On microinstabilities in the foot of high Mach number perpendicular shocks

Matsukiyo, Shuichi
Max-Planck-Institut für extraterrestrische Physik | Earth System Science and Technology, Kyushu University

Scholer, Manfred
Earth System Science and Technology, Kyushu University | Max-Planck-Institut für extraterrestrische Physik

出版情報：Journal of geophysical research. 111, pp.A06104-, 2006. AGU
バージョン：

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. ???, XXXX, DOI:10.1029/,

On Microinstabilities in the Foot of High Mach Number Perpendicular Shocks

S. Matsukiyo[1], and M. Scholer[2]

[1] Earth System Science and Technology, Kyushu University, 816-8580 Kasuga, Japan

[2] Max-Planck-Institut für extraterrestrische Physik, 85741 Garching, Germany

Abstract.

Microinstabilities excited in the foot of a supercritical perpendicular shock wave are investigated. A two-dimensional full particle simulation with periodic boundary conditions in both directions using the physical ion to electron mass ratio is performed as a proxy for the foot region where incoming and specularly reflected ions overlap. The simulation shows that six types of different instabilities are excited in a time period shorter than ion gyroperiod of the reflected ions. The most dominant instability is the modified two-stream instability, which leads to strong parallel electron heating through a so-called two step instability and to ion phase space holes.

1. Introduction

High Mach number collisionless shocks in space and astrophysical plasmas have been studied extensively observationally, numerically, and theoretically [e.g. *Stone and Tsurutani*, 1985; *Tsurutani and Stone*, 1985], since they are a mine of various kinds of wave activities and mechanisms of particle acceleration and heating. It has been recognized for a long time that a variety of microinstabilities play a crucial role in wave excitation, particle heating, and anomalous dissipation mechanisms in the shock transition region (see, for instance, [*Papadopoulos*, 1985] and references therein). Indeed, recent full particle simulation studies revealed that a variety of microinstabilities are generated in the foot of low beta supercritical quasi-perpendicular shocks [e.g., *Shimada and Hoshino*, 2000; *Dieckmann et al.*, 2000; *Hoshino and Shimada*, 2002; *Scholer et al.*, 2003; *Matsukiyo and Scholer*, 2003; *Shimada and Hoshino*, 2004; *Scholer and Matsukiyo*, 2004; *Shimada and Hoshino*, 2005; *Muschietti and Lembège*, in press]. The free energy source of those microinstabilities are the relative drift between incoming electrons and reflected ions and incoming electons and incoming ions, respectively. The presence of reflected ions causes a decrease of the ion bulk velocity in the foot. This implies that the incoming electrons are decelerated so that the zero current condition in the shock normal direction is satisfied. Consequently, a relative bulk velocity between the electrons and the reflected ions or the electrons and the incoming ions arises. Although a relative bulk velocity between two ion species could drive an ion-ion instability [*Forslund and Shonk*, 1970; *Papadopoulos et al.*, 1971], the discussions in the present paper is restricted to electron-ion instabilities.

If the relative bulk velocity between electrons and ions exceeds the electron thermal velocity the Buneman instability [*Buneman*, 1958] is expected to get excited. The electron heating process by counter streaming ion beams in an unmagnetized plasma has been investigated for the first time by *Davidson et al.* [1970] by using a one-dimensional electrostatic simulation and has been extended to the two-dimensional case by *Lampe et al.* [1974]. In a magnetized plasma, on the other hand, the electron cyclotron drift instability (ECDI) is generated across the magnetic field [*Wong*, 1970; *Forslund et al.*, 1970; *Lampe et al.*, 1972]. While these two instabilities are well distinct in the foot of a quasi-perpendicular shock if $\omega_{pe}/\Omega_e \gg 1$, they cannot be separated if $\omega_{pe}/\Omega_e \sim 1$ as will be seen from the present study. Here ω_{pe} and Ω_e denote the electron plasma and the cyclotron frequency, respectively. The modified two-stream isntability (MTSI) is another instability generated in the foot of a quasi-perpendicular shock. This instability occurs even when the bulk velocity is smaller than the electron thermal velocity. The MTSI was initially thought to occur only in the shock ramp [*Lashmore-Davies*, 1971; *Krall and Liewer*, 1971; *Ott et al.*, 1972; *McBride et al.*, 1972; *McBride and Ott*, 1972; *Lemons and Gary*, 1978; *Winske et al.*, 1985; *Winske et al.*, 1987], while in the 1980's it was recognized that the MTSI can also be generated in the foot of a high Mach number quasi-perpendicular shock [*Wu et al.*, 1983; *Wu et al.*, 1984; *Scudder et al.*, 1986; *Gary et al.*, 1987].

Since most of the past simulation studies were performed by using a one-dimensional (1-D) full particle code these instabilities are almost independently discussed each from other. However, according to linear instability analysis, some of these instabilities can get excited simultaneously [e.g., *Wu et al.*, 1984] with different propagation angles. For instance the Buneman instability and the ECDI occur even at $\theta = 90°$, although the MTSI is not unstable at $\theta = 90°$ (θ = wave propagation angle with respect to ambient magnetic field). Furthermore the propagation angle of the MTSI based on electron-reflected ion interactions is different from that of the MTSI based on electron-incoming ion interactions [*Matsukiyo and Scholer*, 2003]. Moreover, if a certain instability is strong enough, other instabilities are subsequently excited in a later stage. Some of these may be due to non-linear processes, while others may be due to a second instability of the so-called two-step instability type in which an instability is excited after a change of the plasma parameters as a result of the first stage instability [*Papadopoulos*, 1988; *Cargill and Papadopoulos*, 1988]. The second stage instabilities do not necessarily have the same propagation angles as the corresponding first stage instabilities. Thus in a real system a variety of instabilities may be simultaneously occur with a variety of propagation angles. Therefore, introducing two-dimensionality is essential to understand the realistic evolution of a system.

The purpose of this paper is to investigate competing processes among instabilities studied in the past by utilizing a two dimensional full particle simulation code with periodic boundary conditions. This periodic system is considered as a proxy for the foot region of a perpendicular shock. Two-dimensionality allows a variety of instabilities to be simultaneously present. It will be shown that the most dominant instability in the parameter regime considered here is the MTSI which results in strong electron heating through a two-step instability (section 2). Summary and conclusions are given in section 3.

2. Two Dimensional Full Particle Simulation

2.1. Initial and Boundary Conditions

In order to study instabilities occurring in the foot region of perpendicular shocks it would be highly desirable to perform 2-D particle-in-cell (PIC) simulations of such shocks with high ion to electron mass ratio. A high ion to electron mass ratio is actually needed since the growth time of many instabilities (in units of the inverse ion gyrofrequency) depends strongly on this parameter. On the other hand the characteristic time in the foot of a perpendicular shock is the gyrotime of specularly reflected ions. Since present computer resources do not yet allow such shock simulations to be performed we will use as a proxy a 2-D PIC simulation of the interactions of electrons and two ion beams in a periodic system. This system is supposed to represent part of the foot region of a perpendicular shock where incoming electrons, incoming ions, and specularly reflected ions overlap. We use periodic boundary conditions in both directions. Since we focus on the time period immediately after specular reflection of incoming ions at the shock ramp, the system is composed of three plasma species, i.e., incoming and specularly reflected ions, and incoming electrons. The simulation is performed in the electron rest frame, and the incoming and the reflected ion beams are streaming parallel and antiparallel to the x-direction, while the ambient magnetic field points in the y-direction, i.e., an exactly perpendicular shock geometry is assumed. The electron and ion beams satisfy zero electrical current condition. Waves are allowed to propagate within the $x - y$ plane (Figure 1(a)).

The parameters used in the simulation run are as follows. The number of grids are 1,024 $\times$ 1,024 with a grid size $\Delta x = \Delta y = 0.5\lambda_{De} \approx 0.04c/\omega_{pe}$, where λ_{De} is the Debye length, and c denotes the speed of light. The time step is $\Delta t \approx 0.02\omega_{pe}^{-1}$. The flow velocities of the incoming and the reflected ions are $u_i/v_A = 2.14$ and $u_r/v_A = -8.57$ respectively, where v_A indicates the upstream Alfvén velocity. The reflection ratio, i.e., the relative density of the reflected to the incoming ions, is $\alpha = 0.25$. The values of ion and electron beta are $\beta_i = 0.04$, $\beta_r = 0.01$, and $\beta_e = 0.05$, which indicates that the temperatures of all the species are identical. Initial distribution functions of the three species in v_x are represented in Figure 1(b). The ion to electron mass ratio is $m_i/m_e = \mu = 1836$, and the squared ratio of the electron plasma to the cyclotron frequency is $\tau = \omega_{pe}^2/\Omega_e^2 = 4$. Using higher values of τ would be highly desirable. However, using both, a realistic ratio of m_i/m_e and a higher value for τ, is due to computational constrains presently not possible. Here, we should note that while the incoming ion Mach number in the shock frame is $[(u_i + u_r)/v_A]/2 \sim 5.4$, the actual Mach number of the corresponding shock would be somewhat higher since the incoming ions are usually pre-decelerated in the foot.

2.2. Simulation Results

2.2.1. Overview of energy time history

First we present in Figure 2 a survey of the time histories of field and particle energies. In all the panels, the energies are normalized to the initial electron thermal energy. A number of growth phases of the electric and magnetic field components can be recognized (we have omitted the E_z component in Figure 2 since its magnitude is negligibly small). The first growth phase of the E_x component, $\omega_{pe}t < 500$, is due to the ECDI (or Buneman instability). The growth of the B_y and the B_z components corresponds to two MTSIs which are discussed later. The second growth phase of the E_x component is also led by the MTSIs, although it is dominated by the ECDI in the earlier stage because of small growth rates of the MTSIs. The E_y and the B_x components suddenly grow after $\omega_{pe}t > 1000$ (labeled as EAI and WI). This is the result of second stage instabilities as will be discussed later. The first growth phase of the B_x component, $\omega_{pe}t < 600$, may be due to an other instability of some sort or a nonlinearity caused by the ECDI. A candidate of the instability is the Weibel instability which is driven by a temperature anisotropy (perpendicular ion beam(s) results in an effective temperature anisotropy).

We do not discuss this phenomenon in detail in the present paper, since its saturation level is quite small. However, the Weibel instability recently have been paid attention in the context of astrophysical applications, since it can generate magnetic field [ref.....]. Therefore, we show in the appendix that the linear growth rate of the Weibel instability in the foot of astrophysical shocks can be quite large in the high Mach number limit. The particle energies including both thermal and bulk energies show rather simple time histories. The electrons (reflected ions) illustrate mainly two growth (decline) phases; one is the gradual phase between $500 < \omega_{pe}t < 1200$ and another one is the rapid phase between $1200 < \omega_{pe}t < 1500$. The energy of the incoming ions exhibits, on the other hand, a single declining phase followed by a small growth phase. It can be seen that a part of the reflected ion energy is efficiently transferred to the electrons. To first order the energy budget in the system is balanced between these two components. More precisely bulk energy of the reflected ions is transferred to thermal energy of the electrons; the thermal energy of the reflected ions stays almost unchanged and the bulk energy of the electrons is almost zero (in the electron rest frame used here) throughout the run. On the other hand, part of the bulk energy of the incoming ions is transformed to thermal energy of these ions via phase space trapping. These trapping oscillations may lead to the small growth phase in the energy-time history of the incoming ions seen in Figure 1.

These features will be discussed in detail in the following subsections. For convenience, the time evolution of the system is divided into three stages according to the time variation of the electron temperature anisotropy (represented by the dashed line in the third panel from top of Figure 2). The anisotropy is defined by $T_\perp/T_\parallel = \left\langle (v_x^2 + v_z^2)/v_y^2 \right\rangle$, where the bracket denotes an ensemble average. Stage I, $\omega_{pe}t < 500$, is the early stage of the run in which $T_\perp/T_\parallel \sim 1$. In stage II ($500 < \omega_{pe}t < 1200$), $T_\perp/T_\parallel > 1$. The remaining phase ($\omega_{pe}t > 1200$) where $T_\perp/T_\parallel < 1$ is stage III.

2.2.2. Stage I ($\omega_{pe}t < 500$)

Figure 3(a) shows a power spectrum of the electric field E_x component in $k_x - k_y$ wave vector space at $\omega_{pe}t = 253.0$. Significant wave power can be seen at $(k_xc/\omega_{pe}, k_yc/\omega_{pe}) \approx (6.8, 0)$. This means that the waves propagate almost perpendicular to the ambient magnetic field. To identify the wave propagation direction (positive or negative x direction) the data are time-integrated along the $x-$axis at fixed $y/(c/\omega_{pe}) = 20$ during the interval $0 < \omega_{pe}t < 404.8$. The $\omega - k_x$ power spectrum obtained in this way is shown in Figure 3(b). Two peaks can be recognized along the line of the reflected ion beam which has a negative slope and originates from the origin. The lower (upper) peak is the result of an interaction between the reflected ion beam and the Bernstein mode of the fundamental (second) harmonics. The wavelength is roughly estimated by putting $k_xu_r \sim n\Omega_e (n = 1, 2, ...)$. This instability is called electron cyclotron drift instability (ECDI). We should like to point out that recently *Muschietti and Lembège*, [in press] also identified the ECDI in a 1-D PIC simulation of a perpendicular shock. In the present run the Bernstein waves propagating in the positive x direction are not generated because the electron thermal velocity, v_{te}, is large compared to the incoming ion beam velocity, u_i. As mentioned in the previous subsection, during this stage the ECDI is dominant.

2.2.3. Stage II ($500 < \omega_{pe}t < 1200$)

In the second stage, the field properties are somewhat more complicated. The top three panels in Figure 4 show from left to right density profiles of the electrons, of the incoming ions, and of the reflected ions at $\omega_{pe}t = 885.4$, respectively. The middle three panels show field profiles of E_x, B_y, and B_z, respectively, and corresponding power spectra in $k_x - k_y$ space are represented in the bottom three panels at the corresponding time.

In the profiles of the electron density, of the reflected ion density, and of the E_x field vertical structures due to the ECDI are seen. This supports our suggestion that the instability is due to an interactions between the electrons and the reflected ions. However, the power in E_x is significantly diffuse in $k_x - k_y$ space (bottom left panel). We will discuss this point later in greater detail.

Another longer wavelength structure can be seen in the B_y and the B_z components, and in the incoming ion density profiles. The structure propagates nearly perpendicular to the ambient magnetic field, which can be seen from the two peaks close to the k_x axis in the B_z power spectrum (bottom right panel). This structure is due to the modified two-stream instability (MTSI) based on the electron-incoming ion interaction which was observed in 1-D full particle quasi-perpendicular shock simulations by *Scholer et al.* [2003] and *Scholer and Matsukiyo* [2004]. The instability was analyzed in detail by *Matsukiyo and Scholer* [2003], who showed its typical wavelength to be of the order of c/ω_{pe}. Henceforth we will refer to this type of MTSI as MTSI-1. Two other more oblique peaks in the B_z power spectrum and corresponding oblique lines in the reflected ion density profile are due to the MTSI based on electron-reflected ion interaction, which we call henceforth MTSI-2. It should be noted that it is difficult to simultaneously observe these two MTSIs in 1-D shock simulations because the two MTSIs have different propagation angles.

The B_y power spectrum contains characteristics similar to both the E_x and the B_z power spectra, i.e., to peaks corresponding to the ECDI and to the two MTSIs. The reason is that while the Bernstein waves have X-mode polarization resulting in compressional magnetic field fluctuations, the MTSIs generate oblique whistler waves which contain fluctuations in all three components of the magnetic field.

Let us address the problem of the diffuse spectrum of the Bernstein waves in more detail. The most plausible interpretation for its occurrence is nonlinear coupling among waves generated by the ECDI and by the MTSI-1. Figure 5(a) shows the $\omega - k_x$ power spectrum of the B_y field for the period of $607.2 < \omega_{pe}t < 1011.9$. Three strong peaks indicated by white arrows correspond to the instabilities discussed above. The low frequency peak with positive (negative) k_x is due to the MTSI-1 (MTSI-2). The fundamental Bernstein mode is seen near $(\omega/\omega_{pe}, k_x c/\omega_{pe}) \approx (0.6, -6.5)$. This peak is wide spread in $\omega - k_x$ space which leads to the diffuse spectrum in $k_x - k_y$ space in the bottom left panel of Figure 4. To clarify the wave-wave interactions through the Bernstein waves a bicoherence analysis has been performed. The bicoherence is defined as the third order correlation between three wave modes [e.g. *Dudok de Wit and Krasnosel'skikh*, 1995]:

$$b^2(k_1, k_2) = \frac{|\langle X(k_1)Y(k_2)Z^*(k_1+k_2)\rangle|^2}{\langle|X(k_1)Y(k_2)Z^*(k_1+k_2)|\rangle^2}, \tag{1}$$

where X, Y, and Z are the Fourier transforms of a given time series, k_1 and k_2 are the wave numbers, the asterisk indicates the complex conjugate, and the bracket denotes ensemble average. The value of b^2 runs from 0 to 1 depending on the strength of the correlation between the wave modes at k_1, k_2, and $k_1 + k_2$. Figure 5(b) exhibits the auto-bicoherence between $B_y(k_1)$, $B_y(k_2)$, and $B_y^*(k_1 + k_2)$ for the same period as Figure 5(a). The high coherence band marked with an ellipse in Figure 5(b) indicates nonlinear couplings between the waves generated by the MTSI-1 and the ECDI, i.e., in Figure 5(a) the range of k_2 corresponds to the unstable modes of the MTSI-1 (spectral peak just below the arrow labeled as k_{2x}), that of k_1 to the amplified modes around the Bernstein branch (spectral peak just below the arrow labeled as k_{1x}), and that of $k_1 + k_2$ is again around the Bernstein branch (spectral peak just above the arrow labeled as $k_{1x} + k_{2x}$). This is why we interpret the broadening of the Bernstein spectrum as being due to the nonlinear interactions between those wave modes. Other unmarked high coherence bands correspond to different types of nonlinear interactions. For instance, the band with $k_1 c/\omega_{pe} \sim k_2 c/\omega_{pe} \approx 6.5$ is due to a coupling of the fundamental Bernstein mode and its higher harmonics.

In this stage, the electron energy gradually increases as the reflected ion energy decreases (Figure 2). The dashed line in the third panel in Figure 2 indicates that perpendicular heating is dominant. The electrons may largely interact with the E_x fields of the broad band Bernstein waves.

2.2.4. Stage III ($\omega_{pe} t > 1200$)

As shown in Figure 6 in a still later stage ($\omega_{pe} t = 2023.9$ or $\Omega_i t = 0.55$ in units of ion gyro time) the Bernstein waves have been damped leading to perpendicular electron heating as discussed above. On the other hand, the two MTSIs still survive. The density profiles of the incoming and the reflected ions are sustained by the MTSI-1 and the MTSI-2, respectively. It is easily confirmed that the MTSI-2 has a more oblique propagation angle to the magnetic field compared with the MTSI-1. We should note that the B_z profile contains both small and large structures corresponding to the MTSI-1 and the MTSI-2, while only the small structure is contained in the E_x profile. This is due to the θ dependence of polarization of whistler waves. Polarization of whistler waves is almost electromagnetic for a wide range of θ (MTSI-2), while the electrostatic nature becomes dominant if θ is close to 90° (MTSI-1). This was also pointed out by *Wu et al.* [1983].

This difference of the polarization affects heating of the two ion species. The response of the ions to the fields is mainly electrostatic, because the ions are almost insensitive to such small amplitude magnetic field fluctuations. Then the MTSI-2 hardly leads to heating of the reflected ions, while the incoming ions are relatively easily heated via the MTSI-1. Indeed, the thermal energy of the incoming ions increases by a factor of 1.9, although that of the reflected ions increases only by a factor of 1.25, which is largely due to the ECDI. The above features are represented in Figure 7(a) which shows the time variations of temperatures of the two ion species. For both ion species, the temperatures increase along the x direction. Figure 7(b) exhibits phase space plots of two ion species in $v_x - x$ phase space at $\omega_{pe}t = 2403.3$. Well-defined phase space holes are recognized for the incoming ions only.

In this stage, parallel electron heating is remarkable, as can be seen from Figure 2 during the interval $1200 < \omega_{pe}t < 1500$. One can easily suspect that this is related to the MTSI-2 since the reflected ion energy decreases correspondingly. On the other hand, the decrease of the incoming ion energy is only minor, which means that the MTSI-1 does not contribute much to the electron heating. This is rather different from the previous 1-D shock simulation results by *Matsukiyo and Scholer* [2003]. According to these authors the main contribution to the parallel electron heating is made by the MTSI-1, since the MTSI-2 is not generated in their 1-D shock simulation. In the next section, we discuss the electron heating process in the context of a two-step instability.

2.2.5. Two-step instabilities

Figure 8(a) represents an electron $v_y - y$ phase space plot at $\omega_{pe}t = 1467.3$ and $19.4 < x/(c/\omega_{pe}) < 21.0$. In addition to the two large electron holes which are produced by the

MTSI-2 small substructures can be recognized. The electrons seem to be thermalized through interactions with these small scale waves in a shorter time than that given by trapping due to the MTSI-2. In this stage, the E_y component is rapidly amplified as seen in Figure 2 and has a broad spectral band width in $\omega - k_y$ space for $1214.3 < \omega_{pe}t < 1416.7$ (Figure 8(b)). Such electrostatic fluctuations are characteristic for electron acoustic waves. Figures 8(c) and (d) denote local electron distribution functions for (b) $17.6 < y/(c/\omega_{pe}) < 19.0$ and (c) $39.1 < y/(c/\omega_{pe}) < 40.5$, respectively. The large electron hole results in locally double peaked distribution functions, which are an ideal free energy source for the electron acoustic instability (EAI). By approximating the distribution function in Figure 8(d) by a bi-Maxwellian the density ratio and the thermal velocities of the two peaks (indicated by subscripts 1 and 2) are estimated as $n_1/n_2 \approx 0.37$, $v_{t1}/v_A \approx 6.7$, and $v_{t2}/v_A \approx 13.7$, respectively. With these values we obtain a linear growth rate for the EAI of $\delta/\omega_{pe} \approx 0.04$, where we have used the dispersion relation: $1 + \sum_j (2\omega_{pj}^2/k_y^2 v_{tj}^2)[1 + \zeta_j Z(\zeta_j)] = 0$. Here ω_{pj} denotes the plasma frequency, $\zeta_j = (\omega - k_y u_j)/k_y v_{tj}$, u_j and v_{tj} the bulk and the thermal velocities for species j, and $Z(\zeta_j)$ indicates the plasma dispersion function. On the other hand if one postulates that the wave is monochromatic and purely electromagnetic (see section 4 for details) the trapping frequency of the electrons by the large MTSI-2 wave is roughly estimated as $\omega_\tau/\omega_{pe} \approx 0.01$. Since $\delta > \omega_\tau$, the EAI can grow sufficiently within the trapping period.

Another two-step instability can be seen from Figure 2 as a rapid growth of the B_x field energy for $1100 < \omega_{pe}t < 1400$. Strong spectral peaks appear along the dispersion curve of parallel whistler waves in the $\omega - k_y$ spectra of the B_x and the B_z components for $910.7 < \omega_{pe}t < 1315.5$ (not shown). Since a temperature anisotropy of the electrons

($T_\perp/T_\parallel > 1$) due to the ECDI arises, the whistler instability (WI) may be generated [*Wu et al.*, 1984].

Both instabilities, the EAI and the WI, may contribute to parallel electron heating. However, it is known that a decrease of the electron temperature anisotropy through the WI is very slow and not very significant when the electron beta is low [*Devine et al.*, 1995]. Actually, the parallel electron energy achieved during this stage is much larger than the free energy due to the temperature anisotropy. Therefore we conclude that parallel electron heating is mainly due to the EAI.

Although the electron heating observed here is rather strong (the final temperature is about 5 times larger than the initial value, see Figure 1), we could not observe well-defined nonthermal electrons like they have been found by *Tanaka and Papadopoulos* [1983]. A possible reason is that unlike as in *Tanaka and Papadopoulos* [1983], the run time of the present simulation is shorter than the ion gyro time, $\Omega_i t \sim 0.65$ ($\Omega_i t \sim 4.2$ (or $\omega_{LH} t = 60$) in *Tanaka and Papadopoulos* [1983], where ω_{LH} denotes the lower-hybrid frequency) and the ions are magnetized. The assumption of unmagnetized ions allows the instability to last without being suppressed by ion gyration.

3. Summary and Conclusions

We have investigated microinstabilities in the foot of a high Mach number perpendicular shock by means of a 2-D particle-in-cell simulation.

A 2-D simulation was performed in order to show that two-dimensionality is quite essential in terms of excitation of waves and of particle heating. We found that three instabilities are excited in the first stage through linear interactions between the background electrons and the two ion beams, and two instabilities are generated in the second stage as two-step instabilities. Depending on the magnitude of the linear growth rates the ECDI grows fastest, leading mainly to perpendicular electron heating, while its saturation level is not so high. After some delay, the two MTSIs become dominant. Even though the two MTSIs have different propagation angles relative to the ambient magnetic field, a 2-D simulation simultaneously results in both of them. In competing processes among those three instabilities a nonlinear coupling between the Bernstein waves and the MTSI-1 waves has been confirmed by a bicoherence analysis. The saturation levels of the two MTSIs are higher than that of the ECDI so that the two MTSIs survive at the end of the simulation in spite of the smaller growth rates of the MTSIs compared to the ECDI. The MTSI-1 leads to the well-defined ion phase space holes in $v_x - x$ phase space. The MTSI-2 provides efficient parallel electron heating through the second stage EAI. We found two kinds of two-step instabilities, one is MTSI-2 $\longrightarrow$ EAI mentioned above and another one is ECDI $\longrightarrow$ WI.

In the present simulation, the ECDI is the fastest growing instability. This is not independent of the fact that an unrealistically small $\tau(= 4)$ has been assumed here. If $\tau \gg 1$, like in most space plasma environments, the Buneman instability is superior to the ECDI, since the maximum growth rate of the Buneman instability (ECDI) is proportional to $(1/\mu)^{1/3}\omega_{pe}$ $((1/\mu)^{1/2}\Omega_e)$ [*Buneman*, 1958; *Dieckmann et al.*, 2000]. When the Buneman instability grows fast enough and strong electron or ion heating occurs, the ECDI may be suppressed by kinetic effects. Actually *Shimada and Hoshino* [2004] confirmed in their simulation that effects of the ECDI are weakened as τ increases. This is the reason why only the Buneman instability is observed in the simulation performed by *Shimada and Hoshino* [2000], who assumed $\tau = 400$. In that sense it might be reasonable to interpret our results by replacing the ECDI by the Buneman instability. In fact we used here $\tau = 4$, resulting in a frequency of the second harmonics of the ECDI close to the upper hybrid frequency. This frequency actually survives in the cold plasma limit and corresponds to the frequency of the Buneman instability.

Let us discuss saturation levels of the three first stage instabilities. When the Mach number is large enough the Buneman instability leads to electron holes (*Shimada and Hoshino* [2000]), and in the present simulation we have seen the large electron holes produced by the MTSI-2. If the saturation mechanism is electron trapping, the saturation levels can be roughly estimated by $\delta \sim \omega_\tau$, where δ and the ω_τ denote the linear growth rate and electron trapping frequency, respectively. For the Buneman instability $\delta \sim (\sqrt{3}/2)\omega_{pe}(m_e/2m_i)^{1/3}$ and $\omega_\tau \sim (ekE/m_e)^{1/2}$, so that $eE/m_ec \sim (3u/4c)\omega_{pe}(m_e/2m_i)^{2/3}$, where e is the elementary charge, E electrostatic field amplitude, and $ku \sim \omega_{pe}$. For the MTSI $\delta \sim (\sqrt{3}/2)(\Omega_e/\sqrt{1+H_e})(m_e/2m_i)^{1/3}(\cos\theta\sin^2\theta/\sqrt{1+H_e})^{1/3}$ (*Matsukiyo and Scholer* [2003]), $\omega_\tau \sim (ekE/m_e)^{1/2}$ in the electrostatic (ES) limit and $\omega_\tau \sim (kv_{te}\Omega_e B/B_0)^{1/2}$ in the electromagnetic (EM) limit, where $H_e = \omega_{pe}^2/k^2c^2$ and B indicates the magnetic field amplitude. Then, $eE/m_ec \sim (3u/4c)(\Omega_e/\cos\theta)(m_e/2m_i)^{2/3}(\cos\theta\sin^2\theta/\sqrt{1+H_e})^{2/3}$ and $eB/m_ec \sim (3u/4v_{te})(\Omega_e/\cos\theta)(m_e/2m_i)^{2/3}(\cos\theta\sin^2\theta/\sqrt{1+H_e})^{2/3}$ are obtained. Over a wide parameter range for quasi-perpendicular shocks $(1/\cos\theta)(\cos\theta\sin^2\theta/\sqrt{1+H_e})^{2/3} \sim O(1)$. The resulting saturation levels are summarized in Table I. It is easily confirmed that the saturation amplitude of the MTSI(ES) is the smallest because $\omega_{pe} \gg \Omega_e$ and $c \gg v_{te}$ in a usual space plasma. Moreover, the saturation amplitude of the MTSI(EM) is the largest if $\beta_e < 1$, since $(v_{te}\omega_{pe}/c\Omega_e) = \sqrt{\beta_e}$. The MTSI-2 is almost electromagnetic as mentioned in section 2 so that this may be the most dominant when those three instabilities simultaneously exist. However, we should note that the MTSI can not be driven when $\sqrt{m_i/m_e}\cos\theta < 4M_A(1-\alpha)/(1+\alpha)$ [*Matsukiyo and Scholer*, 2003], and the Buneman instability may be dominant in that case. In the above discussions the growth rates are obtained by assuming the cold plasma limit, so that the saturation levels are probably upper limits.

Finally, let us consider the influences of the MTSIs on the reformation process of a high Mach number perpendicular shock. In the present study it has been confirmed that two MTSIs get simultaneously excited. If those instabilities are strong enough, not only electron temperature but also ion temperature becomes high in the foot. Thereby, reformation may cease as in the high ion beta shock case or, in other words, reformation time may be rather reduced because of rapid thermalization due to the instabilities. This senario is actually suggested from the simulations by *Shimada and Hoshino* [2005] in which the ion acoustic instability following the Buneman instability leads to strong ion hating in the foot when $M_A \gg 1$.

Acknowledgments. We thank T. Hada and B. Lembège for fruitful discussions. We are also gratefull to the referees for useful comments. The two dimensional simulation was performed by NEC super computer SX6 from ISAS in Sagamihara, Japan, and the code was provided by I. Shinohara.

Appendix

In this appendix, the linear growth rate of the Weibel instability in the foot of a perpendicular shock is derived from the fluid theory. The Weibel instability has been extensively discussed in terms of magnetic field generation in various astrophysical situations [recent examples are *Nishikawa et al.*, 2005, 2003; *Wiersma and Achterberg*, 2004; *Medvedev et al.*, 2004; *Hededal and Nishikawa*, 2004; *Jaroschek et al.*, 2004].

For the simplicity, the following system is supposed. The plasma is composed of magnetized electrons and unmagnetized ions. Only the unmagnetized ions have drift velocity, $\mathbf{u_i}$, perpendicular to the ambient magnetic field, $\mathbf{B_0}$. Waves are allowed to propagate parallel to $\mathbf{B_0}$. If $\mathbf{B_0}$ is in the $y-$direction and $\mathbf{u_i}$ is in the $x-$direction, the zeroth and the first order quantities are discribed as follows.

$$\mathbf{B} = (\delta B_x, B_0, \delta B_z),$$

$$\mathbf{E} = (\delta E_x, \delta E_y, \delta E_z),$$

$$\mathbf{v} = (u + \delta u, \delta v, \delta w),$$

$$n = n_0 + \delta n,$$

where $\nabla = (0, \partial/\partial y, 0)$. The following set of equations are solved.

$$\left(\frac{\partial}{\partial t} + \mathbf{v_j} \cdot \nabla\right) \mathbf{v_j} = \frac{q_j}{m_j}\left(\mathbf{E} + \frac{\mathbf{v_j}}{c} \times \mathbf{B}\right)$$

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v_j}) = 0$$

$$\nabla \cdot \mathbf{E} = 4\pi \sum n_j q_j$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \sum n_j q_j \mathbf{v_j}$$

Here, q and m indicate charge and mass, and the subscript j denotes particle species. Note that $u_e = 0$ and B_0 in the equation of motion for ions is set to zero. We ignored the displacement current. Linearizing and Fourier transforming the above equations, the following expression is obtained after some algebra.

$$\begin{pmatrix} 1+\frac{\omega_{pi}^2}{k^2c^2}+\frac{\omega_{pe}^2}{k^2c^2}\frac{\omega^2}{\omega^2-\Omega_e^2} & -i\frac{\omega_{pe}^2}{k^2c^2}\frac{\omega\Omega_e}{\omega^2-\Omega_e^2} \\ i\frac{\omega_{pe}^2}{k^2c^2}\frac{\omega\Omega_e}{\omega^2-\Omega_e^2} & 1+\frac{\omega_{pi}^2}{k^2c^2}+\frac{\omega_{pe}^2}{k^2c^2}\frac{\omega^2}{\omega^2-\Omega_e^2}+\frac{\omega_{pi}^2}{k^2c^2}\frac{\omega^2-\omega_{pe}^2}{\omega^2-\omega_{pe}^2-\omega_{pi}^2}\frac{k^2u_i^2}{\omega^2} \end{pmatrix}\begin{pmatrix}\delta B_x \\ \delta B_z\end{pmatrix}=0$$

The dispersion relation is given as nontrivial solution of the above equation. It is reduced to a biquadratic equation:

$$\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)^2\omega^4-\left[\left(1+\frac{\omega_{pi}^2}{k^2c^2}\right)\Omega_e^2-\omega_{pi}^2\frac{u_i^2}{c^2}\left(1+\frac{\omega_{pe}^2}{k^2c^2}\right)\right]\omega^2-\omega_{pi}^2\frac{u_i^2}{c^2}\Omega_e^2\left(1+\frac{\omega_{pi}^2}{k^2c^2}\right)=0,$$

where $\omega_{pi}^2 \ll \omega_{pe}^2$ is assumed. In the limit of no magnetic field ($\Omega_e^2 = 0$),

$$\omega = \pm i\frac{u_i\omega_{pi}k}{(k^2c^2+\omega_{pe}^2)^{1/2}}.$$

If the subscript i is replaced by e, this becomes same equation as the one originally obtained by Weibel [1959] who postulated temperature anisotropy in the Vlasov-Maxwell system. It should be noted that the above expression is derived from the two fluid system in the zero temperature limit. However, the cold ion beam gives effective temperature anisotropy in the direction of the beam velocity.

In the case of magnetized electrons two solutions are obtained in the leading order. One solution is stable and describes whistler mode waves, while the second solution describes the Weibel instability:

$$\omega = \pm i\frac{ku_i\omega_{pi}}{(k^2c^2+\omega_{pi}^2)^{1/2}},$$

where $(1+\omega_{pi}^2/k^2c^2)\Omega_e^2 \gg \omega_{pi}^2(1+\omega_{pe}^2/k^2c^2)u_i^2/c^2$ is assumed. The maximum growth rate occurs at $k^2 \to \infty$ as $\omega = i\omega_{pi}u_i/c$. This can be rewritten as

$$\frac{\omega}{\Omega_i} = i\frac{u_i}{v_A},$$

which may be much greater than unity in the limit of a high Mach number shock. The electron Weibel instability may also be possible in some parameter regime [e.g., *Nishikawa* *et al.*, 2005, 2003; *Hededal and Nishikawa*, 2004]; however, a strong electron temperature anisotropy is required in this case.

References

Buneman, O., Instability, turbulence, and conductivity in current-carrying plasma, *Phys. Rev. Lett., 1,* 8–9, 1958.

Cargill, P. J., and Papadopoulos, K., A mechanism for strong shock electron heating in supernova remnants, *Astrophys. J., 329,* L29–L32, 1988.

Davidson, R. C., Krall, N. A., Papadopoulos, K., and Shanny, R., Electron heating by electron-ion beam instabilities, *Phys. Rev. Lett., 24,* 579–582, 1970.

Devine, P. E., Chapman, S. C. and Eastwood, J. W., One- and two-dimensional simulations of whistler mode waves in an anisotropic plasma, *J. Geophys. Res., 100,* 17189–17203, 1995.

Dieckmann, M. E., Chapman, S. C., McClements, K. G., Dendy, R. O., and Drury, L. O., Electron acceleration due to high frequency instabilities at supernova remnant shocks, *Astron. Astrophys., 356,* 377–388, 2000.

Dudok de Wit, T., and Krasnosel'skikh, V. V., Wavelet bicoherence analysis of strong plasma turbulence at the Earth's quasiparallel bow shock, *Phys. Plasmas, 2,* 4307–4311, 1995.

Forslund, D. W., and Shonk, C. R., Numerical simulation of electrostatic counterstreaming instabilities, *Phys. Rev. Lett., 25,* 281–284, 1970.

Forslund, D. W., Morse, R. L., and Nielson, C. W., Electron cyclotron drift instability, *Phys. Rev. Lett., 25,* 1266–1269, 1970.

Gary, S. P., R. L. Toker, and D. Winske, Ion/ion and electron/ion cross-field instabilities near the lower hybrid frequency, *J. Geophys. Res., 92,* 10029–10038, 1987.

Hada, T., M. Onishi, B. Lembège, and P. Savoini, Shock front nonstationarity of supercritical perpendicular shocks, *J. Geophys. Res., 108,* doi:10.1029/2003JA009339,2003.

Hededal, C. B., and K-.I. Nishikawa, The influence of an ambient magnetic field on relativistic collisionless plasma shocks, *Astrophys. J., 623,* L89–L92, 2005.

Hoshino, M., and Shimada, N., Nonthermal electrons at high Mach number shocks: Electron shock surfing acceleration, *Astrophys. J., 572,* 880–887, 2002.

Jaroschek. C. H., H. Lesch and R. A. Treumann, Self-consistent diffusive lifetimes of Weibel magnetic fields in gamma-ray bursts, *Astrophys. J., 616,* 1065–1071, 2004.

Krall, N. A., and Liewer, P. C., Low-frequency instabilities in magnetic pulses, *Phys. Rev. A, 4,* 2094–2103, 1971.

Lampe, M., Manheimer, W. M., McBride, J. B., Orens, J. H., Papadopoulos, K., Shanny, R. and Sudan, R. N., Theory and simulation of the beam cyclotron instability, *Phys. Fluids, 15,* 662–675, 1972.

Lampe, M., Haber, I., Orens, J. H., and Boris, J. P., Two-dimensional study of electron-ion streaming instabilities, *Phys. Fluids, 17,* 428–439, 1974.

Lashmore-Davies C. N., Instability in a perpendicular collisionless shock wave for arbitrary ioin temperature, *Phys. Fluids, 14,* 1481–1484, 1971.

Lemons, D. S., and Gary, Current-driven instabilities in a laminar perpendicular shock, *J. Geophys. Res., 83,* 1625–1632, 1978.

Matsukiyo, S., and Scholer, M., Modified two-stream instability in the foot of high Mach number quasi-perpendicular shocks, *J. Geophys. Res., 108,* doi:10.1029/2003JA010080, 2003.

McBride, J. B., E. Ott, J. P. Boris, and J. H. Orens, Theory and simulation of turbulent heating by the modified two-stream instability, *Phys. Fluids, 15,* 2367–2383, 1972a.

McBride, J. B., and E. Ott, Electromagnetic and finite β_e effects on the modified two-stream instability, *Phys. Lett., 39A,* 363–364, 1972b.

Medvedev, M. V., L. O. Silva, M. Fiore, R. A. Fonseca, and W. B. Mori, Generation of magnetic fields in cosmological shocks, *J. Korean Astron. Soc., 37*, 533–541, 2004.

Muschietti, L., and Lembège, B., Electron cyclotron microinstability in the foot of a perpendicular shock: a self-consistent PIC simulation, *Adv. Space Res.*, in press.

Nishikawa, K.-I., P. Hardee, G. Richardson, R. Preece, H. Sol, and G. J. Fishman, Particle acceleration and magnetic field generation in electron-positron relativistic shocks, *Astrophys. J., 622,* 927–937, 2005.

Nishikawa, K.-I., P. Hardee, G. Richardson, R. Preece, H. Sol, and G. J. Fishman, Particle acceleration in relativistic jets due to Weibel instability, *Astrophys. J., 595,* 555–563, 2003.

Ott, E., J. B. McBride, J. H. Orens, and J. P. Boris, Turbulent heating in computer simulations of the modified plasma two-stream instability, *Phys. Rev. Lett., 28,* 88–91, 1972.

Papadopoulos, K., Davidson, R. C., Dawson, J. M., Haber, I., Hammer, D. A., Krall, N. A., and Shanny, R., Heating of counterstreaming ion beams in an external magnetic field, *Phys. Fluids, 14,* 849–857, 1971.

Papadopoulos, K., Microinstabilities and anomalous transport, *Geophys. Monographs, 34,* American Geophys. Union, Washington, D.C., 59–90, 1985.

Papadopoulos, K., Electron heating in superhigh Mach number shocks, *Astrophys. Space Sci., 144,* 535–547, 1988.

Scholer, M., Shinohara, I., and Matsukiyo, S., Quasi-perpendicular shocks: Length scale of the cross-shock potential, shock reformation, and implication for shock surfing, *J. Geophs. Res., 108,* doi:10.1029/2002JA009515, 2003.

Scholer, M., and Matsukiyo, S., Nonstationarity of quasi-perpendicular shocks: a comparison of full particle simulations with different ion to electron mass ratio, *Ann. Geophs., 22,* 2345–2353, 2004.

Scudder, J. D., A. Mangeney, C. Lacombe, C. C. Harvey, C. S. Wu, and R. R. Anderson, The resolved layer of a collisionless, high β, supercritical, quasi-perpendicular shock wave 3. Vlasov electrodynamics, *J. Geophys. Res., 91,* 11075–11097, 1986.

Shimada, N., and Hoshino, M., Strong electron acceleration at high Mach number shock waves: simulation study of electron dynamics, *Astrophys. J., 543,* L67–L71, 2000.

Shimada, N., and Hoshino, M., Electron heating and acceleration in the shock transition region: Background plasma parameter dependence, *Phys. Plasmas, 11,* 1840–1849, 2004.

Shimada, N., and Hoshino, M., Effect of strong thermalizaion on shock dynamical behaviour, *J. Geophys. Res., 110,* doi:10.1029/2004JA010596, 2005.

Tanaka, M., and K. Papadopoulos, Creation of high-energy electron tails by means of the modified two-stream instability, *Phys. Fluids, 26,* 1697–1699, 1983.

Tsurutani, B. T., and Stone, R. G., Collisionless shocks in the heliosphere: Reviews of current research, *Geophys. Monographs, 35,* American Geophys. Union, Washington, D.C., 1985.

Stone, R. G., and Tsurutani, B. T., Collisionless shocks in the heliosphere: A tutorial review, *Geophys. Monographs, 34,* American Geophys. Union, Washington, D.C., 1985.

Wiersma, J., and A. Achterberg, Magnetic field generation in relativistic shocks, *Astron. Astrophys., 428,* 365–371, 2004.

Winske, D., M. Tanaka, C. S. Wu, and K. B. Quest, Plasma heating at collisionless shocks due to the kinetic cross-field streaming instability, *J. Geophys. Res., 90,* 123–136, 1985.

Winske, D., J. Giacalone, M. F. Thomsen, and M. M. Mellott, A comparative study of plasma heating by ion acoustic and modified two-stream instabilities at subcritical quasi-perpendicular shocks, *J. Geophys. Res., 92,* 4411–4422, 1987.

Wong, H. V., Electrostatic electron-ion streaming instability, *Phys. Fluids, 13,* 757–760, 1970.

Wu, C. S., Zhou, Y. M., Tsai, S. T., Guo, S. C., Winske, D., and Papadopoulos, K., A kinetic cross-field streaming instability, *Phys. Fluids, 26,* 1259–1267, 1983.

Wu, C. S., Winske, D., Zhou, Y. M., Tsai, S. T., Rodoriguez, P., Tanaka, M., Papadopoulos, K., Akimoto, K., Lin, C. S., Leroy, M. M., and Goodrich, C. C., Microinstabilities associated with a high Mach number perpendicular bow shock, *Space Sci. Rev., 36,* 64–109, 1984.

Figure 1. Initial conditions of the simulation run presented in this study. (a) Schematic of the geometry of the periodic system in the electron rest frame. The system is supposed to represent part of the foot region of a perpendicular shock. (b) Initial particle distribution functions of electrons, incoming ions, and reflected ions, respectively, versus v_x.

Figure 2. Time history of various particle and field energies. All energies (solid lines) are normalized to the initial electron thermal energy. The particle energies include both the bulk and the thermal energies. The dashed line in the third panel denotes the electron temperature anisotropy.

Figure 3. Power spectra of the E_x field (a) in $k_x - k_y$ space at $\omega_{pe}t = 253.0$ and (b) in $\omega - k_x$ space for the period of $0 < \omega_{pe}t < 404.8$. The straight line originating in the origin with negative (positive) slope indicates the reflected (incoming) ion beam.

Figure 4. Snapshots at $\omega_{pe}t = 885.4$. From left to right the top three panels show density profiles of electrons, of incoming ions, and of reflected ions. The middle three panels indicate E_x, B_y, and B_z field profiles, and corresponding power spectra in $k_x - k_y$ space are represented in the bottom three panels, respectively.

Table I

instability	saturation amplitude
Buneman inst.	$eE/m_ec \sim (3/4)(m_e/2m_i)^{2/3}(u/c)\omega_{pe}$
MTSI (ES)	$eE/m_ec \sim (3/4)(m_e/2m_i)^{2/3}(u/c)\Omega_e$
MTSI (EM)	$eB/m_ec \sim (3/4)(m_e/2m_i)^{2/3}(u/v_{te})\Omega_e$

Figure 5. (a) $\omega - k_x$ power spectrum of B_y for the period of $607.2 < \omega_{pe}t < 1011.9$ and (b) auto-bicoherence between $B_y(k_1)$, $B_y(k_2)$ and $B_y^*(k_1 + k_2)$ for the corresponding period. Strong three wave couplings are shown by the ellipse in (b) indicating interactions among waves with corresponding wave numbers k_{1x}, k_{2x}, and $k_{1x} + k_{2x}$. Those waves are originally generated by the MTSI-1 at $k_x \sim k_{2x}$ and the ECDI at $k_x \sim k_{1x}$ as confirmed in (a).

Figure 6. Density and field profiles at $\omega_{pe}t = 2023.9$. The top two panels show incoming and reflected ion densities, and the bottom two panels denote E_x and B_z profiles, respectively.

Figure 7. (a) Time variation of temperature for the two ion species. The temperatures are normalized to the initial electron temperature. (b) The $v_x - x$ ion phase space plot at $\omega_{pe}t = 2403.3$.

Figure 8. (a) $v_y - y$ electron phase space plot at $19.4 < x/(c/\omega_{pe}) < 21.0$ and corresponding two local distribution functions for (c) $17.6 < y/(c/\omega_{pe}) < 19.0$ and (d) $39.1 < y/(c/\omega_{pe}) < 40.5$, at $\omega_{pe}t = 1000$. (b) $\omega - k_y$ power spactrum of E_y field for the period of $910.7 < \omega_{pe}t < 1315.5$.