shuichi KAWANO, Daisuke MIIKE & Sadanori KONISHI
Hyper-parameter selection in Bayesian structural equation models
- MI2010-4 Nobuyuki IKEDA & Setsuo TANIGUCHI
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons
- MI2010-5 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and detecting change point via the relevance vector machine
- MI2010-6 Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI
Semi-supervised logistic discrimination via graph-based regularization
- MI2010-7 Teruhisa TSUDA
UC hierarchy and monodromy preserving deformation
- MI2010-8 Takahiro ITO
Abstract collision systems on groups
- MI2010-9 Hiroshi YOSHIDA, Kinji KIMURA, Naoki YOSHIDA, Junko TANAKA & Yoshihiro MIWA
An algebraic approach to underdetermined experiments

- MI2010-10 Kei HIROSE & Sadanori KONISHI
Variable selection via the grouped weighted lasso for factor analysis models
- MI2010-11 Katsusuke NABESHIMA & Hiroshi YOSHIDA
Derivation of specific conditions with Comprehensive Groebner Systems
- MI2010-12 Yoshiyuki KAGEI, Yu NAGAFUCHI & Takeshi SUDOU
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Poiseuille type flow
- MI2010-13 Reiichiro KAWAI & Hiroki MASUDA
On simulation of tempered stable random variates
- MI2010-14 Yoshiyasu OZEKI
Non-existence of certain Galois representations with a uniform tame inertia weight
- MI2010-15 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of a Rotating Flow Driven by Precession of Arbitrary Frequency
- MI2010-16 Yu KAWAKAMI & Daisuke NAKAJO
The value distribution of the Gauss map of improper affine spheres
- MI2010-17 Kazunori YASUTAKE
On the classification of rank 2 almost Fano bundles on projective space
- MI2010-18 Toshimitsu TAKAESU
Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field