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Three-Dimensional Oscillatory Marangoni Flow in Half-Zone Liquid Bridges of Pr=1 Fluid

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Three-dimensional (3-D) numerical simulations of oscillatory Marangoni flow were conducted for half-zone liquid bridges of Pr=1 fluid with different aspect ratios (0.75-1.60) and over a wide range of Marangoni number. Growth rate constants β of 3-D disturbances were determined as functions of the Marangoni number. The critical Marangoni number Ma_c, i.e, the stability limit of the axisymmetric steady flow, was determined by extrapolating β to zero. Thus determined critical Marangoni numbers show good agreement with those of linear stability analyses. A rough estimation predicts a correlation $Ma_a/Ma_c \propto a^2$ for large liquid bridges, where a is the liquid bridge radius and Ma_a is some 'apparent critical Marangoni number' at which the 3-D oscillatory flow can be detected experimentally within a constant observation time t_0. Present result predicts an extremely long observation time for experimental determination of the true critical Marangoni number.

1. INTRODUCTION

Marangoni (thermocapillary) convection in a half-zone liquid bridge of length L and radius a confined between two differentially heated isothermal solid disks has become a typical model for the study of Marangoni flows, their stability, and their bifurcations. Many experimental observations reported multi-morphological features of the 3-D oscillatory flows with various azimuthal wave numbers, m, and different types of oscillation, i.e. *pulsating* or *rotating*, at different aspect ratios ($As=L/a$) and Marangoni numbers (Ma). Linear stability analyses of Neitzel et al.[1], Wanschura et al.[2], and Chen et al.[3] predict that the first instability at large Pr is oscillatory. This feature was also confirmed by nonlinear numerical analyses for high Pr fluids (Rupp et al.[4], Imaishi and Yasuhiro[5], Savino and Monti[6], Yasuhiro et al.[7)8)] and Kuhlmann[9)]). However, there remain open questions related to whether or not the previously reported instability mechanisms do apply, accuracy of the critical Marangoni numbers and how fast and what type of 3-D flow will grow under given conditions. Masud et al.[10] reported that the critical Marangoni number increases with the size of the liquid bridge. This is inconsistent with the linear stability analysis that predicts a critical Marangoni number independent of the bridge size. This issue ought to be discussed based on the growth rates of infinitesimal 3-D disturbances. To date, however, little is known about the growth rate of disturbances under supercritical conditions[11]. The aim of the present paper is to perform a series of numerical simulations on the time-evolution of 3-D oscillatory flows in adiabatic half-zones of $Pr=1.02$ with different aspect ratios ($As=0.75$, 1.0, 1.33, and 1.6) at various Ma. These numerical results clarify the general properties of the oscillatory Marangoni flows, such as the growth rate of the disturbances at supercritical conditions and the size-dependent 'apparent critical Marangoni number'.

2. MATHEMATICAL MODEL

A half-zone liquid bridge with a non-deformable cylindrical surface is sustained between two differentially heated discs in microgravity conditions as shown in **Fig. 1** The temperature differ-

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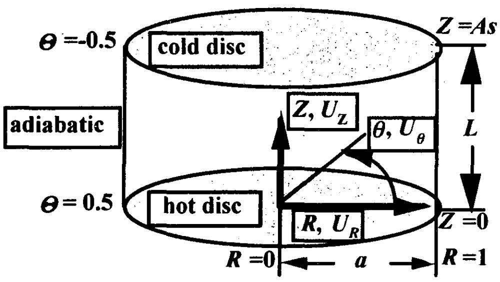

Fig. 1　System coordinate and model.

ence is ΔT. The thermophysical properties of the liquid are assumed constant except for the temperature-dependency of surface tension ($\sigma_T = \partial\sigma/\partial T$). Initially ($\tau<0$), the liquid is at rest and $T=T_c$. At $\tau=0$, the lower disc temperature is instantaneously raised and kept constant at $T_h=T_c+\Delta T$, while the upper colder disc temperature is maintained at T_c. The fundamental equations, boundary and initial conditions are given in non-dimensional form as follows.

Continuity equation : $\nabla\cdot\boldsymbol{U}=0$　(1)

Momentum equation : $\partial\boldsymbol{U}/\partial\tau+(\boldsymbol{U}\cdot\nabla)\boldsymbol{U}=-Pr\nabla P+Pr\nabla^2\boldsymbol{U}$　(2)

Energy equation : $\partial\Theta/\partial\tau+(\boldsymbol{U}\cdot\nabla)\Theta=\nabla^2\Theta$　(3)

Initial conditions : $\boldsymbol{U}=0,\ \Theta=-0.5$　(4)

Boundary conditions : $\boldsymbol{U}_{(R,0,\theta)}=\boldsymbol{U}_{(R,As,\theta)}=0,\ \Theta=_{(R,0,\theta)}=+0.5,\ \Theta_{(R,As,\theta)}=-0.5(\tau>0)$　(5)

at R = 1　$\partial\Theta/\partial R=0,\ \partial U_z/\partial R=-Ma\partial\Theta/\partial Z$　(6)

$R^2\partial(U_\theta/R)/\partial R=-Ma\partial\Theta/\partial\theta,\ U_R=0$　(7)

The non-dimensional variables are defined as $\{R, Z\}=\{r/a, z/a\}$, $P=pa^2/(\alpha\mu)$, $\boldsymbol{U}=\boldsymbol{u}a/\alpha$, $\Theta=(T-T_m)/\Delta T$, $\tau=t\alpha/a^2$; where $T_m=(T_h+T_c)/2$, $\alpha=\lambda/c_p\rho$, $\boldsymbol{u}$: velocity, p : pressure, c_p : heat capacity, ρ : density, λ : thermal conductivity, μ : viscosity, and ν : kinematic viscosity. The dimensionless parameters arising are the Prandtl and the Marangoni numbers defined as $Pr=\nu/\alpha$, and $Ma=-\sigma_T\Delta Ta/\mu\alpha$ respectively.

2. NUMERICAL METHOD

Using cylindrical coordinates, these equations are discretized by a finite difference method with a modified central difference treatment for the convective term [11] and non-uniform staggered grids. The radial velocities on the central axis were calculated by means of the method of Ozoe *et al.*[12]. The HSMAC scheme was used to proceed time evolution of velocity and pressure. The calculations were run on an MPU of Fujitsu VPP700 at the Computer Center of Kyushu University and also on Engineering Work Stations. For numerical calculations, non-uniform grid is adopted. The grid resolution in (r, z, θ) directions is (45, 40, 65) for $As=0.75$, (30,32,49) for $As=1.0$, (26,34,49) for $As=1.33$, and (25,34,49) for $As=1.60$, respectively. Time step $\Delta\tau$ was chosen between 5×10^{-7} and 2×10^{-6}. A two dimensional simulation code with the same scheme and 2-D grids was run in order to obtain a 2-D solution under the same conditions. The thermophysical properties of molten KCl are adopted here as $\alpha=7.2\times10^{-7}$ [m²/s], $\lambda=0.99$ [W/(m·K)], $\mu=1.13$ [mPa·s], $\nu=7.41\times10^{-7}$ [m²/s], $\sigma_T=-7.1\times10^{-5}$ [N/(m·K)] and $Pr=1.02$. Thus $\Delta\tau=1$ corresponds to 13 seconds for a real system in which a is 3.0mm.

3. RESULTS

3.1 Result for As = 1.60 and Ma = 2220

Fig. 2 shows the time evolution of the axial and the azimuthal velocity and the local temperatures at different points in the liquid bridge, as well as a local and the spatially averaged Nusselt number (Nu) on the end plates. As shown in Figure 2-b, periodic azimuthal motions are created within the initial transient stage. The 3-D disturbances are caused by unavoidable round-off errors in numerical calculations. The plot suggests that a perturbation grows exponentially with time in the form, $X_{(\tau)} = F_{X(R,Z)} \exp((\beta + i\omega)\tau) \sin(m\theta)$, where m is the azimuthal wave number, β the growth rate constant, and $\omega = 2\pi f a^2/\alpha$ a non-dimensional frequency of oscillation. In this case study, a 3-D disturbance with $m = 1$ is self-excited and becomes dominant at the early stage of growth. The disturbance increases its amplitude exponentially with time and a *pulsating*, $m = 1$ oscillatory flow with constant oscillation amplitude is established. As time passes, however, the pulsating oscillation is taken over by a rotating oscillation. In this mode, a steadily rotating 3-D structure of temperature and velocity fields is established as shown in **Fig. 3**. This 3-D structure keeps rotating with a constant angular velocity ω/m in counterclockwise direction.

The trajectories of infinitesimal tracer particles over 4 rotation periods (4 periods of local temperature oscillation) are shown in **Fig. 4**. Despite of the obvious counter-clockwise rotation of the 3D structure of the temperature and flow fields, tracer particles fed near the surface show a very long distance azimuthal migration in clock-wise direction. Compared with the previous results for *rotating* $m = 3$ oscillatory flow at $As = 1.0$[7] and *rotating* $m = 1$ oscillatory flow at $As = 1.33$[8], the motion of fluid

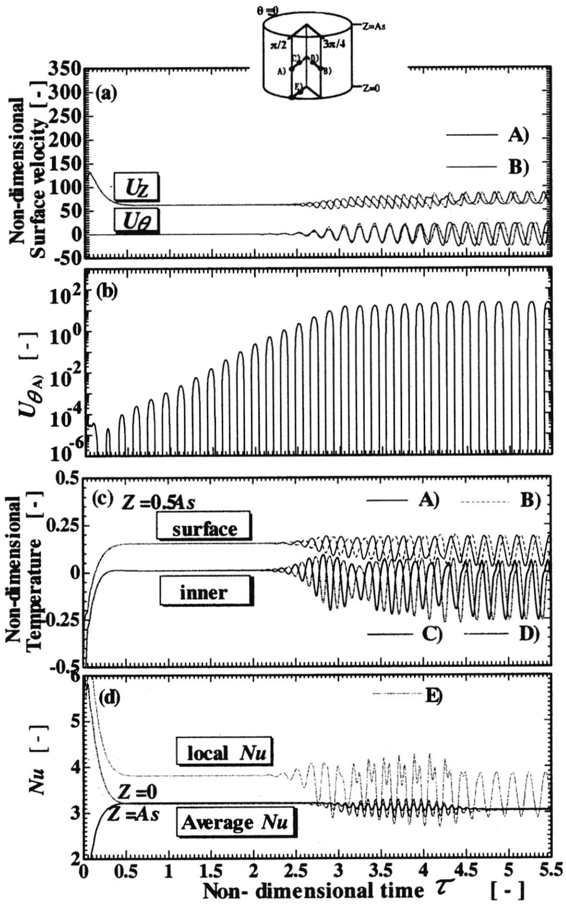

Fig. 2 Simulated results of a 3-D oscillatory Marangoni flow in a half-zone of $Pr = 1.02$ for $As = 1.60$ and $Ma = 2220$.

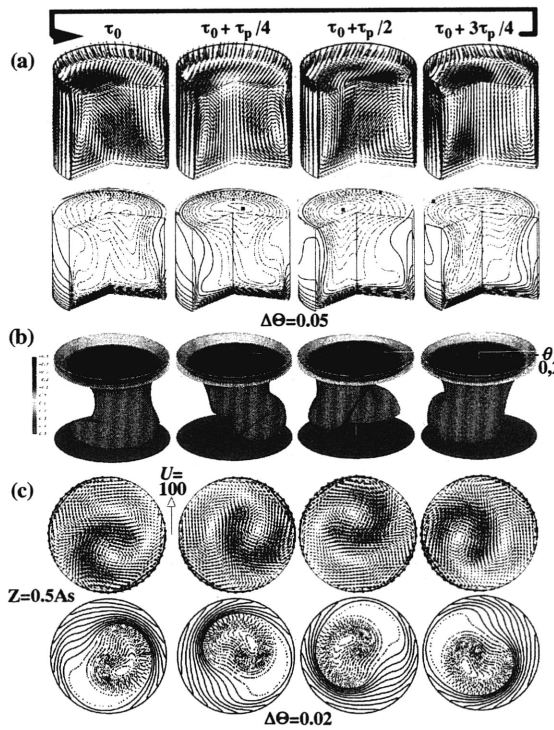

Fig. 3 Snap-shots of a rotating $m = 1$ oscillation: As = 1.60 and $Ma = 2220$.

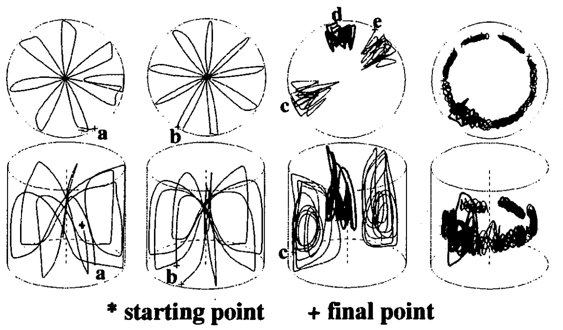

Fig. 4 Trajectories of tracer particles during 4 periods of local temperature oscillation.

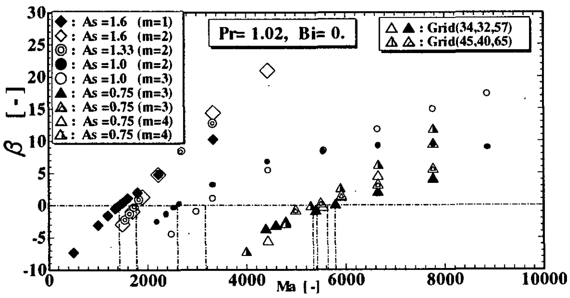

Fig. 5 Growth rate constant β as function of Ma.

elements in this slightly longer half-zone seems less localized. This is caused by the presence of radial velocity on the axis for $m = 1$. 3-D disturbances with $m > 1$ appear not to exhibit such a flow across the axis.

In order to determine the growth rate constant, the Marangoni number was changed at several times in a stepwise manner. The result gives the growth rate constant β as a function of the Marangoni number as shown in **Fig. 5**. From the figure, the critical Marangoni number Ma_c is determined as the Marangoni number at which the growth rate becomes zero; in this case ($As = 1.6$, $m = 1$) $Ma_c = 1430$.

3.2 Results for other aspect ratios

A series of simulations were conducted for other aspect ratios, i.e. $As = 0.75$, 1.0[5)7)], and 1.33[8)]. The critical azimuthal wave numbers (m), the critical Marangoni numbers obtained here Ma_c, the critical Marangoni numbers by linear stability analysis [8)], Ma_{cL}, and the type of oscillations (*pulsating* or *rotating*) are summarized in **Table 1**. The values indicated in parentheses are those obtained by using slightly coarser grid for $As = 0.75$, i.e., (34, 32, 57) grid points in r, z and θ direction, respectively.

The determined critical Marangoni numbers Ma_c are consistent with those of the linear stability analysis [8)], except for $As = 0.75$. The results suggest a mode selection rule, *i.e.* the perturbation which has the largest growth rate constant under a given condition becomes dominant when the Marangoni number is slightly above the critical value.

4. APPARENT CRITICAL MARANGONI NUMBER

Slightly above the threshold, the growth rate constants in **Fig. 5** can be approximated as

$$\beta = B\left\{Ma/Ma_c - 1\right\}^n, \quad Ma > Ma_c \tag{8}$$

The constants B and n were calculated by least-squares. Typically, $B = 10.7$ was obtained for $n = 0.89$. But the parameters depend on As and m. In particular, n may range

Table 1 Critical Marangoni Number and Frequency.

As	m	Present results		Linear stability Theory		Type of oscillation
		Ma_c	ω_c	Ma_{cL}	ω_{cL}	
0.75	3	5424 (5792)	144.2 (149.2)	4944	129.2	pulsating
	4	5350 (5693)	161.3 (165.7)	5775	166.7	pulsating
1.00	2	2615	65.7	2532	62.1	pulsating
	3	3175	82.8	—	—	pulsating-rotating
1.33	2	1722	42.0	1752	41.1	pulsating-rotating
1.60	1	1430	28.3	1413	27.4	pulsating-rotating
	2	1781	36.5	1734	35.5	pulsating

from 0.7 to 1.0. In the previous paper[8], we adopted $B = 20$ for the maximum value of $n = 1.0$. In this paper, $B = 10.7$ was determined for the average of $n = 0.89$.

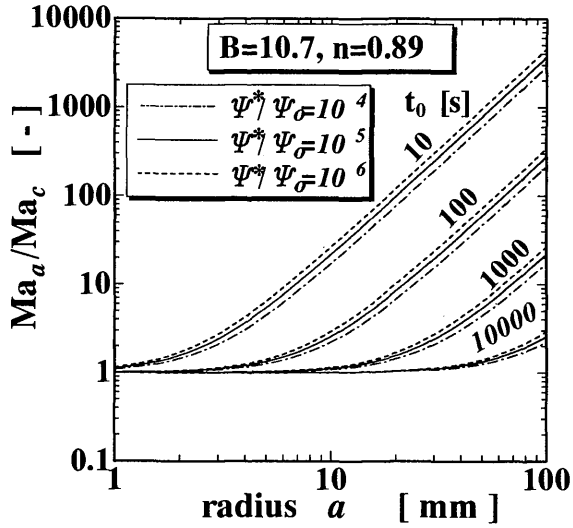

Fig. 6 Apparent critical Marangoni number: effect of t_0 and liquid bridge radius a.

Eq. 8 provides a following discussion on the experimental observability of the incipience of the 3-D oscillatory Marangoni flow in half-zone liquid bridges within a given observation time, t_0. When the Marangoni number is increased beyond its critical value, a disturbance of an initial amplitude Ψ_0 starts its growth exponentially by $\Psi_0 e^{\beta\tau}$ with constant β, and the experimental instrument will detect the perturbation only if a certain threshold amplitude Ψ^* is exceeded. Hence, the apparent critical conditions correspond to

$$Ma_a = Ma_c\left[1+\left(\frac{\ln(\Psi^*/\Psi_0)}{\alpha B t_0}a^2\right)^{1/n}\right] \tag{9}$$

where Ma_a is the apparent critical Marangoni number. Therefore the deviation of the apparent critical Marangoni number from the true critical Marangoni number will be scaled with the square of the radius of the liquid bridge, provided the observation period t_0 is kept constant. Let us assume that Eq.8 holds over a wide range of Ma/Ma_c with $B = 10.7$, $n = 0.89$ and for the fluid KCl. Eq.9 then predicts a size dependence of the apparent critical Marangoni number (Ma_a) as shown in **Fig. 6** for different values of t_0. The parameter Ψ^*/Ψ_0 appears not to give significant effect on Ma_a/Ma_c regardless the liquid bridge size. It should be noted that an extremely long observation time is required to detect the perturbations near the true critical Marangoni number using large liquid bridges. **Fig. 6** explains qualitatively the size-dependent critical Marangoni numbers of Masud et al.[10].

5. CONCLUSION

Three-dimensional simulations over wide ranges of Marangoni number and the aspect ratio ($As = 0.75 - 1.60$) revealed the multi-morphological features of the 3-D oscillatory Marangoni flow in half-zone liquid bridges of $Pr = 1.02$ fluid. The critical Marangoni numbers are consistent with the linear stability analysis, except for $As = 0.75$. The growth-rate constants were correlated as a function of the Marangoni number. And the correlation was used to explain the size-dependent apparent critical Marangoni numbers.

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