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Kagei, Yoshiyuki
Faculty of Mathematics, Kyushu University

Okita, Masatoshi
National Institute of Technology, Kurume College

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Yoshiyuki Kagei
& Masatoshi Okita

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Institute of Mathematics for Industry
Graduate School of Mathematics
Kyushu University
Fukuoka, JAPAN

Asymptotic profiles for the compressible Navier-Stokes equations on the whole space

Yoshiyuki Kagei[1] and Masatoshi Okita[2]

[1] Faculty of Mathematics,
Kyushu University,
Nishi-ku, Motooka 744,
Fukuoka 819-0395, Japan

[2] National Institute of Technology,
Kurume Colloge,
Kurume-shi, Komorino 1-1-1,
Fukuoka 830-8555, Japan

Abstract: This paper is concerned with large time behavior of the strong solutions of the compressible Navier-Stokes equation in the whole space around the motionless state. It was shown by Kawashima-Matsumura-Nishida (1979) and Hoff-Zumbrun (1995) that the perturbation of the motionless state is time-asymptotic to the solution of the linearized problem. In this paper we will give the second-order asymptotics of strong solutions.

Key Words: compressible Navier-Stokes equations; asymptotic profiles ; convergence rate.

2010 Mathematics Subject Classification Numbers. 35Q30, 76N15.

1 Introduction

This paper studies the initial value problem for the compressible Navier-Stokes equation in $\mathbb{R}^n$, $n \geq 3$:

$$\begin{cases} \partial_t \rho + \operatorname{div} m = 0, \\ \partial_t m + \operatorname{div}\left(\frac{m \otimes m}{\rho}\right) + \nabla P(\rho) = \mu \Delta\left(\frac{m}{\rho}\right) + (\mu + \mu')\nabla \operatorname{div}\left(\frac{m}{\rho}\right), \\ (\rho, m)(0, x) = (\rho_0, m_0)(x). \end{cases} \tag{1}$$

Here $t > 0$, $x = {}^T(x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$, and the superscript ${}^T\cdot$ stands for the transposition; the unknown functions $\rho = \rho(t,x) > 0$ and $m = m(t,x) = {}^T(m_1(t,x), m_2(t,x), \cdots, m_n(t,x))$ denote the density and momentum, respectively; $P = P(\rho)$ is the pressure that is assumed to be a function of the density ρ; μ and μ' are the viscosity coefficients satisfying the conditions $\mu > 0$ and $\frac{2}{n}\mu + \mu' \geq 0$; and div , ∇ and Δ denote the usual divergence, gradient and Laplacian with respect to x, respectively. The notation $\operatorname{div}\left(\frac{m \otimes m}{\rho}\right)$ means that its j-th component is given by $\operatorname{div}\left(\frac{m_j m}{\rho}\right)$.

We assume that $P(\rho)$ is smooth in a neighborhood of $\bar{\rho}$ with $P'(\bar{\rho}) > 0$, where $\bar{\rho}$ is a given positive constant.

In this paper we investigate asymptotic properties of strong solutions of problem (1) around the constant stationary solution $(\bar{\rho}, 0)$.

Matsumura and Nishida [6] showed the global in time existence of the solution of (1) for $n = 3$, provided that the initial perturbation $u_0 = {}^T(\gamma(\rho_0 - \bar{\rho}), {}^T m_0)$, with $\gamma = \sqrt{P'(\bar{\rho})}$, is sufficiently small in $H^3(\mathbb{R}^3) \cap L^1(\mathbb{R}^3)$. Furthermore, the following decay estimates were obtained in [6]

$$\|\nabla^k u(t)\|_{L^2} \leq C(1+t)^{-\frac{3}{4}-\frac{k}{2}}, \quad k = 0, 1, \tag{2}$$

where $u(t) = {}^T(\gamma(\rho - \bar\rho), {}^T m)$. (See also [7].) Kawashima, Matsumura and Nishida [4] proved that the solution is time asymptotic to the one of the linearized problem. It was shown in [4] that

$$\|u(t) - G(t) * u_0\|_{L^2} \le CL(t)(1+t)^{-\frac{3}{4}-\frac{1}{2}},$$

if u_0 is sufficiently small in $H^3(\mathbb{R}^3) \cap L^1(\mathbb{R}^n)$ for $n = 3$. Here $G(t) * u_0 = G(t, \cdot) * u_0$, where $G(t, x)$ is Green's matrix for the linearized system at ${}^T(\bar\rho, 0)$ for (1) and $*$ denotes the convolution with respect to x. Hoff and Zumbrun [3] derived a more detailed description of the large-time behavior of $u(t)$ in L^p for all $1 \le p \le \infty$; the following estimates were established:

$$\|u(t)\|_{L^p} \le C \begin{cases} (1+t)^{-\frac{n}{2}(1-\frac{1}{p})}, & 2 \le p \le \infty, \\ (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n-1}{4}(1-\frac{2}{p})} L(t), & 1 \le p < 2, \end{cases}$$

and

$$\|u(t) - G(t) * u_0\|_{L^p} \le CL(t) \begin{cases} (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{1}{2}}, & 2 \le p \le \infty, \\ (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n}{4}(1-\frac{2}{p})-\frac{1}{2}+\eta}, & 1 \le p < 2, \end{cases}$$

where η is any positive number. Moreover, it was proved in [3] that $u(t)$ is time-asymptotic to the solution $\tilde u(t) = {}^T(\tilde\sigma, {}^T\tilde m)$ of the following linear effective artificial viscosity system:

$$\begin{cases} \partial_t \tilde\sigma + \gamma \mathrm{div}\, \tilde m - \frac{1}{2}(\mu_2 + \mu_1)\Delta\tilde\sigma = 0, \\ \partial_t \tilde m - \mu_1 \Delta \tilde m - \frac{1}{2}(\mu_2 - \mu_1)\nabla \mathrm{div}\, \tilde m + \gamma \nabla \tilde\sigma = 0, \end{cases} \tag{3}$$

where $\mu_1 = \frac{\mu}{\bar\rho}$ and $\mu_2 = \frac{\mu + \mu'}{\bar\rho}$. More precisely,

$$\begin{aligned} & \|u(t) - \tilde G(t) * u_0\|_{L^p},\ \|u(t) - \tilde G(t, \cdot) \int u_0 dx\|_{L^p} \\ \le\ & CL(t) \begin{cases} (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{1}{2}}, & 2 \le p \le \infty, \\ (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n}{4}(1-\frac{2}{p})-\frac{1}{2}+\eta}, & 1 \le p < 2, \end{cases} \end{aligned}$$

for any positive constant η, where $\tilde G$ is Green's matrix for (3). Here $L(t) = \log(1+t)$ when $n = 2$, and is otherwise identically one. We also mention that Kobayashi and Shibata [5] proved the following estimates

$$\|\partial_t^j \partial_x^\alpha G_1(t, x)\|_{L^p(\mathbb{R}^n_x)} \le C(1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{j+|\alpha|}{2}}$$

for $2 \le p \le \infty$, and

$$\|\partial_t^j \partial_x^\alpha G_1(t, x)\|_{L^p(\mathbb{R}^n_x)} \le C \begin{cases} (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n-1}{4}(1-\frac{2}{p})-\frac{j+|\alpha|}{2}}, & n \ge 3 \text{ and } n\text{: odd}, \\ (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n}{4}(1-\frac{2}{p})-\frac{j+|\alpha|}{2}}, & n \ge 2 \text{ and } n\text{: even}, \end{cases}$$

for $1 \le p \le \infty$, any positive integer j and any multi-index α. Here G_1 is a low frequency part of G, i.e., $G_1 = \Phi * G$ with $\mathfrak{F}[\Phi] \in C^\infty$, $\mathrm{Supp}\, \mathfrak{F}[\Phi] \subset \{\xi \in \mathbb{R}^n | |\xi| \le \frac{A}{\sqrt{2}}\}$ and $\mathfrak{F}[\Phi](\xi) = 1$ for $|\xi| \le \frac{A}{2}$. Here, $\mathfrak{F}[\Phi]$ denotes the Fourier transform of Φ, $A = \frac{2\gamma}{\mu_1 + \mu_2}$ with $\mu_1 = \frac{\mu}{\bar\rho}$ and $\mu_2 = \frac{\mu + \mu'}{\bar\rho}$.

In this paper we give the second order term in the asymptotic expansion of $u(t)$ as $t \to \infty$. The main result of this paper is stated as follows

Theorem 1.1. *Assume that $n \ge 3$. Then there exists $\epsilon > 0$ such that if*

$$u_0 \in H^{s_0+1} \cap L^1_l \cap L^2_l$$

with $s_0 = [\frac{n}{2}] + 1$ and

$$\|u_0\|_{H^{s_0+1} \cap L^1} \le \epsilon,$$

for $l=\frac{1}{2}$ when $n=3$, and $l=1$ when $n\geq 4$, then

$$\begin{aligned}&\big\|u(t)-G(t)*u_0-\sum_{i=1}^{n}\partial_i G_1(t.\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0 dyds\big\|_{L^2},\\ &\big\|u(t)-G(t)*u_0-\sum_{i=1}^{n}\partial_i \tilde{G}(t.\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0 dyds\big\|_{L^2}\\ \leq\ & C(1+t)^{-\frac{n}{4}-\frac{3}{4}}\begin{cases}\log(1+t), & n=3,\\ (1+t)^{-\frac{1}{4}}\log(1+t), & n=4,\\ (1+t)^{-\frac{1}{4}}, & n\geq 5,\end{cases}\end{aligned}$$

for $t\geq 0$, where

$$\mathcal{F}_i^0=\begin{pmatrix}0\\ F_i^0\end{pmatrix}$$

with $F_i^0=\big(F_{ij}^0\big)_{i,j=1}^n$ being the matrix defined by

$$\begin{aligned}F_{ij}^0 \ =\ & -\delta_{ji}\big\{\frac{1}{2}(\rho-\bar\rho)^2P_{\rho\rho}(\bar\rho)\big\}-\frac{m_jm_i}{\bar\rho}\\ &+\delta_{ji}\big\{\frac{1}{2\gamma^3}\sigma^3\int_0^1(1-\theta)^2\partial_\rho^3P(\frac{1}{\gamma}\sigma\theta+\bar\rho)d\theta\big\}+\frac{\sigma m_jm_i}{\bar\rho(\sigma+\gamma\bar\rho)}.\end{aligned}$$

Remark 1.2. *Let $n\geq 3$. One can also obtain*

$$\big\|u(t)-G(t)*u_0-\sum_{i=1}^{n}\partial_i G_1(t,\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0 dyds\big\|_{L^p}\leq CK(t)(1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{3}{4}}$$

for $2\leq p\leq\infty$, and

$$\begin{aligned}&\big\|u(t)-G(t)*u_0-\sum_{i=1}^{n}\partial_i G_1(t,\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0 dyds\big\|_{L^p}\\ \leq\ & CK(t)\begin{cases}(1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n-1}{4}(1-\frac{2}{p})-\frac{3}{4}}, & n\geq 3 \text{ and } n\text{: odd},\\ (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n}{4}(1-\frac{2}{p})-\frac{3}{4}}, & n\geq 2 \text{ and } n\text{: even}\end{cases}\end{aligned}$$

for $1\leq p<2$, and initial perturbation u_0 sufficiently small in some Sobolev spaces and weighted Lebesgue space. Here $K(t)=\begin{cases}\log(1+t), & n=3,\\ (1+t)^{-\frac{1}{4}}\log(1+t), & n=4,\\ (1+t)^{-\frac{1}{4}}, & n\geq 5.\end{cases}$

Remark 1.3. *Asymptotic expansion of the solution of the linearized equation is given as follows:*

$$\|G(t)*u_0-G_1(t,\cdot)\int u_0dy+\sum_{|\alpha|=1}\partial_x^\alpha G_1(t,\cdot)\int y^\alpha u_0dy\|_{L^2}\leq C(1+t)^{-\frac{n}{4}-1},$$

for $t\geq 0$ and $u_0\in L_2^1\cap L^2$. Hence we have

$$\begin{aligned}&\big\|u(t)-G_1(t,\cdot)\int u_0dy+\sum_{|\alpha|=1}\partial_x^\alpha G_1(t,\cdot)\int y^\alpha u_0dy-\sum_{i=1}^{n}\partial_i G_1(t.\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0 dyds\big\|_{L^2}\\ \leq\ & C(1+t)^{-\frac{n}{4}-\frac{3}{4}}\begin{cases}\log(1+t), & n=3,\\ (1+t)^{-\frac{1}{4}}\log(1+t), & n=4,\\ (1+t)^{-\frac{1}{4}}, & n\geq 5,\end{cases}\end{aligned}$$

for $t\geq 0$ and sufficiently small $u_0\in H^{s_0+1}\cap L_2^1\cap L_l^2$.

This paper is organized as follows. In section 2 we introduce notation and an auxiliary lemma. In section 3 we reformulate the problem; we rewrite the equation into a symmetric one and formulate the problem for a system of low and high frequency parts. Section 4 is devoted to the proof of the main theorem. In section 5 we establish a decay estimate of L^2-moments.

2 Preliminaries

In this section we first introduce the notation which will be used throughout this paper. We then introduce some auxiliary lemmas which will be useful in the proof of the main result.

Let $L^p(1 \le p \le \infty)$ denote the usual L^p-Lebesgue space on $\mathbb{R}^n$. For a nonnegative integer m, we denote by H^m the usual L^2-Sobolev space of order m. For $1 \le p \le \infty$ and $0 \le l$, $L^p\big((1+|x|^l)^p dx\big)$ stands for the weighted L^p spaces over $\mathbb{R}^n$ defined by

$$L^p_l := L^p\big((1+|x|^l)^p dx\big) = \{u : \int_{\mathbb{R}^n} |u(x)|^p (1+|x|^l)^p dx < \infty\}.$$

The inner-product of L^2 is denoted by $(\cdot,\cdot)$.

For any integer $k \ge 0, \nabla^k f$ denotes all of k-th derivatives of f.

For a function f, we denote its Fourier transform by $\mathfrak{F}[f] = \hat{f}$:

$$\mathfrak{F}[f](\xi) = \hat{f}(\xi) = \int_{\mathbb{R}^n} f(x) e^{-ix\cdot\xi} dx \quad (\xi \in \mathbb{R}).$$

The inverse Fourier transform is denoted by $\mathfrak{F}^{-1}[f] = \check{f}$,

$$\mathfrak{F}^{-1}[f](x) = \check{f}(x) = (2\pi)^{-n} \int_{\mathbb{R}^n} f(\xi) e^{i\xi\cdot x} d\xi \quad (x \in \mathbb{R}).$$

We close this section with the Gagliardo-Nirenberg-Sobolev inquality.

Lemma 2.1. *Assume that $u(x) \in L^q(\mathbb{R}^n)$, $\nabla^m u(x) \in L^r(\mathbb{R}^n)$ with $1 \le q, r \le \infty$. Then for each integer $j \in [0, m]$, we have*

$$\|\nabla^j u\|_{L^p} \le C \|\nabla^m u\|^{\alpha}_{L^r} \|u\|^{1-\alpha}_{L^q}.$$

Here

$$\frac{1}{p} = \frac{j}{n} + \alpha\big(\frac{1}{r} - \frac{m}{n}\big) + (1-\alpha)\frac{1}{q}, \qquad \frac{j}{m} \le \alpha \le 1.$$

See, e.g., [2], for the proof.

3 Reformulation of the problem

In this section we first rewrite system (1) into the one for the perturbation. We next state the existence of strong solutions. We then introduce some auxiliary lemmas which will be useful in the proof of the main result.

Let us rewrite the problem (1) in a symmetric form. We define μ_1, μ_2 and γ by

$$\mu_1 = \frac{\mu}{\bar{\rho}},\ \mu_2 = \frac{\mu + \mu'}{\bar{\rho}},\ \gamma = \sqrt{P'(\bar{\rho})}.$$

We introduce the new unknown functions

$$\sigma(t,x) = \gamma\big(\rho(t,x) - \bar{\rho}\big), \quad m(t,x) = m(t,x).$$

The initial value problem (1) is then reformulated as

$$\begin{cases} \partial_t \sigma + \gamma \mathrm{div}\, m = 0, \\ \partial_t m - \mu_1 \Delta m - \mu_2 \nabla \mathrm{div}\, m + \gamma \nabla \sigma = \mathrm{div}\, F(u, \partial_x u), \\ (\sigma, m)(0,x) = (\sigma_0, m_0)(x), \end{cases} \tag{4}$$

where, $u = \begin{pmatrix} \sigma \\ m \end{pmatrix}$, and $F(u, \partial_x u)$ is the $n \times n$ matrix $(F_{jk}(u, \partial_x u))$ defined by

$$\begin{aligned} F_{jk}(u, \partial_x u) \;&=\; -\delta_{jk} \frac{1}{2\gamma^2} \sigma^2 P_{\rho\rho}(\bar{\rho}) - \frac{m_j m_k}{\bar{\rho}} \\ &\quad + \delta_{jk} \big\{ \frac{1}{2\gamma^3} \sigma^3 \int_0^1 (1-\theta)^2 \partial_\rho^3 P(\frac{1}{\gamma}\sigma\theta + \bar{\rho}) d\theta \big\} + \frac{\sigma m_j m_k}{\bar{\rho}(\sigma + \gamma\bar{\rho})} \\ &\quad - \delta_{jk} \mu_2 \mathrm{div}\, \big(\frac{\sigma m}{\sigma + \gamma\bar{\rho}}\big) - \mu_1 \partial_{x_k} \big(\frac{\sigma m_j}{\sigma + \gamma\bar{\rho}}\big), \end{aligned}$$

and the j-th component of $\operatorname{div} F$ is given by $\sum_{k=1}^n \partial_{x_k} F_{jk}$.

The global existence of strong solutions was proved by Matsumura and Nishida [6] by a combination of the spectral analysis and the energy method for the density-velocity formulation for (4). We here restate the global existence in terms of the density and momentum.

Proposition 3.1 (Matsumura-Nishida [6]). *Let $n \geq 2$ and let $u_0 \in H^{s_0+1}$. There exist a positive constant ϵ_1 such that if*

$$\|u_0\|_{H^{s_0+1}} \leq \epsilon_1,$$

then problem (4) has a unique global solution $u \in C([0,\infty); H^{s_0+1})$.

Proposition 3.1 were proved for the case $n = 3$ in [6]. In a similar manner one can see that Proposition 3.1 holds for $n \geq 2$.

We set

$$A = \begin{pmatrix} 0 & -\gamma \nabla \cdot \\ -\gamma \nabla & \mu_1 \Delta + \mu_2 \nabla \nabla \cdot \end{pmatrix}.$$

By using operator A, problem (4) is written as

$$\partial_t u - Au = \operatorname{div} \mathcal{F}, \quad u|_{t=0} = u_0, \tag{5}$$

where

$$\mathcal{F} = \mathcal{F}(u, \partial_x u) = \begin{pmatrix} 0 \\ F(u, \partial_x u) \end{pmatrix}, \quad u_0 = \begin{pmatrix} \sigma_0 \\ m_0 \end{pmatrix}.$$

We introduce a semigroup generated by A. We set

$$E(t)u := \mathfrak{F}^{-1}[e^{\hat{A}(\xi)t}\hat{u}] = G(t) * u \quad \text{for } u \in L^2,$$

where

$$\hat{A}(\xi) = \begin{pmatrix} 0 & -i\gamma^T \xi \\ -i\gamma\xi & -\mu_1|\xi|^2 I_n - \mu_2 \xi^T \xi \end{pmatrix} \quad \text{and} \quad G(t,x) = \mathfrak{F}^{-1}[e^{\hat{A}(\xi)t}](x).$$

Lemma 3.2. *(i) The set of all eigenvalues of $\hat{A}(\xi)$ consists of $\lambda_i(\xi)$ $(i = 1,2,3)$, where*

$$\begin{cases} \lambda_1(\xi) = -\frac{1}{2}(\mu_1+\mu_2)|\xi|^2 + i\gamma|\xi|\sqrt{1 - \frac{(\mu_1+\mu_2)^2}{4\gamma^2}|\xi|^2}, \\ \lambda_2(\xi) = -\frac{1}{2}(\mu_1+\mu_2)^2|\xi|^2 - i\gamma|\xi|\sqrt{1 - \frac{(\mu_1+\mu_2)}{4\gamma^2}|\xi|^2}, \\ \lambda_3(\xi) = -\mu_1|\xi|^2, \end{cases}$$

for all $\xi \in \mathbb{R}^n$.

(ii) $e^{t\hat{A}(\xi)}$ has the spectral resolution

$$e^{t\hat{A}(\xi)} = \sum_{j=1}^3 e^{t\lambda_j(\xi)} P_j(\xi)$$

for all $|\xi| \neq \frac{2\gamma}{\sqrt{\mu_1+\mu_2}}$, where $P_j(\xi)$ is the eigenprojection for $\lambda_j(\xi)$.

For $|\xi| = \frac{2\gamma}{\sqrt{\mu_1+\mu_2}}$, we have $\lambda_1(\xi) = \lambda_2(\xi) = -\frac{\mu_1+\mu_2}{2}|\xi|^2$ and

$$e^{t\hat{A}(\xi)} = e^{t\lambda_1(\xi)}\big(I + t(\hat{A}(\xi) - \lambda_1 I)\big)P_1(\xi) + e^{t\lambda_3(\xi)}P_3(\xi).$$

We see from Lemma 3.2 that

$$\lambda_1(\xi) \sim -\frac{1}{2}(\mu_1+\mu_2)|\xi|^2 + i\gamma|\xi|, \quad \lambda_2(\xi) \sim -\frac{1}{2}(\mu_1+\mu_2)|\xi|^2 - i\gamma|\xi| \qquad (|\xi| \to 0)$$

and

$$\lambda_1(\xi) \sim -(\mu_1+\mu_2)|\xi|^2, \quad \lambda_2(\xi) \sim -\frac{\gamma^2}{\mu_1+\mu_2} \qquad (|\xi| \to \infty),$$

and hence, $G(t) * u$ has different characters in its low and high frequency parts.

We next decompose the solution u of (5) into its low and high frequency parts. Let $\hat{\Phi}$ be a function in $C^\infty(\mathbb{R}^n)$ such that $\hat{\Phi}(\xi) = \begin{cases} 1 & |\xi| \le \frac{A}{2}, \\ 0 & |\xi| \ge \frac{A}{\sqrt{2}}, \end{cases}$ where $A = \frac{2\gamma}{\sqrt{\mu_1+\mu_2}}$. We set

$$G_1(t,x) = \begin{pmatrix} L_{11}(t,x) & L_{12}(t,x) \\ L_{21}(t,x) & L_{22}(t,x) \end{pmatrix},$$

where

$$\begin{aligned}
L_{11}(t,x) &= \mathfrak{F}^{-1}\left[\frac{\lambda_1(\xi)e^{\lambda_2(\xi)t} - \lambda_2(\xi)e^{\lambda_1(\xi)t}}{\lambda_1(\xi)-\lambda_2(\xi)}\hat{\Phi}(\xi)\right](x), \\
L_{12}(t,x) &= -i\gamma\mathfrak{F}^{-1}\left[{}^T\xi\frac{e^{\lambda_1(\xi)t} - e^{\lambda_2(\xi)t}}{\lambda_1(\xi)-\lambda_2(\xi)}\hat{\Phi}(\xi)\right](x), \\
L_{21}(t,x) &= {}^T L_{12}(t,x), \\
L_{22}(t,x) &= K_1(t,x) + K_2(t,x) - K_3(t,x), \\
K_1(t,x) &= \mathfrak{F}^{-1}\left[e^{\lambda_3(\xi)t}\hat{\Phi}(\xi)\right](x)I, \quad I \text{ is unit matrix}, \\
K_2(t,x) &= \mathfrak{F}^{-1}\left[\frac{\lambda_1(\xi)e^{\lambda_1(\xi)t} - \lambda_2(\xi)e^{\lambda_2(\xi)t}}{\lambda_1(\xi)-\lambda_2(\xi)}\frac{\xi_j\xi_k}{|\xi|^2}\hat{\Phi}(\xi)\right](x), \\
K_3(t,x) &= \mathfrak{F}^{-1}\left[e^{\lambda_3(\xi)t}\frac{\xi_j\xi_k}{|\xi|^2}\hat{\Phi}(\xi)\right](x).
\end{aligned}$$

We define the operators S_1 and $\tilde{S}_\infty$ on L^2 by

$$S_1 u := \mathfrak{F}^{-1}\left[\hat{\Phi}\hat{u}\right] \quad \tilde{S}_\infty u := \mathfrak{F}^{-1}\left[(1-\hat{\Phi})\hat{u}\right], \quad u \in L^2.$$

In terms of S_1 and $\tilde{S}_\infty$, we decompose the solution $u(t)$ of (5) as

$$u(t) = u_1(t) + u_\infty(t), \quad u_1(t) = S_1 u(t),\ u_\infty(t) = \tilde{S}_\infty u(t).$$

It then follows that $u_1(t)$ and $u_\infty(t)$ are governed by systems (6) and (7) − (8) given in the following proposition.

Proposition 3.3. *Let $u = {}^T(\sigma, {}^T m)$ be a solution of problem* (5) *on $[0,\infty) \times \mathbb{R}^n$. Then, $u_1(t)$ and $u_\infty(t)$ satisfy*

$$u_1(t) = G_1(t) * u_0 + \int_0^t G_1(t-s) * \operatorname{div} \mathcal{F}(u, \partial_x u)(s)ds \tag{6}$$

and

$$\partial_t u_\infty - A u_\infty = \tilde{S}_\infty \operatorname{div} \mathcal{F}(u, \partial_x u), \tag{7}$$

$$u_\infty|_{t=0} = u_{0\infty}, \tag{8}$$

where $u_{0\infty} = \tilde{S}_\infty u_0$.

See, e.g., [8], for the proof.

4 Proof of main result

In this section we prove Theorem 1.1. We have the following decay estimate for high frequency part.

Proposition 4.1 ([8]). *There exists $\epsilon > 0$ such that if*

$$u_0 \in H^{s_0} \cap L^1$$

and

$$\|u_0\|_{H^{s_0}\cap L^1} \leq \epsilon,$$

then we have

$$\|\nabla^k u(t)\|_{L^2} \leq C\epsilon(1+t)^{-\frac{n}{4}-\frac{k}{2}}$$

for $k = 1, 2, \cdots, s_0$, and

$$\|u_\infty(t)\|_{L^2} \leq C\epsilon(1+t)^{-\frac{n}{4}-1}.$$

In view of Proposition 4.1 we expect that the large-time behavior of solution is described by the behavior of the low frequency part. Hence, we investigate asymptotic properties of the low frequency part of solution in the form of the integral equation (6). We first consider the linear term $G_1(t) * u_0$ in (6). We have the following estimates for $G_1(t) * u_0$.

Lemma 4.2 (Kobayashi-Shibata [5]). *Let $n \geq 2$ and let j and α be any positive integer and any multi-index, respectively. Then, for any $t \geq 0$, we have*

$$\|\partial_t^j \partial_x^\alpha G_1(t,x)\|_{L^p(\mathbb{R}^n_x)} \leq C(1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{j+|\alpha|}{2}}$$

for $2 \leq p \leq \infty$, and

$$\|\partial_t^j \partial_x^\alpha G_1(t,x)\|_{L^p(\mathbb{R}^n_x)} \leq C \begin{cases} (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n-1}{4}(1-\frac{2}{p})-\frac{j+|\alpha|}{2}}, & n \geq 3 \text{ and } n\text{: odd,} \\ (1+t)^{-\frac{n}{2}(1-\frac{1}{p})-\frac{n}{4}(1-\frac{2}{p})-\frac{j+|\alpha|}{2}}, & n \geq 2 \text{ and } n\text{: even,} \end{cases}$$

for $1 \leq p < 2$.

On the other hand, the high frequency part of $G(t) * u_0$ decays exponentially. In fact, we have the following estimate.

Proposition 4.3. *There exists constants $C, c_0 > 0$ independent of t such that*

$$\|G_1(t) * u_0 - G(t) * u_0\|_{L^2} \leq Ce^{-c_0 t}\|u_0\|_{L^2}.$$

Proof. By Plancherel's theorem, we have

$$\begin{aligned} \|G_1(t) * u_0 - G(t) * u_0\|_{L^2} &\leq C\Big(\int_{|\xi|\geq\frac{A}{2}} \big|e^{\hat{A}(\xi)t}\hat{u_0}\big|^2 d\xi\Big)^{\frac{1}{2}} \\ &\leq Ce^{-c_0 t}\Big(\int_{\mathbb{R}^n} |\hat{u_0}(\xi)|^2 d\xi\Big)^{\frac{1}{2}} \\ &\leq Ce^{-c_0 t}\|u_0\|_{L^2}. \end{aligned}$$

□

The following estimates show that $\tilde{G}(t) * u_0$ is time-asymptotic to $G_1(t) * u_0$.

Proposition 4.4 (Hoff-Zumbrun [3]). *Let $\tilde{G}$ be Green's matrix for the linear artificial viscosity system* (3)*. Then for each nonnegative integer k, $u_0 \in L^1 \cap H^k$, and $|\alpha| \leq k$, there is a positive constant C such that*

$$\|\partial_x^\alpha \tilde{G}(t) * u_0\|_{L^2} \leq C(1+t)^{\frac{n}{4}-\frac{|\alpha|}{2}}\|u_0\|_{L^1\cap H^k},$$

$$\|\partial_x^\alpha [G_1(t) * u_0 - \tilde{G}(t) * u_0]\|_{L^2} \leq C(1+t)^{-\frac{n}{4}-\frac{|\alpha|}{2}-\frac{1}{2}}\|u_0\|_{L^1\cap H^k}.$$

The linear term $G_1(t) * u_0$ has the following expansion.

Proposition 4.5. *Suppose $u_0 \in L^1_2$. Then we obtain*

$$\|G_1(t) * u_0 - G_1(t,\cdot)\int u_0(y)dy + \sum_{|\alpha|=1} \partial_x^\alpha G_1(t,\cdot)\int y^\alpha u_0 dy\|_{L^2} \le C(1+t)^{-\frac{n}{4}-1}$$

Proof. Applying Taylor's formula we have

$$\begin{aligned} G_1(t) * u_0 &= \int G_1(t, x-y)u_0(y)dy \\ &= G_1(t,x)\int u_0(y)dy - \sum_{|\alpha|=1} \partial_x^\alpha G_1(t,x)\int y^\alpha u_0(y)dy \\ &\quad + \sum_{|\alpha|=2}\int\int_0^1 (\partial_x^\alpha G)(x-\theta y)y^\alpha u_0(y)d\theta dy. \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} \|G_1(t) * u_0 - G_1(t,\cdot)\int u_0(y)dy + \sum_{|\alpha|=1} \partial_x^\alpha G_1(t,\cdot)\int y^\alpha u_0 dy\|_{L^2} &\le \|\nabla^2 G_1(t)\|_{L^2}\|u_0\|_{L^1_2} \\ &\le C(1+t)^{-\frac{n}{4}-1}\|u_0\|_{L^1_2}. \end{aligned}$$

□

We next give the asymptotic leading term for the nonlinearity of (6).

Theorem 4.6. *Assume that $n \ge 3$. Let $l = \frac{1}{2}$ when $n = 3$ or $l = 1$ when $n \ge 4$. There exists $\epsilon > 0$ such that if*

$$u_0 \in H^{s_0+1} \cap L^1_l \cap L^2_l$$

and

$$\|u_0\|_{H^{s_0+1}\cap L^1} \le \epsilon,$$

then we have

$$\begin{aligned} &\Big\|\int_0^t G_1(t-s) * \operatorname{div} \mathcal{F}(u, \partial_x u)(s)ds - \sum_{i=1}^n \partial_i G_1(t.\cdot)\int_0^\infty \int_{\mathbb{R}^n} \mathcal{F}_i^0 dyds\Big\|_{L^2} \\ \le\ & C(1+t)^{-\frac{n}{4}-\frac{3}{4}} \begin{cases} \log(1+t), & n = 3, \\ (1+t)^{-\frac{1}{4}}\log(1+t), & n = 4, \\ (1+t)^{-\frac{1}{4}}, & n \ge 5 \end{cases} \end{aligned}$$

for $t \ge 0$.

To prove Theorem 4.6 we introduce the following decay estimates of L^2-moments of solutions of problem (4).

Proposition 4.7. *Let $l = \frac{1}{2}$ or $l = 1$ for $n \ge 3$. There exists $\epsilon > 0$ such that if*

$$u_0 \in H^{s_0+1} \cap L^1_l \cap L^2_l$$

and

$$\|u_0\|_{H^{s_0}} \le \epsilon,$$

then the solution $u(t)$ of (4) satisfies

$$\big\||x|^l u(t)\big\|_{L^2} \le C(1+t)^{-\frac{n}{4}+l}.$$

We will give a proof of Proposition 4.7 in section 5.

Proof of Theorem 4.6. Direct calculation gives

$$
\begin{aligned}
&\int_0^t G_1(t-s)*\operatorname{div}\mathcal{F}(u,\partial_x u)(s)ds-\sum_{i=1}^n\partial_i G_1(t.\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0dyds\\
=\;&\int_0^{\frac{t}{2}}\int\sum_{i=1}^n\big(\partial_{x_i}G_1(t-s,x-y)-\partial_{x_i}G_1(t,x)\big)\mathcal{F}_i^0dyds\\
&+\int_0^{\frac{t}{2}}\int\sum_{i=1}^n\partial_{x_i}G_1(t-s,x-y)\mathcal{F}_i^1dyds\\
&+\int_{\frac{t}{2}}^t G_1(t-s)*\operatorname{div}\mathcal{F}(u,\partial_x u)(s)ds+\sum_{i=1}^n\partial_{x_i}G_1(t,\cdot)\int_{\frac{t}{2}}^\infty\int\mathcal{F}_i^0dyds\\
=:\;&J_1+J_2+J_3+J_4,
\end{aligned}
$$

where $\mathcal{F}_i^1=\begin{pmatrix}0\\ F_i^1(u,\partial_x u)\end{pmatrix}$ with $F_i^1(u,\partial_x u)=\big(F_{ij}^1(u,\partial_x u)\big)_j^n$, $F_{ij}^1(u,\partial_x u)=-\delta_{ji}\mu_2\operatorname{div}\big(\frac{\sigma m}{\sigma+\gamma\bar\rho}\big)-\mu_1\partial_{x_i}\big(\frac{\sigma m_j}{\sigma+\gamma\bar\rho}\big)$.

We first estimate the L^2 norms of J_3 and J_4:

$$
\begin{aligned}
\|J_3\|_{L^2}=\Big\|\int_{\frac{t}{2}}^t G_1(t-s)*\operatorname{div}\mathcal{F}ds\Big\|_{L^2}&\le C\int_{\frac{t}{2}}^t\|u\|_{L^\infty}\|\nabla u\|_{H^1}\\
&\le C\int_{\frac{t}{2}}^t(1+s)^{-\frac{3n}{4}-\frac12}\le C(1+t)^{-\frac{n}{4}-1},
\end{aligned}
$$

$$
\begin{aligned}
\|J_4\|_{L^2}=\Big\|\sum_{i=1}^n\partial_{x_i}G_1(t,\cdot)\Big\|_{L^2}\int_{\frac{t}{2}}^\infty\int\mathcal{F}_i^0dyds&\le C(1+t)^{-\frac{n}{4}-\frac12}\int_{\frac{t}{2}}^t\|u\|_{L^2}^2ds\\
&\le C(1+t)^{-\frac{n}{4}-1}.
\end{aligned}
$$

We next estimate J_2. We put $F_i^2(u,\partial_x u)=\big(F_{i,jk}^2(u,\partial_x u)\big)_{j,k=1}^n$ with $F_{i,jk}^2(u,\partial_x u)=-\delta_{ji}\mu_2\big(\frac{\sigma m_k}{\sigma+\gamma\bar\rho}\big)$ for $1\le i\le n$ and $F^3(u,\partial_x u)=\big(F_j^3(u,\partial_x u)\big)_{j=1}^n$ with $F_j^3(u,\partial_x u)=-\mu_1\big(\frac{\sigma m_j}{\sigma+\gamma\bar\rho}\big)$; and we set $\mathcal{F}_i^2=\begin{pmatrix}0\\ F_i^2(u,\partial_x u)\end{pmatrix}$ and $\mathcal{F}^3=\begin{pmatrix}0\\ F^3(u,\partial_x u)\end{pmatrix}$. By using $\mathcal{F}_i^2$ and $\mathcal{F}^3$, we can write $\mathcal{F}_i^1=\sum_{k=1}^n\partial_{x_k}\mathcal{F}_{i,k}^2+\partial_{x_i}\mathcal{F}^3$, where $\mathcal{F}_{i,k}^2=\begin{pmatrix}0\\ F_{ik}^2(u,\partial_x u)\end{pmatrix}$ with $F_{ik}^2(u,\partial_x u)=\big(F_{i,jk}^2(u,\partial_x u)\big)_{j=1}^n$. Hence we obtain

$$
\begin{aligned}
\|J_2\|_{L^2}&=\Big\|\int_0^{\frac{t}{2}}\int\sum_{i=1}^n\partial_{x_i}G_1(t-s,x-y)\mathcal{F}_i^1dyds\Big\|_{L^2}\\
&=\Big\|\int_0^{\frac{t}{2}}\int\sum_{i=1}^n\partial_{x_i}G_1(t-s,x-y)\{\sum_{k=1}^n\partial_{x_k}\mathcal{F}_{i,k}^2+\partial_{x_i}\mathcal{F}^3\}dyds\Big\|_{L^2}\\
&=\Big\|\int_0^{\frac{t}{2}}\int\{\sum_{i=1}^n\sum_{k=1}^n\partial_{x_i}\partial_{x_k}G_1(t-s,x-y)\mathcal{F}_{i,k}^2\}\\
&\qquad+\{\sum_{i=1}^n\partial_{x_i}^2G_1(t-s,x-y)\mathcal{F}^3\}dyds\Big\|_{L^2}\\
&\le C\int_0^{\frac{t}{2}}\|\nabla^2G_1(t-s)\|_{L^2}\|u(s)\|_{L^2}^2ds\\
&\le C\int_0^{\frac{t}{2}}(1+t-s)^{-\frac{n}{4}-1}(1+s)^{-\frac{n}{2}}ds\\
&\le C(1+t)^{-\frac{n}{4}-1}.
\end{aligned}
$$

Let us next consider J_1. We decompose J_1 as

$$\begin{aligned} J_1 &= \int_0^{\frac{t}{2}} \int \sum_{i=1}^n \big(\partial_{x_i} G_1(t-s,x-y) - \partial_{x_i} G_1(t,x)\big)\mathcal{F}_i^4 dyds \\ &\quad + \int_0^{\frac{t}{2}} \int \sum_{i=1}^n \big(\partial_{x_i} G_1(t-s,x-y) - \partial_{x_i} G_1(t,x)\big)\mathcal{F}_i^5 dyds \\ &=: I_1 + I_2, \end{aligned}$$

where $\mathcal{F}_i^4 = \begin{pmatrix} 0 \\ F_i^4(u) \end{pmatrix}$, and $\mathcal{F}_i^5 = \begin{pmatrix} 0 \\ F_i^5(u) \end{pmatrix}$ with $F_i^4(u) = \big(F_{ij}^4(u)\big)_{j=1}^n$, $F_{ij}^4(u) = -\delta_{ji}\big\{\frac{1}{2\gamma^2}\sigma^2 P_{\rho\rho}(\bar\rho)\big\} - \frac{m_j m_i}{\bar\rho}$, and $F_i^5(u) = \big(F_{ij}^5(u)\big)_{j=1}^n$ with $F_{ij}^5(u) = \delta_{ji}\big\{\frac{1}{2\gamma^3}\sigma^3 \int_0^1 (1-\theta)^2 \partial_\rho^3 P(\frac{1}{\gamma}\sigma\theta + \bar\rho)d\theta\big\} + \frac{\sigma m_j m_i}{\bar\rho(\sigma+\gamma\bar\rho)}$.

Moreover, we decompose I_1 as

$$\begin{aligned} I_1 &= \int_0^{\frac{t}{2}} \int \sum_{i=1}^n \big(\partial_{x_i} G_1(t-s,x-y) - \partial_{x_i} G_1(t,x)\big)\mathcal{F}_i^4 dyds \\ &= \int_0^{\frac{t}{2}} \int \sum_{i=1}^n \big(\partial_{x_i} G_1(t-s,x-y) - \partial_{x_i} G_1(t-s,x)\big)\mathcal{F}_i^4 dyds \\ &\quad + \int_0^{\frac{t}{2}} \int \sum_{i=1}^n \big(\partial_{x_i} G_1(t-s,x) - \partial_{x_i} G_1(t,x)\big)\mathcal{F}_i^4 dyds \\ &=: I_{11} + I_{12}. \end{aligned}$$

We first estimate I_{11} and I_{12} when $n=3$. By using Lemma 2.1 and Lemma 4.2, we have

$$\begin{aligned} \|I_{11}\|_{L^2} &= \Big\| \int_0^{\frac{t}{2}} \int \sum_{i=1}^3 \big(\partial_{x_i} G_1(t-s,x-y) - \partial_{x_i} G_1(t-s,x)\big)\mathcal{F}_i^4(s,y) dyds \Big\|_{L^2} \\ &\le C \int_0^{\frac{t}{2}} \int \big\|\nabla\big\{G_1(t-s,x-y) - G_1(t-s,x)\big\}\big\|_{L^2} \sum_{i=1}^3 \big|\mathcal{F}_i^4(s,y)\big| dyds \\ &\le C \int_0^{\frac{t}{2}} \int \big\|G_1(t-s,x-y) - G_1(t-s,x)\big\|_{L^2}^{\frac12} \big\|\nabla^2\big\{G_1(t-s,x-y) - G_1(t-s,x)\big\}\big\|_{L^2}^{\frac12} \\ &\quad \times \sum_{i=1}^3 \big|\mathcal{F}_i^4(s,y)\big| dyds \\ &\le C \int_0^{\frac{t}{2}} \int \Big\| \int_0^1 \big|\nabla G_1(t-s,x-\tau y)\big| d\tau \Big\|_{L^2}^{\frac12} \big\|\nabla^2\big\{G_1(t-s,x-y) - G_1(t-s,x)\big\}\big\|_{L^2}^{\frac12} |y|^{\frac12} \\ &\quad \times \sum_{i=1}^3 \big|\mathcal{F}_i^4(s,y)\big| dyds \\ &\le C \int_0^{\frac{t}{2}} (1+t-s)^{-\frac12(\frac34+\frac12)} (1+t-s)^{-\frac12(\frac34+1)} \sum_{i=1}^3 \big\||x|^{\frac12}\mathcal{F}_i^4(s,x)\big\|_{L^1} ds \\ &\le C \int_0^{\frac{t}{2}} (1+t-s)^{-\frac32} \||x|^{\frac12} u\|_{L^2} \|u\|_{L^2} ds \\ &\le C(1+t)^{-\frac32} \int_0^{\frac{t}{2}} (1+s)^{-1} ds \le C(1+t)^{-\frac32} \log(1+t), \end{aligned}$$

and

$$
\begin{aligned}
\|I_{12}\|_{L^2} &= \Big\| \int_0^{\frac{t}{2}} \int \sum_{i=1}^{3} \big(\partial_{x_i} G_1(t-s,x) - \partial_{x_i} G_1(t,x)\big) \mathcal{F}_i^4(s,y)dyds \Big\|_{L^2} \\
&\leq C \int_0^{\frac{t}{2}} \int \big\|G_1(t-s,x) - G_1(t,x)\big\|_{L^2}^{\frac{1}{2}} \big\|\nabla^2\big\{G_1(t-s,x) - G_1(t,x)\big\}\big\|_{L^2}^{\frac{1}{2}} \\
&\qquad\qquad \times \sum_{i=1}^{3} \big|\mathcal{F}_i^4(s,y)\big|dyds \\
&\leq C \int_0^{\frac{t}{2}} \int \Big\| \int_0^1 \big|\partial_t G_1(t-\tau s,x)\big| d\tau \Big\|_{L^2}^{\frac{1}{2}} \\
&\quad \times \big\|\nabla^2\big\{G_1(t-s,x) - G_1(t,x)\big\}\big\|_{L^2}^{\frac{1}{2}} s^{\frac{1}{2}} \sum_{i=1}^{3} \big|\mathcal{F}_i^4(s,y)\big|dyds \\
&\leq C(1+t)^{-\frac{3}{2}} \int_0^{\frac{t}{2}} s^{\frac{1}{2}} \|u\|_{L^2} \|u\|_{L^2} ds \\
&\leq C(1+t)^{-\frac{3}{2}} \int_0^{\frac{t}{2}} (1+s)^{-1} ds \leq C(1+t)^{-\frac{3}{2}} \log(1+t).
\end{aligned}
$$

Hence we obtain the estimate of $\|I_1\|_{L^2}$ for $n=3$.

We next consider the case $n \geq 4$. We have

$$
\begin{aligned}
\|I_{11}\|_{L^2} &= \Big\| \int_0^{\frac{t}{2}} \int \sum_{i=1}^{n} \big(\partial_{x_i} G_1(t-s,x-y) - \partial_{x_i} G_1(t-s,x)\big) \mathcal{F}_i^4(s,y)dyds \Big\|_{L^2} \\
&= \Big\| \int_0^{\frac{t}{2}} \int \sum_{i=1}^{n} \int_0^1 \partial_{x_i} \nabla G_1(t-s,x-\tau y) d\tau y \mathcal{F}_i^4(s,y)dyds \Big\|_{L^2} \\
&\leq C \int_0^{\frac{t}{2}} (1+t-s)^{-\frac{n}{4}-1} \||x|u\|_{L^2} \|u\|_{L^2} ds \\
&\leq C(1+t)^{-\frac{n}{4}-1} \int_0^{\frac{t}{2}} (1+s)^{-\frac{n}{2}+1} ds \\
&\leq C(1+t)^{-\frac{n}{4}-1} \begin{cases} \log(1+t), & n=4, \\ 1, & n \geq 5. \end{cases}
\end{aligned}
$$

Similarly, we obtain

$$
\begin{aligned}
\|I_{12}\|_{L^2} &= \Big\| \int_0^{\frac{t}{2}} \int \sum_{i=1}^{n} \big(\partial_{x_i} G_1(t-s,x) - \partial_{x_i} G_1(t,x)\big) \mathcal{F}_i^4(s,y)dyds \Big\|_{L^2} \\
&= \Big\| \int_0^{\frac{t}{2}} \int \sum_{i=1}^{n} \int_0^1 \partial_{x_i} \partial_t G_1(t-\tau s,x) d\tau \, s \mathcal{F}_i^4(s,y)dyds \Big\|_{L^2} \\
&\leq C(1+t)^{-\frac{n}{4}-1} \int_0^{\frac{t}{2}} (1+s)^{-\frac{n}{2}+1} ds \\
&\leq C(1+t)^{-\frac{n}{4}-1} \begin{cases} \log(1+t), & n=4, \\ 1, & n \geq 5, \end{cases}
\end{aligned}
$$

and the estimate of I_1 is obtained.

As for I_2, we can estimate similarly to the proof of the estimate of I_1:

$$
\begin{aligned}
\|I_2\|_{L^2} &\le C\Big\|\int_0^{\frac{t}{2}}\int\sum_{i=1}^n\big(\partial_{x_i}G_1(t-s,x-y)-\partial_{x_i}G_1(t-s,x)\big)\mathcal{F}_i^5dyds\Big\|_{L^2}\\
&\quad +C\Big\|\int_0^{\frac{t}{2}}\int\sum_{i=1}^n\big(\partial_{x_i}G_1(t-s,x)-\partial_{x_i}G_1(t,x)\big)\mathcal{F}_i^5dyds\Big\|_{L^2}\\
&\le C\int_0^{\frac{t}{2}}(1+t-s)^{-\frac{n}{4}-1}\||x|^{\frac{1}{2}}u\|_{L^2}^2\|\sigma\|_{L^\infty}ds\\
&\quad +C\int_0^{\frac{t}{2}}(1+t-s)^{-\frac{n}{4}-1}s\|u\|_{L^2}^2\|\sigma\|_{L^\infty}ds\\
&\le C(1+t)^{-\frac{n}{4}-1}\int_0^{\frac{t}{2}}(1+s)^{-n+1}ds\le C(1+t)^{-\frac{n}{4}-1}.
\end{aligned}
$$

This completes the proof. □

Proof of Theorem 1.1. It follows from Proposition 4.1, Proposition 4.3 and Theorem 4.6 that

$$
\begin{aligned}
&\big\|u(t)-G(t)*u_0-\sum_{i=1}^n\partial_iG_1(t,\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0dyds\big\|_{L^2}\\
=\ &\big\|u_1(t)+u_\infty(t)-G(t)*u_0-\sum_{i=1}^n\partial_iG_1(t,\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0dyds\big\|_{L^2}\\
\le\ &\big\|G_1(t)*u-G(t)*u_0\big\|_{L^2}\\
&+\big\|\int_0^tG_1(t-s)*\operatorname{div}\mathcal{F}(u,\partial_xu)(s)ds-\sum_{i=1}^n\partial_iG_1(t,\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0dyds\big\|_{L^2}\\
&+\big\|u_\infty(t)\big\|_{L^2}\\
\le\ &C(1+t)^{-\frac{n}{4}-\frac{3}{4}}\begin{cases}\log(1+t), & n=3,\\ (1+t)^{-\frac{1}{4}}\log(1+t), & n=4,\\ (1+t)^{-\frac{1}{4}}, & n\ge 5.\end{cases}
\end{aligned}
$$

Moreover, in a similar manner to the proof of Theorem 4.6, we have

$$
\begin{aligned}
&\big\|\int_0^tG_1(t-s)*\operatorname{div}\mathcal{F}(u,\partial_xu)(s)ds-\sum_{i=1}^n\partial_i\tilde{G}(t,\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0dyds\big\|_{L^2}\\
\le\ &\big\|\int_0^tG_1(t-s)*\operatorname{div}\mathcal{F}(u,\partial_xu)(s)ds-\int_0^t\tilde{G}(t-s)*\operatorname{div}\mathcal{F}(u,\partial_xu)(s)ds\big\|_{L^2}\\
&+\big\|\int_0^t\tilde{G}(t-s)*\operatorname{div}\mathcal{F}(u,\partial_xu)(s)ds-\sum_{i=1}^n\partial_i\tilde{G}(t,\cdot)\int_0^\infty\int_{\mathbb{R}^n}\mathcal{F}_i^0dyds\big\|_{L^2}\\
\le\ &C(1+t)^{-\frac{n}{4}-\frac{3}{4}}\begin{cases}\log(1+t), & n=3,\\ (1+t)^{-\frac{1}{4}}\log(1+t), & n=4,\\ (1+t)^{-\frac{1}{4}}, & n\ge 5.\end{cases}
\end{aligned}
$$

Here we used proposition 4.4. We thus obtain the desired decay estimate in Theorem 1.1. □

5 Proof of Proposition 4.7

In this section we will give a proof of Proposition 4.7. Let us reformulate the problem (1) by using the velocity. We set $v = \frac{m}{\rho}$ and $v_0 = \frac{m_0}{\rho_0}$. Then (1) is rewritten as

$$\begin{cases} \partial_t \rho + \mathrm{div}\,(\rho v) = 0, \\ \partial_t v + (v \cdot \nabla) v + \frac{\nabla P(\rho)}{\rho} = \frac{\mu}{\rho} \Delta v + \frac{\mu + \mu'}{\rho} \nabla(\mathrm{div}\, v), \\ (\rho, v)(0, x) = (\rho_0, v_0)(x). \end{cases} \tag{9}$$

Moreover we rewrite the system (9) into the one for perturbation. By using the new unknown function

$$\phi(t,x) = \frac{\rho(t,x) - \bar{\rho}}{\bar{\rho}}, \quad w(t,x) = \frac{1}{\gamma} u(t,x),$$

the initial value problem (9) is reformulated as

$$\begin{cases} \partial_t \phi + \gamma \mathrm{div}\, w = F_1(U), \\ \partial_t w - \mu_1 \triangle w - \mu_2 \nabla(\mathrm{div}\, w) + \gamma \nabla \phi = F_2(U), \\ (\phi, w)(0, x) = (\phi_0, w_0)(x), \end{cases} \tag{10}$$

where $U = \begin{pmatrix} \phi \\ w \end{pmatrix}$,

$$F_1(U) = -\gamma(w \cdot \nabla \phi + \phi \mathrm{div}\, w),$$

$$\begin{aligned} F_2(U) \quad &= \quad -\gamma(w \cdot \nabla) w - \mu_1 \frac{\phi}{\phi + 1} \triangle w - \mu_2 \frac{\phi}{\phi + 1} \nabla(\mathrm{div}\, w) \\ &\quad + \Big(\frac{\bar{\rho}\gamma}{\phi + 1} - \frac{\bar{\rho}}{\gamma} \frac{\int_0^1 P''(s\bar{\rho}\phi + \bar{\rho}) ds}{\phi + 1} \Big) \phi \nabla \phi. \end{aligned}$$

We have the following estimates for the solution U of (10) which were obtaind by Matsumura and Nishida [6, 7].

Proposition 5.1. *There exists a $\epsilon > 0$ such that if*

$$\|U_0\|_{H^{s_0+1}} \le \epsilon.$$

Then we obtain

$$\Big(\int_0^\infty \|\nabla U\|_{H^{s_0}}^2 ds \Big)^{\frac{1}{2}} + \Big(\int_0^\infty \|\nabla w\|_{H^{s_0+1}}^2 ds \Big)^{\frac{1}{2}} \le C\epsilon$$

See, for instance, [6, 7] for a proof of Proposition 5.1.

By using operator A, problem (10) is written as

$$\partial_t U - AU = F(U), \quad U|_{t=0} = U_0, \tag{11}$$

where

$$F(U) = \begin{pmatrix} F_1(U) \\ F_2(U) \end{pmatrix}, \quad U_0 = \begin{pmatrix} \phi_0 \\ w_0 \end{pmatrix}.$$

In terms of G, solution U is written as

$$\begin{aligned} U(t) \quad &= \quad G(t) * U_0 + \int_0^t G(t-s) * F(U) ds \\ &= \quad G_1(t) * U_0 + G_\infty(t) * U_0 + \int_0^t G_1(t-s) * F(U) ds + \int_0^t G_\infty(t-s) * F(U) ds \end{aligned} \tag{12}$$

where $G_\infty(t)$ is the high frequency part of G, i.e., $G_\infty(t)$ is given by $G_\infty(t) = G(t) - G_1(t)$.

To prove Proposition 4.7, we introduce some notation. We set

$$M_l(t) := \sup_{0\le\tau\le t}(1+\tau)^{\frac{n}{4}-l}\big\||x|^l U(\tau)\big\|_2.$$

Let us introduce the dyadic partition of unity. We take $\phi \in C_0^\infty$ satisfying

$$\operatorname{Supp}\phi \subset \{x\in\mathbb{R}^n | \frac{3}{4}\le |x| \le \frac{8}{3}\},$$

$$\sum_{j\in\mathbb{Z}}\phi(2^{-j}x) = 1 \quad \textit{for } x\in\mathbb{R}^n\backslash\{0\},$$

$$\operatorname{Supp}\phi(2^{-j}\cdot)\cap\operatorname{Supp}\phi(2^{-j'}\cdot) = \emptyset \quad \textit{for } |j-j'|\ge 2.$$

We set $h = \mathfrak{F}[\phi]$ and define the dyadic blocks by

$$\dot{\triangle}_j\hat{u} = 2^{jn}\int_{\mathbb{R}^n}h(2^j\eta)\hat{u}(\xi-\eta)d\eta \quad \text{if } j\in\mathbb{Z}.$$

The cut-off operator $\dot{S}_j$ is defined by

$$\dot{S}_j\hat{u} = \sum_{k\le j-1}\dot{\triangle}_k\hat{u}.$$

The paraproducts of $\hat{u}$ and $\hat{v}$ are defined by

$$\dot{T}_{\hat{u}}\hat{v} := \sum_j \dot{S}_{j-1}\hat{u}\dot{\triangle}_j\hat{v}, \qquad \dot{R}(\hat{u},\hat{v}) = \sum_{|k-j|\le 1}\dot{\triangle}_k\hat{u}\dot{\triangle}_j\hat{v}.$$

The Bony decomposition

$$\hat{u}\hat{v} = \dot{T}_{\hat{u}}\hat{v} + \dot{T}_{\hat{v}}\hat{u} + \dot{R}(\hat{u},\hat{v})$$

is well known. See, for instance, [1]

We have the following estimate for $|x|^{\frac{1}{2}}u*v$.

Lemma 5.2. *Let $\nabla^k\hat{u}\in L^\infty$ for $k = 0$, 1, and $(1+|x|^{\frac{1}{2}})v\in L^2$. We have*

$$\big\||x|^{\frac{1}{2}}u*v\big\|_{L^2} \le C\big\{\|\nabla_\xi\hat{u}\|_{L^\infty}+\|\hat{u}\|_{L^\infty}\big\}\big\{\|v\|_{L^2}+\||x|^{\frac{1}{2}}v\|_{L^2}\big\}.$$

Proof. We obtain

$$\begin{aligned}
\big\||x|^{\frac{1}{2}}u*v\big\|_{L^2}^2 &= \big\|\sum_j\phi(2^{-j}x)|x|^{\frac{1}{2}}u*v\big\|_{L^2}^2 \\
&\le C\sum_j\{2^{\frac{1}{2}j}\big\|\phi(2^{-j}x)u*v\big\|_{L^2}\}^2 \\
&\le C\sum_j\{2^{\frac{1}{2}j}\big\|\dot{\triangle}_k\hat{u}\hat{v}\big\|_{L^2}\}^2 \\
&\le C\sum_j\{2^j\|\dot{\triangle}_j\dot{T}_{\hat{u}}\hat{v}\|_{L^2}^2 + 2^j\|\dot{\triangle}_j\dot{T}_{\hat{v}}\hat{u}\|_{L^2}^2 + 2^j\|\dot{\triangle}_j\dot{R}(\hat{u},\hat{v})\|_{L^2}^2\}.
\end{aligned}$$

Each term on the right-hand side is estimated as

$$\begin{aligned}
\sum_j 2^j\|\dot{\triangle}_k\dot{T}_{\hat{u}}\hat{v}\|_{L^2}^2 &= \sum_j 2^j\|\dot{\triangle}_j\sum_k\dot{S}_{k-1}\hat{u}\dot{\triangle}_k\hat{v}\|_{L^2}^2 \\
&= \sum_j 2^j\|\dot{\triangle}_j\sum_{|j-k|\le 5}\dot{S}_{k-1}\hat{u}\dot{\triangle}_k\hat{v}\|_{L^2}^2 \\
&\le C\sum_j 2^j\sum_{|j-k|\le 5}\|\dot{S}_{k-1}\hat{u}\|_{L^\infty}^2\|\dot{\triangle}_k\hat{v}\|_{L^2}^2 \\
&\le C\|\hat{u}\|_{L^\infty}^2\sum_j 2^j\|\dot{\triangle}_j\hat{v}\|_{L^2}^2 \\
&\le C\|\hat{u}\|_{L^\infty}^2\||x|^{\frac{1}{2}}v\|_{L^2}^2,
\end{aligned}$$

$$
\begin{aligned}
\sum_j 2^j \|\dot{\triangle}_k \dot{T}_{\hat v}\hat u\|_{L^2}^2 &\le C\sum_j 2^j \sum_{|j-k|\le 5} \|\dot S_{k-1}\hat v\|_{L^2}^2 \|\dot\triangle_k \hat u\|_{L^\infty}^2\\
&\le C\|v\|_{L^2}^2 \sum_j 2^j \|\triangle_j \hat u\|_{L^\infty}^2\\
&\le C\|v\|_{L^2}^2\big\{\|\hat u\|_{L^\infty}^2 \sum_{j<0} 2^j + \sup_j (2^j\|\triangle_j \hat u\|_{L^\infty})^2 \sum_{j\ge 0} 2^{-j}\big\}\\
&\le \|v\|_{L^2}^2\big\{\|\nabla_\xi \hat u\|_{L^\infty}^2 + \|\hat u\|_{L^\infty}^2\big\},
\end{aligned}
$$

and

$$
\begin{aligned}
\sum_j 2^j\|\dot\triangle_j \dot R(\hat u,\hat v)\|_{L^2}^2 &\le C\sum_j 2^j \sum_{k>j-2} \|\tilde\triangle_k \hat u \dot\triangle_k \hat v\|_{L^2}^2\\
&\le C\sum_j \sum_{k>j-2} 2^{j-k}\|\tilde\triangle_k \hat u\|_{L^\infty}^2 2^k \|\dot\triangle_k \hat v\|_{L^2}^2\\
&\le C\|\hat u\|_{L^\infty}^2 \sum_k 2^k \|\dot\triangle_k \hat v\|_{L^2}^2 \sum_{j<k+2} 2^{j-k}\\
&\le C\|\hat u\|_{L^\infty}^2 \||x|^{\frac12} v\|_{L^2}^2,
\end{aligned}
$$

where $\tilde\triangle_k = \triangle_{k-1} + \triangle_k + \triangle_{k+1}$. This completes the proof. □

$G_1(t)$ and $G_\infty(t)$ satisfy the following estimate.

Lemma 5.3. *Let k be a nonnegative integer. Then there hold*

$$
\big\||x|^l \nabla^k G_1(t) * U_0(t)\big\|_{L^2} \le C(1+t)^{-\frac n4 - \frac k2 + l}\|U_0\|_{L^1} + C(1+t)^{-\frac n4 - \frac k2}\big\||x|^l U_0\big\|_{L^1},
$$

$$
\big\||x|^l \nabla^k G_\infty(t) * U_0(t)\big\|_{L^2} \le Ce^{-c_0 t}\big\{\|\nabla^k U_0\|_{L^2} + \big\||x|^l\nabla^k U_0\big\|_{L^2}\big\}
$$

for $l = 0, \frac12, 1$ and $t \ge 0$, where c_0 is a positive constant independent of t.

Proof. The proof for the case of $l = 0$ is well known. When $l = 1$, we have, for any α with $|\alpha| = k$,

$$
\begin{aligned}
\big\||x|\partial_x^\alpha G_1(t)\big\|_{L^2} &\le C\Big\{\int_{|\xi|\le 2A} \big|\nabla_\xi\big((i\xi)^\alpha e^{\hat A(\xi)t}\hat\Phi\big)\big|^2 d\xi\Big\}^{\frac12}\\
&\le C\Big\{\int_{|\xi|\le 2A} \big(t|\xi|^k e^{-\beta|\xi|^2 t}\big)^2 d\xi\Big\}^{\frac12} + C\Big\{\int_{|\xi|\le 2A}\big(|\xi|^{k-1}e^{-\beta|\xi|^2 t}\big)^2 d\xi\Big\}^{\frac12}\\
&\quad + C\Big\{\int_{|\xi|\le 2A}\big(|\xi|^k e^{-\beta|\xi|^2 t}\big)^2 d\xi\Big\}^{\frac12}\\
&\le C(1+t)^{-\frac n4 - \frac k2 + 1},
\end{aligned}
$$

where β is positive constant. When $l = \frac12$ we obtain

$$
\begin{aligned}
\big\||x|^{\frac12}\nabla^k G_1(t)\big\|_{L^2} &\le C\big\||x|\nabla^k G_1(t)\big\|_{L^2}^{\frac12}\big\|\nabla^k G_1(t)\big\|_{L^2}^{\frac12}\\
&\le C(1+t)^{-\frac n4 - \frac k2 + \frac12}.
\end{aligned}
$$

Hence we have

$$
\begin{aligned}
\big\||x|^l\nabla^k G_1(t) * U_0\big\|_{L^2} &\le \big\||x|^l \nabla^k G_1(t)\big\|_{L^2}\|U_0\|_{L^1} + \big\|\nabla^k G_1(t)\big\|_{L^2}\big\||x|^l U_0\big\|_{L^1}\\
&\le C(1+t)^{-\frac n4 - \frac k2 + l}\|U_0\|_{L^1} + C(1+t)^{-\frac n4 - \frac k2}\big\||x|^l U_0\big\|_{L^1}
\end{aligned}
$$

for $l = \frac12, 1$.

We next consider the estimate of the high frequency part. A direct computation gives the following estimates for $\hat{G}_\infty(t,\xi)$:

$$|\nabla_\xi^k \hat{G}_\infty(t,\xi)| \le C(k)\big(e^{-\theta t}|\xi|^{-k-1} + e^{-\theta|\xi|^2 t}|\xi|^{-k}\big), \tag{13}$$

where θ is a positive constant. By using Lemma 5.2 and (13), we have

$$\begin{aligned} &\big\||x|^l \nabla^k G_\infty(t) * U_0\big\|_{L^2} \\ \le\ & \big\{\|\nabla_\xi \hat{G}_\infty\|_{L^\infty} + \|\hat{G}_\infty\|_{L^\infty}\big\}\big\{\||x|^l U_0\|_{L^2}\|U_0\|_{L^2}\big\} \\ \le\ & Ce^{-c_0 t}\|\nabla^k U_0\|_{L^2} + Ce^{-c_0 t}\big\||x|^l \nabla^k U_0\big\|_{L^2} \end{aligned}$$

for $l = \frac{1}{2}$, 1, where c_0 is a positive constant. This completes the proof. □

We are in a position to complete the proof of Proposition 4.7.

Proposition 5.4. *Let $n \ge 3$ and let $l = \frac{1}{2}$ or $l = 1$. There exists $\epsilon > 0$ such that if*

$$U_0 \in H^{s_0+1} \cap L_l^1 \cap L_l^2$$

and

$$\|U_0\|_{H^{s_0+1}\cap L^1} \le \epsilon,$$

then we have

$$M_l(t) \le C\|U_0\|_{L_l^1 \cap L_l^2} + C\epsilon$$

for $t \in [0,\infty)$.

Proof. By Proposition 4.1, Proposition 4.3, Lemma 5.3 and (12), we see that

$$\begin{aligned} \big\||x|^l U(\tau)\big\|_{L^2} \le\ & C\big\||x|^l G_1(\tau) * U_0\big\|_{L^2} + C\big\||x|^l G_\infty(\tau) * U_0\big\|_{L^2} \\ & + C\int_0^\tau \||x|^l G_1(\tau - s) * F(U)\|_{L^2} ds + C\int_0^\tau \||x|^l G_\infty(\tau - s) * F(U)\|_{L^2} ds \\ \le\ & C(1+\tau)^{-\frac{n}{4}+l}\|U_0\|_{L^1} + C(1+\tau)^{-\frac{n}{4}}\big\||x|^l U_0\big\|_{L^1} + Ce^{-c_0\tau}\big\|(1+|x|^l)U_0\|_{L^2} \\ & + C\int_0^\tau \big\{(1+\tau-s)^{-\frac{n}{4}+l}\|F(U)\|_{L^1} + (1+\tau-s)^{-\frac{n}{4}}\big\||x|^l F(U)\big\|_{L^1}\big\} ds \\ & + C\int_0^\tau e^{-c_0(\tau-s)}\|(1+|x|^l)F(U)\|_{L^2} ds \\ \le\ & C(1+\tau)^{-\frac{n}{4}+l}\|U_0\|_{L_l^1\cap L_l^2} + C\int_0^\tau (1+\tau-s)^{-\frac{n}{4}+l}\|U\|_{L^2}\|\nabla U\|_{H^1} ds \\ & + C\int_0^\tau (1+\tau-s)^{-\frac{n}{4}}\big\||x|^l U\big\|_{L^2}\|\nabla U\|_{H^1} ds \\ & + C\int_0^\tau e^{-c_o(\tau-s)}\big\{\big\||x|^l U\big\|_{L^2} + \big\|U\big\|_{L^2}\big\}\big\{\|\nabla U\|_{H^{s_0}} + \|\nabla w\|_{H^{s_0+1}}\big\} ds \\ \le\ & C(1+\tau)^{-\frac{n}{4}+l}\|U_0\|_{L_l^1\cap L_l^2} + C\epsilon^2\int_0^\tau (1+\tau-s)^{-\frac{n}{4}+l}(1+s)^{-\frac{n}{2}-\frac{1}{2}} ds \\ & + C\epsilon M_l(t)\int_0^\tau (1+\tau-s)^{-\frac{n}{4}}(1+s)^{-\frac{n}{2}-\frac{1}{2}+l} ds \\ & + C\big\{M_l(t)+\epsilon\big\}\Big\{\Big(\int_0^\tau e^{-2c_0(\tau-s)}(1+s)^{-\frac{n}{2}+2l} ds\Big)^{\frac{1}{2}}\Big(\int_0^\tau \|\nabla U\|_{H^{s_0}}^2 + \|\nabla w\|_{H^{s_0+1}}^2 ds\Big)^{\frac{1}{2}}\Big\} \\ \le\ & C(1+\tau)^{-\frac{n}{4}+l}\|U_0\|_{L_l^1\cap L_l^2} + C\epsilon(1+\tau)^{-\frac{n}{4}+l} + C\epsilon(1+\tau)^{-\frac{n}{4}+l}M_l(t) \end{aligned}$$

for $k = 0, 1$ and $0 \le \tau \le t$. Hence we have

$$(1+\tau)^{\frac{n}{4}-l}\big\||x|^l U\big\|_{L^2} \le C\|U_0\|_{L_l^1\cap L_l^2} + C\epsilon + C\epsilon M_l(t).$$

Taking the supremum in $\tau \in [0,t]$, we obtain the desired estimate if ϵ is sufficiently small. □

The desired decay estimate in Proposition 4.7 now follows from Proposition 5.4.

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