A Supplement to Mathematical Structure Common to Animal Growth Analysis and Space Expansion Analysis

Shimojo, Masataka
Laboratory of Regulation in Metabolism and Behavior, Division of Animal and Marine Bioresource Sciences, Department of Bioresource Sciences, Faculty of Agriculture, Kyushu University

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A Supplement to Mathematical Structure Common to Animal Growth Analysis and Space Expansion Analysis

Masataka SHIMOJO*

Laboratory of Regulation in Metabolism and Behavior, Division of Animal and Marine Bioresource Sciences, Department of Bioresource Sciences, Faculty of Agriculture, Kyushu University, Fukuoka 812–8581, Japan

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This study was conducted to add some explanations to the previous study on the mathematical structure common to animal growth analysis and space expansion analysis. The results obtained were as follows. (A) In the construction of differential equations, the way of combining the Bertalanffy equation with its first and second derivatives in animal growth analysis was the same as the way of combining the Friedmann equations with its first and second derivatives in space expansion analysis. This commonness was related to the feature of differential calculation of exponential function. The animal growth analysis was based on the balance between anabolism and catabolism, and the space expansion analysis was based on the balance between attractive force and repulsive force. Those two phenomena might look like each other. (B) The space expansion was discussed in terms of two different redshifts. There was a duality in the interpretation of space expansion history using cosmic scale factor (cosmological redshift) and toy space model (relativistic Doppler redshift). This might be related to the hypothesis that the flat FLRW space with gravity might look like the Minkowski space without gravity.

Key words: animal growth, common structure, duality, redshift, space expansion

INTRODUCTION

Tegmark (2014) showed a similarity between the growth of a human baby and the expansion of our baby universe under the inflationary theory that was proposed by some studies (Starobinsky, 1980; Kazanas, 1980; Sato, 1981; Guth, 1981; Linde, 1982; Albrecht and Steinhardt, 1982). Shimojo (2016) suggested a mathematical structure common to animal growth analysis and space expansion analysis, but there was an insufficiency of the explanation. The present study was designed to add some explanations to this issue.

ANIMAL GROWTH ANALYSIS AND SPACE EXPANSION ANALYSIS

Bertalanffy equation for animal growth analysis

Bertalanffy equation (1949, 1957) is given by

$$\frac{dW}{dt} = \alpha W^{m} - \beta W, \tag{1}$$

$$W = (\alpha/\beta - (\alpha/\beta - W_0^{1-m}) \exp(-\beta(1-m)t))^{1/(1-m)}, \tag{2}$$

where W = weight, t = time, α = anabolic constant, β = catabolic constant, m = constant, W_0^{1-m} = weight at $t = 0$.

Operating (1) and (2) gives (3) ~ (5),

$$\left(\frac{1}{W} \cdot \frac{dW}{dt}\right)^2 = (\alpha W^{m-1} - \beta)^2, \tag{3}$$

$$\frac{1}{W} \cdot \frac{d^2W}{dt^2} = (\alpha W^{m-1} - \beta)(\alpha m W^{m-1} - \beta), \tag{4}$$

$$\frac{W(d^2W/dt^2)}{(dW/dt)^2} = \frac{\alpha m W^{m} - \beta W}{\alpha W^{m} - \beta W} . \tag{5}$$

The more appropriate description of the right–hand side of (3) ~ (5) is given by replacing W with the right–hand side of (2).

FLRW metric and Friedmann equations for space expansion analysis

FLRW metric is given by

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{6}$$

where s = space–time interval, c = speed of light in vacuum, t = time, a = cosmic scale factor, K = curvature of space, (r, θ, ϕ) = spherical polar coordinates.

The cosmic scale factor is used as follows in the Friedmann equations for space expansion analysis,

$$\left(\frac{1}{a} \cdot \frac{da}{dt}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{c^2K}{a^2} + \frac{c^2\Lambda}{3} , \tag{7}$$

$$\frac{1}{a} \cdot \frac{d^2a}{dt^2} = -\frac{4\pi G}{3c^2}(\rho + 3p) + \frac{c^2\Lambda}{3} , \tag{8}$$

* E–mail: mshimojo@agr.kyushu–u.ac.jp
This study was conducted mostly at Agri–Bio Research Laboratory on Ito Campus, Kyushu University, Motooka, Fukuoka 819–0395

$$\frac{a(d^2a/dt^2)}{(da/dt)^2} = -q, \qquad (9)$$

where π = circular constant, G = gravitational constant, ρ = energy density, Λ = cosmological constant, p = pressure, q = deceleration parameter.

Mathematical structure common to Bertalanffy equation and Friedmann equations

The mathematical structure of left–hand side of equations is common to animal growth analysis [(3), (4), (5)] and space expansion analysis [(7), (8), (9)], which is related to the feature of differential calculation of exponential function.

If animal growth rate and space expansion rate are constants respectively, then the mathematical structure of the equation is common to animal growth analysis (10) and space expansion analysis (11),

$$\frac{1}{W}\cdot\frac{dW}{dt} = r\,(\text{constant}), \;\rightarrow\; W(t)\propto\exp(r\cdot t), \quad (10)$$

$$\frac{1}{a}\cdot\frac{da}{dt} = H\,(\text{constant}), \;\rightarrow\; a(t)\propto\exp(H\cdot t), \quad (11)$$

where r = relative growth rate of animal growth, H = Hubble parameter of space expansion.

The animal growth depends on the balance between anabolism and catabolism, and the space expansion depends on the balance between attractive force and repulsive force. Those two phenomena might look like each other.

SPACE EXPANSION ANALYSIS BASED ON REDSHIFTS

As is defined, the redshift caused by the space expansion is the cosmological redshift, not the relativistic Doppler redshift (Weinberg, 2008). However, there are many studies proposing the relationship beween the relativistic Doppler redshift and the space expansion (for example, Macleod, 2004; Chodorowski, 2006; Sitnikov, 2006; Bunn and Hogg. 2009; Davis, 2010a, 2010b; Farley, 2010; May and Yu, 2013; Tsujikawa, 2013; Li, 2014; Pierseaux, 2014; Tatum *et al.*, 2015).

Subsequent studies (Shimojo, 2014, 2015a, 2015b, 2016) investigated the space expansion using a toy space model related to the relativistic Doppler redshift. This issue is taken up again in this chapter by adding some explanations.

Cosmological redshift, relativistic Doppler redshift, and space expansion

The cosmological redshift (Z_C) is given by

$$Z_C + 1 = \frac{\lambda_o}{\lambda_E} = \frac{a(t_o)}{a(t_E)}, \qquad (12)$$

where λ_E = emitted wavelength, λ_o = observed wavelength, $a(t_E)$ = size of the cosmic scale factor when light was emitted, $a(t_o)$ = size of the cosmic scale factor when light was observed.
Normalizing the cosmic scale factor [$a(t_o)$ = 1] in (12) gives

$$\frac{\lambda_o}{\lambda_E} = \frac{1}{a(t_E)}. \qquad (13)$$

The relativistic Doppler redshift (Z_R) is given by

$$Z_R + 1 = \frac{\lambda_o}{\lambda_E} = \frac{1+v_E/c}{\sqrt{1-(v_E/c)^2}} = \sqrt{\frac{c+v_E}{c-v_E}}, \qquad (14)$$

where v_E = receding velocity of the object when light was emitted.
Transforming (14) gives

$$\frac{\lambda_o}{\lambda_E} = \frac{1}{\sqrt{\dfrac{c-v_E}{c+v_E}}}. \qquad (15)$$

Thus, relating (13) and (15) gives (16) ~ (18),

$$\frac{1}{a(t_E)} = \frac{\lambda_o}{\lambda_E} = \frac{1}{\sqrt{\dfrac{c-v_E}{c+v_E}}}. \qquad (16)$$

$$a(t) = \sqrt{\frac{c-v}{c+v}}, \;(17) \quad \rightarrow \quad a(t) = \sqrt{\frac{1-v}{1+v}}, \;(18)$$

where $c \geq v \geq 0$ in (17), $c = 1$ and $1 \geq v \geq 0$ in (18).
The right–hand side of (17) and (18) might be interpreted as a toy model [M] of the space due to the equality with the cosmic scale factor. In addition, there might be a correspondence between (7) and (19) and a correspondence between (8) and (20),

$$\left(\frac{1}{a}\cdot\frac{da}{dt}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{c^2K}{a^2} + \frac{c^2\Lambda}{3}, \qquad (7)$$

$$\left(\frac{1}{M}\cdot\frac{dM}{dv}\right)^2 = \frac{c^2}{(c-v)^2(c+v)^2}, \qquad (19)$$

$$\frac{1}{a}\cdot\frac{d^2a}{dt^2} = -\frac{4\pi G}{3c^2}(\rho+3p) + \frac{c^2\Lambda}{3}, \qquad (8)$$

$$\frac{1}{M}\cdot\frac{d^2M}{dv^2} = \frac{c\,(c-2v)}{(c-v)^2(c+v)^2}. \qquad (20)$$

Despite the same redshift (16), the cause of the flatness is completely different between the FLRW space (a) and the Minkowski space (M). The FLRW space is flattened by the presence of gravity (baryonic matter + dark matter) and anti–gravity (dark energy), a change of the curvature from negative to zero. However, the flatness of the Minkowski space is ascribed to the absence of gravity. Since the gravity curves the space, the flat FLRW space might look like the Minkowski space. Due to the same redshift (16), the object that is seemingly moved by the straight expansion of the flat FLRW space might look like the object that really moves straight in the Minkowski space. The FLRW metric, when it shows the flat universe ($K = 0$), equals the Minkowski metric that shows a change with time (Nagashima, 2008). Due to the same redshift (16), the toy space model (M) might have the energy that corresponds to the energy that the flat FLRW space has. This energy issue will be taken up in the next section on the expansion history of toy space model.

Despite the opposite causes of the flatness, the cosmic scale factor (a) and the toy space model (M) describe the same cosmic expansion history [(16) ~ (18)]. The expansion history of the toy space model might play some role in the estimation of the components of the cosmic scale factor and of the chronological changes in the proportion of baryonic matter, dark matter and dark energy. Maldacena (2005) proposed AdS/CFT correspondence between anti–de Sitter spaces with quantum gravity and conformal field theories without gravity. Bekenstein (2003) described that two universes of different dimension and obeying disparate physical laws are rendered completely equivalent by the holographic principle.

Due to the same redshift, there is a correspondence between t in the cosmic scale factor and v in the toy space model [(16) ~ (18)]. This shows that the space–time in the general relativity is locally the Minkowski space–time. Carmeli (2006) proposed a correspondence between the cosmic time (t) in the cosmological transformation and the velocity (v) in the Lorentz transformation. Milne proposed a kinematic–special relativistic cosmology as an alternative to the general relativistic cosmology (Kragh, 2007).

Expansion history of toy space model

This issue was investigated in previous reports (Shimojo, 2014, 2015a, 2015b, 2016) using the following toy models; space (21), energy density of space (22), and conservation of energy in toy space model (23),

$$M(v) = \sqrt{\frac{c-v}{c+v}}\ , \quad (21) \qquad E(v) = \sqrt{\frac{c+v}{c-v}}\ , \quad (22)$$

$$M(v) \cdot E(v) = \sqrt{\frac{c-v}{c+v}} \cdot \sqrt{\frac{c+v}{c-v}} = 1. \qquad (23)$$

In this section, the description of the expansion history of toy space model is taken up again [(i) ~ (ix)]. (i) The proudct of the size of toy space model (21) and the energy density (22) gives the conservation of energy (23) in this toy model. This is different from the standard cosmic model that shows that dark energy density is not changed by the space expansion. (ii) The space is born from the singularity [$v = c$, space size = 0, energy density = ∞, quantum phenomenon: {ln(0 · ∞) = ln(1)} → {− ∞ + ∞ = $2n\pi \boldsymbol{i} = 2n\pi\,(pq\text{–}qp)/\hbar$}, $\boldsymbol{i}$ = imaginary unit]. Hawking and Penrose (1970) showed the big–bang singularity. (iii) Immediately after the birth, the rapid expansion of space occurs under the decelerated condition. Linde (1982) and Albrecht and Steinhardt (1982) proposed the slow–roll inflation occurring under the very slow decrease of the Hubble parameter. (iv) Decelerated expansion in the domain of [$c \geq v \geq c/2$] with a large decrease in energy density [∞→1.732]. Dark matter is not identified yet. (v) Inflection point [$v = c/2$, space size = 0.577, energy density = 1.732]. (vi) Accelerated expansion in the domain of [$c/2 \geq v \geq 0$] with a small decrease in energy density [1.732→1]. Dark energy is not identified yet. (vii) Present point [$v = 0$, space size = 1, energy density = 1]. (viii) Comparing (vii) with (ii) shows that the energy density of space is much lower in the present point [= 1] than in the birth point [= ∞]. At present, the observed quantity of dark energy is much lower than the theoretical quantity of quantum vacuum energy. The chronological changes in the energy density (22) of toy space model (21) might correspond to the chronological changes in the density of baryonic matter, dark matter and dark energy in the cosmic scale factor [(7), (8)]. (ix) The expansion history of toy space model under the condition of [$c = 1$ and $1 \geq v \geq 0$] is shown in Fig. 1. This might look like the expansion history using cosmic scale factor shown by some studies (Perlmutter *et al.*, 1999; Barrow, 2011; Perlmutter, 2011, 2015). There might be a duality in the interpretation of space expansion history using cosmic scale factor and toy space

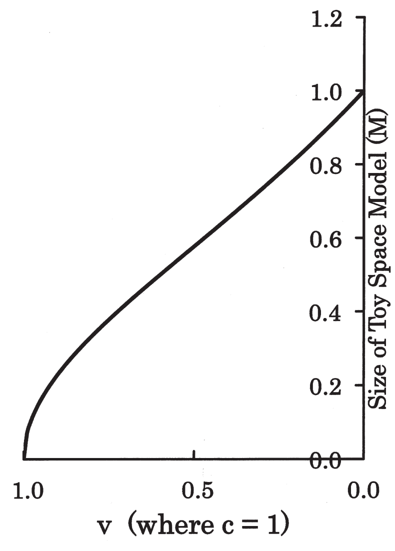

Fig. 1. Space expansion history using toy space model (M) under $c = 1$ and $1 \geq v \geq 0$.

model, namely a duality between the flat FLRW metric and the Minkowski metric.

Singularity of toy space model with energy density

This issue was investigated in previous reports (Shimojo, 2015a, 2015b, 2016), and is taken up again in this section by adding a few explanations.

When $v = c$ in the toy space model with energy density (23), the singularity is given by (24), namely space size = 0 and energy density = ∞ as shown in (25),

$$\sqrt{\frac{c-c}{c+c}} \cdot \sqrt{\frac{c+c}{c-c}} = 1, \tag{24}$$

$$0 \cdot \infty = 1. \tag{25}$$

The natural logarithm of (25) gives (26) and thus (27), and in addition, (28) ~ (31),

$$\ln(0) + \ln(\infty) = \ln(1), \tag{26}$$

$$-\infty + \infty$$

$$= 2n\pi \boldsymbol{i}, \tag{27}$$

$$= 2n\pi \cdot \frac{\psi(x,t) - \psi^*(x,t)}{2A\sin(x,t)}, \tag{28}$$

$$= 2n\pi \cdot \frac{pq - qp}{h/2\pi}, \tag{29}$$

$$= 2n\pi \cdot \lim_{v \to \infty} \sqrt{\frac{c-v}{c+v}}, \tag{30}$$

$$= 2n\pi \cdot \lim_{\Delta t \to 0} \sqrt{\frac{c-(\Delta x/\Delta t)}{c+(\Delta x/\Delta t)}}, \tag{31}$$

where $\boldsymbol{i}$ = imaginary unit.

Something like a renormalization of infinities (27) might be related to quantum phenomena [(28), (29)] through the imaginary unit [$\boldsymbol{i}$ in (27)]. The imaginary unit might be mathematically related to the infinite velocity (30), and the state of $\Delta x/0$ in (31) might be related to the simultaneous existence of different states that the quantum probability wave shows. This might be a deeper level that is not Lorentz invariant behind the apparent Lorentz invariant, as John Bell told about quantum phenomena (Davies and Brown, 1986). In short, the micro–distance [Δx in (31)] and quantum phenomena [(28), (29)] might be born from the singularity (25) by something like a renormalization of infinities (27).

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