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Long Time Evolution of Electromagnetic Waves driven by the Relativistic Ring Distribution

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Ring distribution of particles in a plasma in momentum space generates electromagnetic waves via cyclotron resonance. In addition to the well-known temperature anisotropy driven instability with finite wavenumber \( k \), a wave mode around \( k = 0 \) is excited due to relativistic mass variation of ring particles. We studied in detail long time evolution of this so-called cyclotron maser instability at \( k = 0 \), by performing particle simulations using a plasma which consists of relativistic ring electrons, background positrons, and electrons. A phase transition like behaviour is observed as initial ring energy is varied.

1 Introduction

The relativistic effect in a plasma appears as the downshift of the particle cyclotron frequency due to the relativistic mass variation. Consequently, the resonance condition between particles and waves is modified by the Lorentz factor,

\[
\omega - k_{||}v_{||} - \Omega_o/\gamma = 0, \tag{1}
\]

where \( \omega \) is the wave frequency, \( k_{||} \) the parallel wavenumber (wavenumber component parallel to the ambient magnetic field), \( v_{||} \) the parallel particle drift velocity, and \( \Omega_o \) is the non-relativistic cyclotron frequency, respectively. The instability driven by this effect is known as the cyclotron maser instability and the application of this instability is well discussed related to the laboratory (especially in the gyrotron) and the space plasma (the auroral kilometric radiation (AKR) or the relativistic shocks) environments.

The mechanism of the electromagnetic radiation due to the cyclotron maser instability was originally proposed in the late 1950's by Twiss', Schneider' and Gaponov', independently. The waves are generated as a result of azimuthal bunching of the particles driven by the relativistic mass dependence of the particle cyclotron frequency. Chu and Hirshfield' analyzed linear dispersion relation of the cyclotron maser instability assuming the delta function relativistic ring distribution as a free energy source. They compared two unstable modes for parallel propagation. While one of them, corresponding to the usual temperature anisotropy driven instability, is also observed in non-relativistic case, the other one, which corresponds to the cyclotron maser instability, is excited only in the relativistic case. Winglee discussed more in detail the linear growth rate of the latter instability using a Dory-Guest-Harris (DGH) distribution function [Dory et al. 6'] including finite temperature effect. On the other hand, the saturation levels and their mechanisms in the nonlinear stage of the instability are also well discussed by both theoretical and numerical ways (e.g. Queau, Mourenas et al., Pritchett, Queau et al.). These results are briefly summarized as follows. There are two candidates of saturation mechanisms. When the excited waves have large bandwidth of the frequency, the quasi-linear diffusion ceases the wave growth. In the other case, the narrow frequency band leads particle trapping in the momentum space. The saturation levels for each mechanisms are

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obtained by the conditions of $\partial f_0 / \partial p_\perp = 0$ for quasi-linear diffusion and $\delta_i = \Omega_{\text{trap}}$ for trapping respectively, where $f_0$ and $p_\perp$ are the zeroth order distribution function of particles and perpendicular component of the momentum, $\delta_i$ and $\Omega_{\text{trap}}$ denote the linear growth rate of the wave and trapping frequency of the particles.

Most of the previous discussions are focused on the parameter range $k_\perp c / \Omega_0 \sim 1$ ($k_\perp$ is the perpendicular wavenumber and $c$ is the speed of light) and around $90^\circ$ propagation to the ambient magnetic field. This is because the cyclotron maser instability has a peak of the wave energy density near above parameters. The basic mechanism for the instability is, however, the relativistic mass dependence of the cyclotron frequency as mentioned before. This means that the propagation effect of the waves is not necessary for the instability. Since the basic process of the instability driven by the relativistic effect is an interesting subject from the pure physical point of view, we investigate this instability in detail by neglecting the spatial dependence of the system in this paper. This enables us to discuss the nonlinear evolution of the instability with quite high accuracy including the full relativistic effect, while most of the previous works are done for the weakly relativistic case.

2 Linear Theory

We assume that the plasma has three components, background electrons, positrons and rarefied ring electrons. In this section, for simplicity, all the components are assumed to have no thermal spread. Then the distribution function of each component is written as

$$f_{j0} = \frac{n_{j0}}{2\pi p_\perp} \delta(p_{j\parallel}) \delta(p_\perp),$$

$$f_{r0} = \frac{n_{r0}}{2\pi p_\perp} \delta(p_{r\parallel}) \delta(p_\perp - p_0),$$

where the subscript $j$ and $r$ represent the plasma species ($j$ is the background electrons and positrons, and $r$ is the ring electrons), $n_{j0}$ denotes the plasma density, $p_\perp$ and $p_{\parallel}$ are the perpendicular and parallel momenta, and $p_0$ indicates the initial ring momentum, respectively.

For the non-relativistic background plasmas and the relativistic ring electrons, the electromagnetic dispersion relation parallel to the ambient magnetic field is given as

$$0 = 1 - \frac{k^2 c^2}{\omega^2} - \frac{\omega_0^2}{\omega(\omega + \Omega_0)} - \frac{(1 - \alpha) \omega_0^2}{\omega(\omega - \Omega_0)} - \frac{\alpha \omega_0^2}{\gamma_0 \omega - \Omega_0} - \frac{p_0^2 (\omega^2 - k_\perp^2 c^2)}{2 m_0 c^2 (\gamma_0 \omega - \Omega_0)^2},$$

(2)

where $\omega_0$ is the plasma frequency, $\alpha$ is the density ratio of the ring electrons to the background positrons, $\gamma_0 = \sqrt{1 + p_0^2 / m_0 c^2}$ is the Lorentz factor of a particle with momentum $p_0$ and rest mass $m_0$. Hereafter we put $\Omega_0 = c = m_0 = 1$ as normalization factors and introduce $\alpha = \omega_0^2 / \Omega_0^2$. In eq. (2) the fifth and sixth terms of the right hand side indicate the dielectric permittivity of the ring electrons and other terms represent the contribution from the vacuum (the first and second terms) and the background positrons (the third term) and the electrons (the forth term), respectively. The fifth term does not contribute to the instability but behaves as a part of the background plasma. Hence the dispersion relation for the modified background plasma including the fifth term is the same as that of the multi-component plasma. The solutions of eq. (2) for the cyclotron waves are shown in Fig. 1. In Fig. 1a, the solid lines denote the background plasma dispersion relation (the solution of eq. (2) with $\alpha = 0$) and the dashed lines correspond to the modified background plasma dispersion relation. The addition of the fifth term splits the mode of the cyclotron wave into two branches. The cut off frequency of the upper branch ($\omega_c$) is approximately estimated as
This mode is observed only in the relativistic case, because the mode for the cyclotron wave in the modified background plasma no longer splits and the sixth term at \( k = 0 \) vanishes in the non-relativistic limit. Here we assumed \( \omega = 1/\gamma + \delta_i \) and the ordering of parameters are \( O(\alpha) = O(\rho_0) = \epsilon^5 \), \( O(\alpha) = \epsilon^2 \) and \( O(\delta_i) = \epsilon^4 \), where \( \epsilon \) represents the smallness of the parameter. We will discuss more in detail the analytical expressions for a variety of parameter regimes in the next section.

The excitation mechanism of this mode is as follows. Let us consider the moment which the ring electrons deviate a little from the initial distribution. Then the electric field arises in the opposite direction to the deviation of ring electrons. The electrons satisfying \( p'E > 0 \) have a slightly smaller values of momenta than the electrons satisfying \( p'E < 0 \). This momentum difference between former and latter electrons causes the difference of cyclotron frequency due to their different relativistic masses. The former electrons give their energy to the wave electric field while the latter absorb the energy from the wave. When the electric field resonates with the former electrons, it gains energy on the average, since the frequency of the wave electric field

\[
\omega \approx \frac{1}{\gamma_0} + \frac{\alpha \rho_0^5}{2a + (1 + 2a) \rho_0^2}.
\]  

When the sixth term, which is the driving term of the instability, is included, the dispersion curves are modified as in Fig. 1b for \( \alpha = 0.01, a = 1, \rho_0 = 2 \). The solid lines correspond to the real frequency, and the dashed lines are the growth rate. The dispersion curves degenerate around the resonant frequency, \( \omega \approx 1/\gamma_0 \), and it is obvious that there are two unstable modes. The mode with short wavelength \( (k_0/\omega^2 > 1) \) is so called, the temperature anisotropy driven instability, and its maximum growth rate \( (\eta_i) \) is obtained as

\[
\delta_i = \sqrt{\frac{aa}{2} \frac{\rho_0}{\gamma_0^{3/2}}} \sqrt{\frac{p_0^2}{2a + (1 + 2a) \rho_0^2}}.
\]  

This mode is also observed in a non-relativistic plasma \( (\gamma_0 = 1) \). On the other hand, the mode with long wavelength \( (k_0/\omega^2 < 1) \) has the maximum growth rate at \( k_0/\omega = 0 \),

\[
\delta_i = \sqrt{\frac{aa}{2} \frac{\rho_0}{\gamma_0^{3/2}}} \sqrt{\frac{p_0^2}{2a + (1 + 2a) \rho_0^2}}.
\]  

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**Fig. 1** Parallel dispersion relation (\( \omega - k \) diagram) of the electromagnetic waves. (a) The solid line denotes the dispersion relation of the background plasma (the solution of eq. (2) with \( \alpha = 0 \)), and dashed lines indicate the modified background plasma dispersion relation (the solution of eq. (2) with no sixth term). (b) Solution of eq. (2) for \( \alpha = 0.01, a = 1, \rho_0 = 2 \). The solid and dashed lines show the real and the imaginary parts of the frequency.
differs from that of the latter electrons. This frequency difference between former and latter electrons just comes from the relativistic effect. In this study, we consider only this mode, especially at \( k = 0 \), in order to focus on the waves excited by the purely relativistic effect.

### 2.1 Parametric Survey

The stability of the \( k = 0 \) mode is examined for various values of \( a, p_0 \) and \( \alpha \). The behaviour of the real and imaginary parts of the frequency is classified into three regimes by \( p_0 \) as seen in Fig. 2 in which the numerical solutions of eq. (2) with \( k = 0 \) is shown, where the solid and the dashed lines denote the real and the imaginary parts of the wave frequency as a function of initial ring momentum \( p_0 \). The upper straight lines in Fig. 2a–d correspond to the cut off frequency of the high frequency mode of electromagnetic wave \( (\approx \sqrt{1 + 2a}) \). The analytical approximation for the unstable roots is obtained by expanding eq. (2) with \( k = 0 \) around the ring cyclotron frequency. Some examples are shown below for which \( a \) and \( \alpha \) are of the order of \( \epsilon^0 \) and \( \epsilon^2 \) respectively. This ordering is appropriate to the case of Fig. 2c.

#### 2.1.1 Regime i) \((p_0 < p_c)\)

We assume that \( p_0 \) is small and of the order of \( \epsilon \), so that \( \omega \) is written as \( \omega \approx 1/\gamma + \omega_1 \approx 1 - p_0^2/2 + \omega_1 \cdots \), where the ordering of \( \omega_1 \) is required by inspection of the mathematical form of eq. (2) as \( \epsilon^3 \). From the lowest order terms in eq. (2), \( \omega_1 \) is obtained as

\[
\omega_1 = \pm i \frac{\sqrt{\alpha p_0^2}}{2}.
\]

---

**Fig. 2** Stability of the \( k = 0 \) mode for various values of \( a, p_0 \) and \( \alpha \). The solid and dashed lines represent the real and the imaginary parts of the frequency as a function of initial ring momentum \( p_0 \) for (a) \( a = 0.01, \alpha = 0.01 \), (b) \( a = 0.01, \alpha = 0.1 \), (c) \( a = 0.01, \alpha = 0.01 \), (d) \( a = 1, \alpha = 0.1 \), (e) \( a = 100, \alpha = 0.01 \) and (f) \( a = 100, \alpha = 0.1 \).
Therefore the expression of the wave frequency is
\[
\omega \approx 1 - \frac{p_0^2}{2} \pm i\frac{\sqrt{\alpha p_0^2}}{2}.
\] (6)

Eq. (6) gives a good approximation for small \(p_0\) as seen in Fig. 2c, in which the dotted lines show the analytical solution. This expression does not give a good approximation for \(p_0 \sim 1\) because the ordering adopted here is no longer appropriate for such values of \(p_0\). The upper limit of the regime i), \(\omega_i\), is determined later.

2.1.2 Regime ii) \((p_0 < p_0 < p_c)\)

As \(p_0\) increases to \(p_0 \gg 1\), we have to adopt another ordering. Assuming \(p_0\) and \(\gamma_0\) are of the order of \(e^{-2}\), \(\omega\) itself is the order of \(e^2\) and written as \(\omega \approx \omega_1\). Setting the lowest terms in eq. (2) to zero gives the approximate dispersion relation,
\[
\omega_1 = \frac{A + p_0 B}{p_0 A} \left(1 + \frac{1}{\sqrt{1 - \frac{A(A + 3p_0 B)}{(A + p_0 B)^2}}}ight).
\] (7)

where \(A = 2 (1 + 2a)\) and \(B = \alpha a\). One can easily confirm that there exists a critical value of \(p_0\) which indicates the marginal stability. It is written as \(p_c\) and obtained by putting the inside of the square root to zero,
\[
p_c = \frac{2(1 + 2a)}{\alpha a}.
\]

For the case of Fig. 2c \((a = 1, \alpha = 0.01), p_c = 600\). When \(p_0 < p_c\) \((A > p_0 B)\),
\[
\omega \approx \omega_1 = \frac{A + p_0 B}{p_0 A} \left(1 \pm i\frac{p_0 B(A - p_0 B)}{(A + p_0 B)^2}\right) \approx \frac{1}{p_0} + \frac{\alpha a}{2(1 + 2a)} \pm i\frac{\alpha a}{2(1 + 2a)p_0}.
\] (8)

Comparing the imaginary parts of eq. (6) and eq. (8),
\[
p_i = \left(\frac{2a}{\alpha (1 + 2a)}\right)^{1/3}.
\]

Eq. (8) is in good agreement with numerical solution of eq. (2), except when \(p_0 \sim 1\) (we should use eq. (5) for the regime between i) and ii)).

2.1.3 Regime iii) \((p_c < p_0)\)

In this regime the system becomes stable and the solution, eq. (8), has two real roots,
\[
\omega_1 \approx \frac{1}{2p_0} + \frac{\alpha a}{1 + 2a} \frac{3}{2p_0}.
\] (9)

This stabilization takes place as follows. The cut off frequency of the modified background plasma, \(\omega_i\) in eq. (3), is close to the ring cyclotron frequency \((1/\gamma_0)\) for \(p_0 \ll p_c\), so that the eigenmode of the ring electrons formed by the sixth term in eq. (2) can interact with the modified background plasma through the resonant condition \(\omega \approx 1/\gamma_0\). However, when \(p_0\) is close to \(p_c\) \((p_0 \gg 1)\) the cut off frequency of the modified background plasma deviates from the ring cyclotron frequency (the second term in eq. (3) is more dominant than the first term). Finally at \(p_0 = p_c\) the modified background plasma mode splits from the eigenmode of ring component. As a result the ring electrons have no channel to give their free energy to the wave supported by the modified background plasma.
3 Numerical Simulation

As mentioned above, in this study we drop spatial dependence of the system (i.e. zero-dimension) since we are interested in the time evolution of the instability driven by only the relativistic mass variation; this enables us to look at nonlinear evolution and saturation of the system with extremely high accuracy. The relativistic electromagnetic full particle code is used here. The equations solved in this simulation code are the equation of motion for each particle and the Maxwell equation for Ampere’s law with \( \nabla = 0 \). We confirmed this instability is observed for wide range of parameters as the linear theory predicts. The simulation parameters of each runs shown in this paper are summarized in Table 1.

### 3.1 Run 1

Here we show the time evolution of the system for one of the typical runs (run 1) for \( p_0 < p_c \). Fig. 3 shows the snapshots of distribution of ring electrons for run 1. The figures on the left side correspond to the distribution in the perpendicular momentum space \( (p_x - p_y) \). In the figures on the right side, the horizontal axes \( (\Delta \phi_x) \) are the particle phase to the electric field and the vertical axes \( (\Delta \rho) \) indicate the deviation of momentum from the initial value. In the linear stage \( (t = 100) \), the distribution remains almost unchanged. As time has passed \( (t = 300, 500) \), the ring structure is gradually broken and steepens around the stationary point \( (\Delta \phi_x = \pi/2) \), and finally \( (t = 4,000) \) the ring electrons are bunched there. In Fig. 4, the time history of electric field energy for run 1 is shown. It is confirmed that the time development of the system is divided into two stages, the linear growth and the nonlinear saturation stages respectively. The oscillation in the nonlinear stage corresponds to the trapped oscillation of the ring electrons as seen in the Fig. 3 \( (t = 4,000) \).

#### 3.1.1 Saturation Level

The saturation mechanism of this instability is the phase bunching of the ring electrons. The saturation level of each physical quantity can be estimated by modeling the final state of simulation results. The model is shown in Fig. 5. After sufficiently long time, the amplitude
Fig. 3 Time evolution of the ring distribution. The panels on the left side represent the distributions in the perpendicular momentum space \((p_x - p_y)\). In the panels on the right side, the horizontal axes indicate the particle phase to the wave electric field, and the vertical axes are the deviation of momentum from their initial values.
of electric field is assumed to be constant. In addition, the phase relations between wave and each plasma component is as follows: 1) the ring electrons concentrate on the stationary point \( \Delta \phi_r = \pi/2 \) and construct single cluster, 2) the background electrons and positrons move cancelling out the electric force \( \Delta \phi_{e,p} = -\pi/2 \). Solving the equations of motion for three components, Maxwell equation and the equation of energy conservation, i.e.,

\[
\begin{align*}
\dot{\mathbf{p}} &= q_0 \mathbf{E} + \frac{\mathbf{p}}{\gamma_s} \times \Omega_0 \mathbf{z}, \\
0 &= \frac{1}{n_0} j + \dot{\mathbf{E}}, \\
W_{\text{kinetic}}(t) &= W_{\text{field}}(t_{\text{sat}}) + W_{\text{kinetic}}(t_{\text{sat}}),
\end{align*}
\]

we can obtain the saturation levels of all the physical quantities. Here the subscript \( s \) represents the plasma species (e.g., background electrons and positrons, \( r \); ring electrons), \( q_{e,r} = \Omega_{oe,r} = -1 \), \( q_0 = \Omega_{op} = 1 \) and \( \dot{} \) shows the time derivative. For instance assuming the quantities \( p_0 \) and \( a \) are of the order of \( \varepsilon^0 \), the saturation levels of electric field \( (E_{\text{sat}}) \), particle momenta \( (p_{e,sat}, p_{p,sat} \text{ and } \delta p_r = p_0 - p_{r,sat}) \) and \( \alpha \) are of the order of \( \varepsilon^2 \). \( E_{\text{sat}} \) is obtained by expanding eqs. (10), (11) and (12) around the stationary point mentioned above as

\[
E_{\text{sat}} = \frac{\alpha a p_0^3}{2a + (1 + 2a)p_0^2}.
\]

Generally, when the saturation mechanism is attributed to the bunching, the saturation level is roughly derived from the relation \( \delta t = \beta \Omega_{\text{trap}} \), where \( \delta t \) is the linear growth rate, \( \Omega_{\text{trap}} = \sqrt{E_{\text{sat}}p_0/\gamma_0^2} \) indicates the trapping frequency and \( \beta \) denotes a factor close to unity. Here, we can obtain more exact expression without using the uncertain factor \( \beta \). Now one can easily confirm \( \beta = 2 \) by using eq. (5) for \( \delta t \). \textbf{Fig. 6} represents the

![Fig. 4 Time history of the electric field energy for run 1.](image1)

![Fig. 5 Model of the saturation state (single cluster).](image2)

![Fig. 6 Saturation level of the electric field energy as a function of initial ring momentum. The solid line is obtained by solving eqs. (10), (11) and (12), the dots indicate the results of numerical simulations.](image3)
saturation level of the electric field energy as a function of initial ring momentum \((p_0)\). The solid line corresponds to the solution based on the model, and the dots indicate the simulation results. As the ring momentum is gradually increased, the wave saturation level increases too. However, if the ring momentum is further increased to exceed the critical value \(p_c\) introduced in the linear theory, the saturation level drops significantly as expected but it is still finite compared to the noise level (the normalized energy noise level is typically \(10^{-7}\)). In the next section, we analyze the results of another simulation run (run 15) in which \(p_0\) just exceeds \(p_c\).

3.2 Run 15 \((p_0 > p_c)\)

Fig. 7 represents the time history of electric field energy for run 15. It is emphasized that the wave energy explicitly grows for \(t > 60000\) while the system is linearly stable for \(p_0 > p_c\). In contrast to run 1, the time development of the system is divided into three stages. The stage I newly appears in this case and the stage II and III again represent the exponential growth and saturation stages as seen in run 1. The distribution of ring electrons in the stage I is shown in Fig. 8. In this stage the ring electrons have a double cluster structure which has not been observed in run 1. When \(p_0\) exceeds \(p_c\), the stage I continues for long time (this stage is metastable state) since each cluster is separated by almost \(\pi\) in the phase space from others, i.e., they move as roughly keeping the symmetry in the momentum space. Therefore, the energy levels for \(p_0 > p_c\) in Fig. 6 correspond to those of the stage I. We are now investigating how the double clusters are produced.

4 Summary and Discussion

The long time evolution of the waves generated by the relativistic ring distributions is investigated. In this article, we ignored the thermal spread of the particle distributions. Generally the thermal spread of the distribution suppresses the wave growth. Winglee discussed the thermal effect for the cyclotron maser instability and concluded that the thermal spread of velocities destroys the phase synchronism between the wave and ring electrons and allows the presence of cyclotron damping. In addition, the modes with finite \(k\) is ignored in this paper. The free energy must also be relaxed by wave propagation in the real space. Indeed the cyclotron maser instability mainly drives perpendicularly propagating waves and their harmonics. However, the \(k = 0\) mode is an ideal case to investigate the basic process of the instability driven by the relativistic mass variation.

We performed the parametric survey of the linear stability. The characteristics of the real and imaginary parts of the frequency depend on \(p_0\) and are divided into three regimes. The
The nonlinear evolution of the system is studied by using the relativistic full particle simulation code. The saturation levels of the wave electric field are estimated. The saturation levels obtained by the simulations are in good agreement with the single cluster model for $p_0 < p_c$. When $p_0 > p_c$, however, the wave energy levels are finite while the system is linearly stable. Furthermore, the stage I for $p_0 > p_c$ gives the double cluster state which is not observed in the case $p_0 < p_c$. The mechanism of the double cluster formation is a subject which should be addressed and is being studied now.

The transition from the single to double cluster states corresponds to a phase transition. In interpreting this transition on the analogy of the Landau theory, there is only one potential minimum corresponding to the single cluster state during $p_0 < p_c$ (Fig. 9a), while the one more local minimum appears as $p_0$ exceeds $p_c$ (Fig. 9b) and it corresponds to the double cluster state and is metastable. Thus, the exclusion of finite $k$ effects enabled us to find another local minimum of the potential, which is inherent in the system but not recognized including the finite $k$ effects.

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