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ON THE CLASSIFICATION OF RANK 2 ALMOST FANO BUNDLES ON PROJECTIVE SPACE

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Abstract. An almost Fano bundle is a vector bundle on a smooth projective variety that its projectivization is an almost Fano variety. In this paper, we prove that almost Fano bundles exist only on almost Fano manifolds and study rank 2 almost Fano bundles over projective spaces.

Introduction

An almost Fano variety is a smooth projective variety whose anti-canonical line bundle is nef and big. This is a natural generalization of Fano varieties and often appears in the study of deformation of a Fano variety ([12], [15]). Almost Fano surfaces were completely classi ed by Demazure [3]. Recently Jahnke, Peternel and Radlo classi ed almost Fano threefolds with picard number 2 whose pluri-anti-canonical morphism is divisorial in [8]. In [19], Takeuchi studied almost Fano threefolds with del pezzo bration structure whose pluri-anti-canonical morphism is small. In higher dimensional case, Jahnke and Peternell classi ed almost del Pezzo varieties, which are almost Fano n-folds with index $n-1$ i.e. its anti-canonical line bundle is divisible by $n-1$ in the Picard group.

The aim of this paper is to study ruled almost Fano varieties M of dimension $n \geqslant 3$ over nonsingular variety S i.e. there is a vector bundle $\mathcal{E}$ on S such that M is isomorphic to its projectivization $\mathbb{P}_S(\mathcal{E})$.

Szurek and Wisniewski introduced the notion of Fano bundle in [16]. As an almost Fano version, we introduce the notion of almost Fano bundle as below.

Definition 0.1. Let $\mathcal{E}$ be a vector bundle on a smooth complex projective variety M. We say that $\mathcal{E}$ is almost Fano if its projectivization $\mathbb{P}_M(\mathcal{E})$ is an almost Fano variety.

Such bundles always exist on an almost Fano variety M. In fact, we notice that the trivial rank r vector bundle is almost Fano since $\mathbb{P}_M(\mathcal{O}_M^{\oplus r}) \cong M \times \mathbb{P}^{r-1}$ is also an almost Fano variety. In [16, Theorem 1.6], it is shown that Fano bundles are only on Fano manifolds. We consider the almost Fano case and obtain the following theorem.

Theorem A. If $\mathcal{E}$ is an almost Fano bundle over a smooth complex projective variety M, then M is an almost Fano variety.

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On projective spaces, rank 2 Fano bundles are completely classi ed in [1], [16] and [17]. Using their methods, we study the classi cation of rank 2 almost Fano bundles on projective spaces and have the list mentioned below.

Theorem B. Let $\mathcal{E}$ be a rank 2 normalized (i.e. $c_1(\mathcal{E}) = 0$ or -1) almost Fano bundle on $\mathbb{P}^n$. Assume that $\mathcal{E}$ is not Fano. Then, $\mathcal{E}$ is isomorphic to one of the following :
(1) $\mathcal{O}_{\mathbb{P}^n}(\lfloor \frac{n}{2} \rfloor) \oplus \mathcal{O}_{\mathbb{P}^n}(\lfloor -\frac{n}{2} \rfloor)$, where $\lfloor \frac{n}{2} \rfloor$ is the largest integer $\leqslant \frac{n}{2}$.
(2) a stable bundle on $\mathbb{P}^3$ with $c_1 = 0$, $c_2 = 2$.
(3) a stable bundle on $\mathbb{P}^3$ with $c_1 = 0$, $c_2 = 3$.
(4) a vector bundle on $\mathbb{P}^2$ determined by the exact sequence : $0 \to \mathcal{O}_{\mathbb{P}^2} \to \mathcal{E} \to \mathcal{I}_p(-1) \to 0$, where $\mathcal{I}_p$ is the ideal sheaf of a point p.
(5) a stable bundle on $\mathbb{P}^2$ with $c_1 = -1$, $2 \leqslant c_2 \leqslant 5$.
(6) a stable bundle on $\mathbb{P}^2$ with $c_1 = 0$, $4 \leqslant c_2 \leqslant 6$.

Moreover, we show that all cases stated above really exist, except the case when $c_2 = 6$ in (6). Note that these varieties are of index 1 or 2. On three dimensional projective space, the most di cult part is a construction of almost Fano bundles satisfying the condition in (3). To obtain this, we use Maruyama's theory of elementary transformation of vector bundles. On projective plane, the case $c_1 = -1$ was classi ed in [7]. Therefore we treat the case $c_1 = 0$ i.e. $\mathbb{P}(\mathcal{E})$ is of index 1. In particular, we study almost Fano threefolds of index 1 whose pluri-anti-canonical morphism is small, having $\mathbb{P}^1$-bundle structure over $\mathbb{P}^2$.

Ruled varieties play an important role in the classi cation theory of projective varieties. So we may expect that our results also have applications.

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Notation

Throughout this paper $\mathcal{E}$ is a vector bundle on a smooth complex projective variety M and $\xi_{\mathcal{E}}$ is the tautological line bundle on $X = \mathbb{P}_M(\mathcal{E})$. By π we denote the projection $\pi : \mathbb{P}_M(\mathcal{E}) \to M$ and by H the pull-back of hyperplane if $M = \mathbb{P}^n$ (i.e. $\mathcal{O}_{\mathbb{P}_M(\mathcal{E})}(H) \cong \pi^*\mathcal{O}_{\mathbb{P}^n}(1)$). For a curve C in M, we denote by $[C]$ the numerical equivalence class of C in M.

1. Proof of Theorem A

In this section we prove Theorem A. Before starting the proof, we prepare some facts.

Definition 1.1. Let X be a normal projective variety and Δ an e ective $\mathbb{Q}$-divisor on X. Let $\varphi : Y \to X$ be a log resolution of (X, Δ). We set

$$K_Y = \varphi^*(K_X + \Delta) + \sum a_i E_i,$$

where E_i is a prime divisor. The pair (X, Δ) is called kawamata log terminal (klt, for short) if $a_i > -1$ for all i.

Definition 1.2. Let X be a normal projective variety and Δ an e ective $\mathbb{Q}$-divisor on X. We say that the pair (X, Δ) is a log Fano variety if (X, Δ) is klt and $-(K_X + \Delta)$ is an ample $\mathbb{Q}$-divisor.

Lemma 1.3. *If X is an almost Fano manifold, there is an effective $\mathbb{Q}$-divisor Δ such that (X, Δ) is a log Fano variety.*

Proof. For any ample divisor A, there are an integer m and an e ective divisor E such that $-mK_X = A + E$ by [9, Lemma 2.60]. Put $\Delta = \frac{1}{l}E$ for $l \gg 0$, then (X, Δ) is klt from [9], corollary 2.35 and

$$-lm(K_X + \Delta) = m(l - m)(-K_X) + mA$$

is ample. $\square$

Using this lemma, we get the following results by [9] and [21].

Theorem 1.4. *Let X be an almost Fano manifold. Then,*
(1)(*Basepoint-free Theorem*)
Any nef divisor D on X is semiample (i.e. bD is basepoint free for $b \gg 0$).
(2)(*Cone Theorem*)
There are finitely many rational curves C_j on X such that

$$\overline{NE(X)} = \sum_{finite} \mathbb{R}_{\geqslant 0}[C_j].$$

(3) *X is rationally connected i.e. for any two points in X there exists a rational curve which passes through them.*

The next lemma is also needed.

Lemma 1.5. (c.f. [20, Lemma 3.3]). *Let $\pi : X = \mathbb{P}_M(\mathcal{E}) \to M$ be the projectivization of a* rank r *almost Fano bundle $\mathcal{E}$ and C an extremal rational curve on X not contracted by π. Then, we have $0 \leqslant -K_X.C \leq -K_M.\pi(C)$.*

Proof. Let C be an extremal rational curve on X not contracted by π and φ_C the corresponding elementary contraction map. Then φ satis es the assumption in [20, Lemma 3.3]. Hence we obtain the inequality in the lemma. $\square$

Proof of Theorem A. Put $X = \mathbb{P}_M(\mathcal{E})$. From Theorem 1.4, we can nd nitely many extremal rational curves C_0, C_1, $\cdots$, C_ρ in X which generate the Kleiman-Mori cone $\overline{NE(X)}$. Let C_0 be contained in a ber of the projection π. Then we see that $\overline{NE(M)} = \sum_{i=1}^{\rho} \mathbb{R}_{\geq 0}[\pi(C_i)]$. From Lemma 1.5, it follows that

$$-K_M.\pi(C_i) \geq -K_X.C_i \geq 0$$

for $1 \leqslant i \leqslant \rho$. Therefore $-K_M$ is nef. Next we show the bigness of $-K_M$. Applying Theorem 1.4 (1) to $D := \pi^*(-K_M)$, we know D is semiample. Because π is projective space bundle, $-K_M$ is also semiample. Let $\varphi = \varphi_{|-lK_M|} : M \to W$ be a morphism induced by $-lK_M$ for $l \gg 0$. Suppose that $\dim M > \dim W$. Take the Stein factorization, we may assume a ber of φ is connected. We denote its general ber by F. Then F is smooth and we see that $-K_M|_F = -K_F$ holds. From this,

$$\begin{aligned} -K_X|_{\pi^{-1}(F)} &= (r\xi_{\mathcal{E}} - \pi^*(K_M + c_1(\mathcal{E})))|_{\pi^{-1}(F)} \\ &= r\xi_{\mathcal{E}|_F} - \pi^*(K_F + c_1(\mathcal{E}|_F)) = -K_{\mathbb{P}_F(\mathcal{E}|_F)}. \end{aligned}$$

Therefore we may only consider $\varphi(M)$ is a point. In this case, Kodaira dimension $\kappa(M)$ of M is equal to 0. On the other hand, X is rationally connected due to Theorem 1.4 (3). Since π is surjective, M is also rationally connected. Hence we have $\kappa(M) = -\infty$. This is a contradiction. □

Remark 1.6. (1) This theorem is proved in [2] if $\dim X = 2$ and $\mathrm{rank}\mathcal{E} = 2$.

(2) Recently Fujino and Gongyo prove if X is almost Fano and $f : X \to Y$ is a smooth morphism, then Y should be almost Fano [5].

2. Proof of Theorem B

In this section, we study the structure of almost Fano bundles on projective space.

First we consider almost Fano bundles which are decomposed into a direct sum of line bundles. In this case, we can characterize almost Fano bundles for any rank.

Proposition 2.1. *Let $\mathcal{E} \cong \mathcal{O} \oplus \mathcal{O}(a_1) \oplus \mathcal{O}(a_2) \oplus \cdots \mathcal{O}(a_r)$ be a vector bundle on $\mathbb{P}^n$, where $0 \leqslant a_1 \leqslant a_2 \leqslant \cdots \leqslant a_r$. Then, $\mathcal{E}$ is almost Fano if and only if $0 \leqslant c_1(\mathcal{E}) = \sum_{i=1}^{r} a_i \leqslant n+1$. Moreover $\mathcal{E}$ is not Fano if and only if $c_1(\mathcal{E}) = n+1$.*

Proof. Put $X = \mathbb{P}_{\mathbb{P}^n}(\mathcal{E})$. Then, we have $-K_X = r\xi_{\mathcal{E}} - (n+1-c_1(\mathcal{E}))H$. From the choice of $\mathcal{E}$, we can check naturally that $\mathcal{E}$ is Fano if and only if $0 \leqslant c_1(\mathcal{E}) \leqslant n$. Next we will establish the latter part. It is easy to see that $-K_M$ is nef but not ample if and only if $c_1(\mathcal{E}) = n+1$. Therefore it is su cient to show that $-K_M$ is big if $c_1(\mathcal{E}) = n+1$. In this case, $H^0(\xi_{\mathcal{E}} - H) \cong H^0(\mathcal{E}(-1)) \neq 0$. By Kodaira's lemma, $-K_M = r\xi_{\mathcal{E}} = ((r-1)\xi_{\mathcal{E}} + H) + (\xi_{\mathcal{E}} - H)$ is big. □

From now on, we give a proof of Theorem B. The proof is divided into three parts, (I) $n \geqslant 4$, (II) $n = 3$ and (III) $n = 2$.

(I) $n \geqslant 4$.

At rst, we consider the case where $n \geqslant 4$. The claim is as follows.

Proposition 2.2. *Let $\mathcal{E}$ be an almost Fano 2-bundle on $\mathbb{P}^n$, $n \geq 4$. Then $\mathcal{E}$ is a direct sum of two line bundles.*

These bundles are classi ed in Proposition 2.1. To show this, we need the next two lemmata.

Lemma 2.3. *Let $\mathcal{E}$ be a normalized* rank *2 almost Fano bundle on $\mathbb{P}^n$. If $n \geqslant 4$, then $\mathcal{E}(n)$ is generated by its global sections.*

Proof. The proof is in the similar fashion as in [1, Proposition 2.6]. We give an outline of the proof in the case where n is even and $c_1 = -1$. Put $n = 2k$ and $X = \mathbb{P}_{\mathbb{P}^n}(\mathcal{E})$, then we have

$$-K_X = 2\xi_{\mathcal{E}} + (2k+2)H = 2(\xi_{\mathcal{E}} + (k+1)H)$$

is nef and big. Therefore $\mathcal{E}(k+2)$ is ample vector bundle. By Le Potier vanishing theorem,

$$H^i(\mathcal{E}(k+2+j) \otimes K_{\mathbb{P}^n}) = H^i(\mathcal{E}(j-k+1)) = 0$$

for any $i \geqslant 2$ and $j \geqslant 0$. Especially letting $j = 3k - i - 1$, we have $H^i(\mathcal{E}(n-i)) = 0$ for $i \geqslant 2$. Moreover

$$H^1(\mathbb{P}_{\mathbb{P}^n}(\mathcal{E}), 3(\xi_{\mathcal{E}} + (k+1)H) + (k-2)H + K_{\mathbb{P}_{\mathbb{P}^n}(\mathcal{E})}) = H^1(\mathbb{P}^n, \mathcal{E}(n-1)) = 0$$

from Kawamata-Vieweg vanishing theorem. Combining above, then we see that $H^i(\mathcal{E}(n-i)) = 0$ for $i \geqslant 1$ namely $\mathcal{E}$ is $n-$regular. By means of Castelnuovo-Mumford lemma, $\mathcal{E}(n)$ is generated by its global sections. Other cases are proved in the same way. □

Lemma 2.4. [1] *Let $\mathcal{E}$ be a globally generated 2-bundle on $\mathbb{P}^n$. Then we have*

(1) *If $\mathcal{E}$ is not stable and $c_2(\mathcal{E}) < (n-1)(c_1(\mathcal{E}) - n + 2)$, then $\mathcal{E}$ is split into a direct sum of two line bundles.*

(2) *If $n \geqslant 6$ and $c_1(\mathcal{E})^2 < 4c_2(\mathcal{E})$, then we have $c_1(\mathcal{E}) \geq 2n + 3$.*

Proof of Proposition 2.2. Applying Lemma 2.4 to $\mathcal{E}(n)$, we can show immediately that $\mathcal{E}$ is split except for $n = 4$ *and* 5 essentially in the same as in the proof of Proposition 3.1 and Proposition 5.1 in [1]. If $n = 4$ (resp. $n = 5$), then $\mathcal{E}(3)$ (resp. $\mathcal{E}(4)$) is nef. From [1, Proposition 9.2] (resp. [1, Proposition 9.4]), $\mathcal{E}$ is split. □

(II) $n = 3$.

Next, we consider the case where $n = 3$. To start with, we demonstrate rank 2 almost Fano bundle on $\mathbb{P}^3$ is one of vector bundles below.

PROPOSITION 2.5. *Let $\mathcal{E}$ be a normalized almost Fano 2-bundle on $\mathbb{P}^3$. Then $\mathcal{E}$ is isomorphic to a direct sum of two line bundles or one of the following :*

(1) *stable vector bundle with $c_1 = 0$, $c_2 = 1$.*
(2) *stable vector bundle with $c_1 = 0$, $c_2 = 2$.*
(3) *stable vector bundle with $c_1 = 0$, $c_2 = 3$.*

PROOF. We shall discuss the two cases $c_1 = 0$ and $c_1 = -1$ separately.

First we treat $c_1 = -1$. Since $-K_X = 2\xi_{\mathcal{E}} + 5H$ is nef and big, we have that $\mathcal{E}(3)$ is ample. We can apply the argument in [16, Theorem 2.2], to this case and we can show that $\mathcal{E}$ is decomposed into a direct sum of two line bundles.

Next we treat $c_1 = 0$. In this case, $\mathcal{E}(2)$ is nef. If $H^0(\mathcal{E}(-2)) \neq 0$, then we can take a non-zero section $s \in H^0(\mathcal{E}(-2))$. If $Z := \{s = 0\} = \emptyset$, then $\mathcal{E}$ is decomposed into a direct sum of line bundles. If $Z \neq \emptyset$, then for a line L meeting Z in a nite number of points we would have

$$\mathcal{E}(-2)|_L \cong \mathcal{O}_L(d) \oplus \mathcal{O}_L(-4-d), (d \geq 1)$$

which contradicts to the nefness of $\mathcal{E}(2)$. If $H^0(\mathcal{E}(-2)) = 0$ and $H^0(\mathcal{E}(-1)) \neq 0$, then we can take a non-zero section $s \in H^0(\mathcal{E}(-1))$. If $Z := \{s = 0\} = \emptyset$, then $\mathcal{E}$ is decomposed into a direct sum of line bundles. If $Z \neq \emptyset$, then Z is a curve. Suppose that $\deg Z \geq 2$, we can take a line L intersecting with Z at least two points. Then

$$\mathcal{E}(-1)|_L \cong \mathcal{O}_L(d) \oplus \mathcal{O}_L(-2-d), (d \geq 2)$$

and contradict to the nefness of $\mathcal{E}(2)$. If $\deg Z = 1$, then Z is a line. But,

$$\deg K_Z = \deg(K_{\mathbb{P}^3} + c_1(\mathcal{E}(-1)))|_Z = -6.$$

This is a contradiction. If $H^0(\mathcal{E}(-1)) = 0$ and $H^0(\mathcal{E}) \neq 0$, then we can take a non-zero section $s \in H^0(\mathcal{E})$. If $Z := \{s = 0\} = \emptyset$, then $\mathcal{E}$ is decomposed into a direct sum of two line bundles. If $Z \neq \emptyset$, then Z is a curve and $\deg Z = c_2 \geq 1$. On the other hand, $\xi_{\mathcal{E}}(-K_X)^3 = 8 - 6c_2 \geq 0$. Therefore $\deg Z = 1$ i.e. Z is a line. But,

$$\deg K_Z = \deg(K_{\mathbb{P}^3} + c_1(\mathcal{E}))|_Z = -4.$$

This is a contradiction. Finally we assume $H^0(\mathcal{E}) = 0$ i.e. $\mathcal{E}$ is stable. In this case, $c_1^2 < 4c_2$ and $(-K_X)^4 = 128(4 - c_2) > 0$ hold. Hence $1 \leq c_2 \leq 3$. □

It is shown [16] that all stable bundles satisfying $c_1 = 0$, $c_2 = 1$ are Fano. If $c_2 = 2$, then $\mathcal{E}$ is 2-regular by [6]. Therefore $-K_X = 2(\xi_{\mathcal{E}} + 2H)$ is nef and big i.e. $\mathcal{E}$ is almost Fano. Such $\mathcal{E}$ is not Fano bundle [16]. The case $c_2 = 3$ is more complicated. First we show such an almost Fano bundle really exists.

PROPOSITION 2.6. *There is an almost Fano stable bundle on $\mathbb{P}^3$ with $c_1 = 0$, $c_2 = 3$.*

To show this, we use the following result.

THEOREM 2.7. [13, Proposition 6] *There is a nonsingular elliptic curve C on a smooth quartic surface $S \subset \mathbb{P}^3$ and a very ample divisor A on S such that*

(1) $Pic(S) \cong \mathbb{Z}[A] \oplus \mathbb{Z}[C]$.
(2) $A^2 = 4,\ A.C = 7,\ C^2 = 0$.
(3) *C is base point free.*
(4) *S does not contain any rational curve.*

Proof of Proposition 2.6. Let (S, C) be a pair in Theorem 2.7. Using the theory of elementary transformation [10] and [11], we can construct a rank 2 regular vector bundle $\mathcal{F}$ on $\mathbb{P}^3$ where $c_1(\mathcal{E}) = S,\ c_2(\mathcal{E}) = C$ modulo numerical equivalence. We will prove that $\mathcal{E} := \mathcal{F}(-2)$ is the bundle what we want. Since $\mathcal{F}$ has a global sections, we have a following exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^3} \to \mathcal{F} \to \mathcal{I}_C(4) \to 0.$$

Twist by $\mathcal{O}_{\mathbb{P}^3}(-2)$, we obtain

$$0 \to \mathcal{O}_{\mathbb{P}^3}(-2) \to \mathcal{F}(-2) \to \mathcal{I}_C(2) \to 0.$$

Because C is not contained in any quadric surface, we see that $H^0(\mathcal{I}_C(2)) = 0$. Therefore $\mathcal{F}$ is stable since $H^0(\mathcal{F}(-2)) = 0$ and $c_1(\mathcal{F}(-2)) = 0$. Next we show $\mathcal{F}$ is nef. Note that $\mathcal{F}$ has 2 global sections which induce the generically surjective morphism $\varphi : \mathcal{O}^{\bigoplus 2} \to \mathcal{F}$ where φ is isomorphic outside S by the construction. Consequently $\mathcal{F}$ is nef over curves not contained in S. Over S, we get an exact sequence $0 \to \mathcal{O}_S(C) \to \mathcal{F}|_S \to \mathcal{O}_S(4A - C) \to 0$. From the choice of C, $\mathcal{O}_S(C)$ is nef. We have only to check the nefness of $\mathcal{O}_S(4A - C)$. Since $(aA + bC)(4A - C) = 9a + 28b$, we must prove that $9a + 28b \geq 0$ if $aA + bC$ is e ective. But this is true since $(28A - 9C)^2 = -17 < 0$ and in view of Kleiman-Mori cone. Therefore $-K_X = 2\xi_{\mathcal{F}}$ is nef and big. Namely $\mathcal{F}$ is almost Fano. Hence $\mathcal{E} = \mathcal{F}(-2)$ is a stable 2-bundle with $c_1 = 0,\ c_2 = 3$ which is almost Fano. □

Let $\mathcal{M}(0, 3)$ be the moduli space of stable rank 2 vector bundles on $\mathbb{P}^2$ with $c_1 = 0$ and $c_2 = 3$. From Theorem in [4], we see that $\mathcal{M}(0, 3)$ has two irreducible components $\mathcal{M}_0(0, 3)$ and $\mathcal{M}_1(0, 3)$ where $\mathcal{M}_\alpha(0, 3)$ is the moduli space of vector bundles $\mathcal{E}$ satisfying the condition $\dim H^1(\mathcal{E}(-2)) \equiv \alpha\ (mod\ 2)$. The dimension of each components are 21. Almost Fano bundles constructed in Proposition 2.6 are contained in $\mathcal{M}_0(0, 3)$. The author does not know whether $\mathcal{M}_1(0, 3)$ contains almost Fano bundles or not.

Next we show that each components contain the member which is not almost Fano.

EXAMPLE 2.8. From Proposition in [14], we see that vector bundles in $\mathcal{M}_0(0, 3)$ which have a maximal order jumping line is of dimension 20. Such a bundle $\mathcal{E}$ is decomposed into $\mathcal{O}_L(3) \oplus \mathcal{O}_L(-3)$ over some line L. These bundles cannot be almost Fano since $\mathcal{E}(2)$ is not nef.

EXAMPLE 2.9. Let Y be a disjoint union of a plane cubic and a nonsingular space elliptic curve in $\mathbb{P}^3$. By Serre construction, we can construct a rank 2 bundle $\mathcal{F}$ on $\mathbb{P}^3$

with $c_1 = 4$, $c_2 = 7$. Then, we can check $H^0(\mathcal{F}(-2)) = 0$ due to the exact sequence $0 \to \mathcal{O}_{\mathbb{P}^3} \to \mathcal{F} \to \mathcal{I}_Y(4) \to 0$. Hence $\mathcal{F}$ is stable. Since every nonsingular space elliptic curve is a complete intersection of two quadrics, we have $H^0(\mathcal{I}_Y(3)) = H^0(\mathcal{F}(-1)) \neq 0$. From easy computation, $(\xi_{\mathcal{F}} - H).(-K_{\mathbb{P}(\mathcal{F})})^3 = -1$. Thus $\mathcal{E} := \mathcal{F}(-2)$ is a stable vector bundle with $c_1 = 0$, $c_2 = 3$ which is not almost Fano. We can check

$$\dim H^1(E(-2)) = \dim H^1(\mathcal{I}_Y) = \dim H^0(\mathcal{O}_Y) - 1 = 1$$

using the exact sequence $0 \to \mathcal{I}_Y \to \mathcal{O}_{\mathbb{P}^3} \to \mathcal{O}_Y \to 0$. Hence $\mathcal{E}$ is contained in $\mathcal{M}_1(0,3)$.

(Ⅲ) $n = 2$.

Finally, we consider the case where $n = 2$. If $c_1 = -1$, then $X = \mathbb{P}_{\mathbb{P}^2}(\mathcal{E})$ is an almost del Pezzo 3-fold and completely classi ed in [7]. So we may only study bundles with $c_1 = 0$. The statement is as follows.

Proposition 2.10. *Let $\mathcal{E}$ be a* rank *2 almost Fano bundle on $\mathbb{P}^2$ with $c_1 = 0$. Then, $\mathcal{E}$ is isomorphic to one of the folowing*
(1) $\mathcal{O}_{\mathbb{P}^2}(1) \bigoplus \mathcal{O}_{\mathbb{P}^2}(-1)$,
(2) $\mathcal{O}_{\mathbb{P}^2} \bigoplus \mathcal{O}_{\mathbb{P}^2}$,
(3) *$\mathcal{E}$ is determined by $0 \to \mathcal{O}_{\mathbb{P}^2} \to \mathcal{E} \to \mathcal{I}_p \to 0$, where $\mathcal{I}_p$ is the ideal sheaf of a point,*
(4) *stable vector bundle with $2 \leqslant c_2 \leqslant 6$.*

Proof. In this case, $\mathcal{E}(2)$ is ample. If $H^0(\mathcal{E}(-1)) \neq 0$, we take a non-zero section $s \in H^0(\mathcal{E}(-1))$. If $Z := \{s = 0\} = \emptyset$, then $\mathcal{E}$ is decomposed into a direct sum of line bundles. If $Z \neq \emptyset$, then for a line L meeting Z in a nite number of points we would have

$$\mathcal{E}(-1)|_L \cong \mathcal{O}_L(d) \oplus \mathcal{O}_L(-2-d),\ d \geqslant 1.$$

This contradict to the ampleness of $\mathcal{E}(2)$.

If $H^0(\mathcal{E}(-1)) = 0$ and $H^0(\mathcal{E}) \neq 0$, take a non-zero section $s \in H^0(\mathcal{E})$. If $Z := \{s = 0\} = \emptyset$, then $\mathcal{E}$ is decomposed into a direct sum of line bundles. If $Z \neq \emptyset$ and $\deg Z \geq 2$, then for a line L intersecting with Z at least two points we would have

$$\mathcal{E}|_L \cong \mathcal{O}_L(d) \oplus \mathcal{O}_L(-d),\ d \geq 2.$$

This is a contradiction.

If $\deg Z = 1$, $\mathcal{E}$ has an exact sequence $0 \to \mathcal{O}_{\mathbb{P}^2} \to \mathcal{E} \to \mathcal{I}_p \to 0$, where $\mathcal{I}_p$ is the ideal sheaf of a point p. In this case $\mathcal{E}$ is Fano bundle by [17, Proposition 2.3]. Finally we consider the case $H^0(\mathcal{E}) = 0$ i.e. $\mathcal{E}$ is stable. Then $2 \leq c_2 \leq 6$ since $(-K_X)^3 = 54 - 8c_2 > 0$. □

We have some comments of Fano bundles with $c_1 = 0$. If $c_2 = 2$, all stable bundles are Fano bundle from [17]. In the situation $c_2 = 3$, there is a stable Fano bundles by [17]. Moreover, we have the following result.

PROPOSITION 2.11. *If $\mathcal{E}$ is a stable almost Fano bundle on a projective plane with $c_1 = 0$, $c_2 = 3$. Then $\mathcal{E}$ is Fano bundle.*

PROOF. Let $\mathcal{E}$ be a stable almost Fano bundle on a projective plane with $c_1 = 0$, $c_2 = 3$. Using Riemann-Roch theorem, we have $\dim H^0(\mathcal{E}(1)) > 0$. Therefore we get an exact sequence $0 \to \mathcal{O}_{\mathbb{P}^2} \to \mathcal{E}(1) \to \mathcal{I}_Z(2) \to 0$ where Z is 4 points in $\mathbb{P}^2$ and $\mathcal{I}_Z$ is the ideal sheaf of Z. If $\mathcal{E}$ is not Fano, then the linear system $|\xi_{\mathcal{E}}+H|$ has one dimensional base locus B by [17], Claim 2.7. By virtue of Claim 2.10 and 2.11 in [17], we have $H.B \leqslant 2$ and $(\xi_{\mathcal{E}} + H).B \leqslant -1$. Since $0 \leqslant -K_{\mathbb{P}_{\mathbb{P}^2}(\mathcal{E})}.B = 2(\xi_{\mathcal{E}} + H).B + H.B \leqslant 0$, we obtain $H.B = 2$ and $(\xi_{\mathcal{E}} + H).B = -1$. If $\pi(B)$ is a line L, we have $\mathcal{E}(1)|_L \cong \mathcal{O}(d) \oplus \mathcal{O}(2-d)$, $d \geqslant 3$. This contradicts the ampleness of $\mathcal{E}(2)$. If $\pi(B)$ is a two line, take a irreducible component L. In this case $\mathcal{E}(1)$ is not nef over $\pi(B)$, so over L. Therefore we have $\mathcal{E}(1)|_L \cong \mathcal{O}(d) \oplus \mathcal{O}(2-d)$, $d \geqslant 3$. This contradicts the ampleness of $\mathcal{E}(2)$. Finally we consider the case where $\pi(B)$ is nonsingular conic C. Since $(\xi_{\mathcal{E}} + H).B = -1$, we obtain the splitting $\mathcal{E}(1)|_C \cong \mathcal{O}_C(d) \oplus \mathcal{O}_C(4-d), d \geqslant 5$. This is impossible because Z is only 4 points. $\square$

COROLLARY 2.12. *Let $\mathcal{E}$ be a stable vector bundle on a projective plane with $c_1 = 0$, $c_2 = 3$. If $\mathcal{S}^2(\mathcal{E})(3)$ is nef, then $\mathcal{E}(1)$ is generated by global sections.*

PROOF. If $\mathcal{S}^2(\mathcal{E})(3)$ is nef, then $\mathcal{E}$ is almost Fano. From Proposition 2.11, $\mathcal{E}$ is Fano bundle. By means of Proposition 2.6 in [17], $\mathcal{E}(1)$ is generated by global sections. $\square$

When $c_2 = 4$, no stable 2-bundle is Fano [17]. We can constract almost Fano 2 bundle with $c_1 = 0, c_2 = 4$ as follows.

EXAMPLE 2.13. Let Y be 5 points in general position and C is a smooth conic containing Y. Then the pair (C, Y) yields us a rank 2 regular vector bundle $\mathcal{F}$ with $c_1 = C$, $c_2 = Y$ by virtue of an elementary transform by [10] and [11]. We have a following exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^2} \to \mathcal{F} \to \mathcal{I}_Y(2) \to 0$$

where $\mathcal{I}_Y$ is the ideal sheaf of Y. Twist by $\mathcal{O}_{\mathbb{P}^2}(-1)$, we get

$$0 \to \mathcal{O}_{\mathbb{P}^2}(-1) \to \mathcal{F}(-1) \to \mathcal{I}_Y(1) \to 0.$$

Because there is no line containing Y, we have $H^0(\mathcal{I}_Y(1)) = 0$. Therefore $\mathcal{F}$ is stable since $H^0(\mathcal{F}(-1)) = 0$ and $c_1(\mathcal{F}(-1)) = 0$. We check $-K_{\mathbb{P}_{\mathbb{P}^2}(\mathcal{F})} = 2\xi_{\mathcal{F}} + H$ is nef. First we remark that $\mathcal{F}$ has 2 global sections which induce the generically surjective morphism $\varphi : \mathcal{O}^{\bigoplus 2} \to \mathcal{F}$ where φ is isomorphic outside C by the construction. Hence we notice that $2\xi_{\mathcal{F}} + H$ is nef outside $\pi^{-1}(C)$. On C, we have that $\mathcal{F}|_C \cong \mathcal{O}_C(5) \oplus \mathcal{O}_C(-1)$ from the theory of elementary transformation. From this fact, we can check that $(2\xi_{\mathcal{F}} + H).D \geqslant 0$ for every curves D contained in Hirzebruch surface $\mathbb{P}_C(\mathcal{F}|_{\mathcal{C}})$. The equality holds only for the minimal section associated with the quotient line bundle $\mathcal{F}|_C \to \mathcal{O}_C(-1) \to 0$.

Therefore $-K_{\mathbb{P}_{\mathbb{P}^2}(\mathcal{F})}$ is nef and big. Hence $\mathcal{E} := \mathcal{F}(-1)$ is a stable almost Fano bundle with $c_2 = 4$.

There exists an almost Fano stable bundle with $c_1 = 0$, $c_2 = 5$ from Theorem 0.19(C) in [18]. Finally we construct stable vector bundles with $c_1 = 0$, $3 \leqslant c_2 \leqslant 6$ which are not almost Fano.

Example 2.14. Let $Y_k = \{p_0, p_1, \cdots, p_k\}$ be the $k+1$ points ($4 \leqslant k \leqslant 7$) in $\mathbb{P}^2$. We assume that p_0, p_1, p_2 are lying in a line L and other points are not on L. By Serre construction, we have rank 2 vector bundles $\mathcal{E}_k$ on $\mathbb{P}^2$ with $c_1 = 2$, $c_2 = k+1$. $\mathcal{E}_k$ has an exact sequence $0 \to \mathcal{O}_{\mathbb{P}^2} \to \mathcal{E}_k \to \mathcal{I}_{Y_k}(2) \to 0$. Becuase there is no line containing Y_k, we see $\dim H^0(\mathcal{E}_k(-1)) = \dim H^0(\mathcal{I}_{Y_k})$=0. Combining with $c_1(\mathcal{E}_k(-1)) = 0$, $\mathcal{E}_k$ is stable bundle. Restricting each bundles into L, we get $\mathcal{E}_k|_L \cong \mathcal{O}_L(3) \oplus \mathcal{O}_L(-1)$. Since $\mathcal{E}_k(1)$ is not ample, $\mathcal{E}_k$ is not almost Fano.

References

[1] V. Ancona, T. Peternell and J.A. Wiśniewski, Fano bundles and splitting theorems on projective spaces and quadrics, Pacific J. Math. 163 (1994), no.1, 17–42.

[2] C. Casagrande, P. Jahnke and I. Radloff, On the Picard number of almost Fano threefolds with pseudo-index> 1, Internat. J. Math. 19 (2008), no.2, 173–191.

[3] M. Demazure, Surfaces de del Pezzo I-V, 21–69, Lecture Notes in Math., 777, Springer, Berlin, 1980.

[4] G. Ellingsrud and S. A. Stromme, Stable rank 2 vector bundles on $\mathbb{P}^3$ with $c_1 = 0$ and $c_2 = 3$, Math. Ann. 255 (1981), no. 1, 123–135.

[5] O. Fujino and Y. Gongyo, On images of weak Fano manifolds. preprint

[6] R. Hartshorne, Stable vector bundles of rank 2 on $\mathbb{P}^3$, Math. Ann. 238 (1978), no.3, 229–280.

[7] P. Jahnke and T. Peternell, Almost del Pezzo manifolds, Adv. Geom. 8 (2008), 387–411.

[8] P. Jahnke and T. Peternell, I. Radloff, Threefolds with big and nef anticanonical bundle. I, Math. Ann. 333 (2005), no.3, 569–631.

[9] J. Kollár and S. Mori, Birational geometry of algebraic varieties, Cambridge Tracts in Mathematics 134 (Cambridge University Press, MA, 1998).

[10] M. Maruyama, On a family of algebraic vector bundles, Number Theory, Algebraic Geomtry and Commutative Algebra (1973), 95–149, Kinokuniya.

[11] M, Maruyama, Elementary transformations in the theory of algebraic vector bundles. 241–266, Lecture Notes in Math., 961, Springer, Berlin, 1982.

[12] T, Minagawa, On classification of weakened Fano 3-fold with $B_2 = 2$, in Proc. of algebraic geometry symposium (Kinosaki, Oct. 2000)

[13] S. Mori, On degrees and genera of curves on smooth quatic surfaces in $\mathbb{P}^3$, Nagoya. Math. J. 96 (1984), 127–132.

[14] P. Rao, Mathematical instantons with maximal order jumping lines, Pacific. J. 178 (1997), no.2, 331–344.

[15] H. Sato, The classification of smooth toric weakened Fano 3-folds, Manuscripta Math. 109 (2002), no.1, 73–84.

[16] M. Szurek and J.A. Wiśniewski, Fano bundles over $\mathbb{P}^3$ and Q_3, Pacific J. Math. 141 (1990), 197–208.
[17] M. Szurek and J.A. Wiśniewski, Fano bundles of rank 2 on surfaces, Compositio. Math. 76 (1990), no.1-2, 295–305.
[18] H. Takagi, On classification of $\mathbb{Q}$ Fano 3-folds of Gorenstein index 2. *I*, *II*, Nagoya. Math. 167 (2002), 117–155, 157–216.
[19] K. Takeuchi, Weak Fano threefolds with del Pezzo fibration. preprint
[20] J.A. Wiśniewski, On contractions of extremal rays of Fano manifolds, J. Reine Angew. Math. 417 (1991), 141–157.
[21] Q. Zhang, Rational connectedness of log $\mathbb{Q}$-Fano varieties, J. Reine Angew. Math. 590 (2006), 131–142.

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