A higher-order asymptotic theory for motion of a counter-rotating vortex pair of finite thickness

ハビバ, ウミュ

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氏　　名：ハビバ　ウミュ

論文題目： A higher-order asymptotic theory for motion of a counter-rotating vortex pair of finite thickness
（有限太さの逆回転渦対の運動に対する高次漸近理論）

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論　文　内　容　の　要　旨

A counter-rotating vortex pair is relevant to the wake vortices behind the wings of an airplane and controlling it has been demanded since jet planes started their commercial flight. We establish the traveling speed of a counter-rotating vortex pair moving in an incompressible fluid. The solution of the Navier-Stokes is constructed by use of the matched asymptotic expansions in a small parameter $\varepsilon = 1/\sqrt{\mathrm{Re}}$ where $\mathrm{Re} = \Gamma/\nu$ is the Reynolds number with ν being the kinematic viscosity of the fluid and $\pm\Gamma$ being the circulation of the vortices. The parameter ε is a measure for ratio of core radius σ to the half distance d between the vortices. The radius of vortex core is assumed to be much smaller than the distance between two centroids $(\varepsilon = 1)$. The Biot-Savart law is valid in the outer region, and its inner limit provides the boundary condition on the inner solution. Adapting Dyson's technique to two dimensions facilitates the evaluation the inner limit for an arbitrary of vorticity distribution.

In Chapter 3, the asymptotic expansions are performed for the solution of the Navier-Stokes equation for a counter-rotating vortex pair up to seventh order. The 0^{th}-order solution represents a circular vortex. The 1st-order solution produces the translation speed which is coincident with of the pair of point vortices. The 2nd, 3rd and 4th orders incorporate pure shear induced by the companion vortex where the core of a vortex pair deforms to an ellipse, and the perturbation vorticity at 2nd, 3rd and 4th orders represent quadrupole, hexapole and octapole structures respectively. At the 5th order, a correction due to the effect of finite thickness of the vortices makes the first appearance to the traveling speed for both of the viscous and the inviscid fluids. Moreover, a formula is established in surprisingly simple form to $O(\varepsilon^5)$ for the traveling speed of counter-rotating vortex pair for a general vorticity distribution. This general formula is backed by the analytical proof. The general formula is powerful since we can calculate fifth order traveling speed of a counter-rotating vortex pair only from second-order solution which is numerically calculated by shooting method. The next correction the general formula for a counter-rotating vortex pair is given at seventh order. Furthermore the lateral motion of the centroids of vorticity are given through the hydrodynamics impulse theory, and the end product is the general formula of the lateral position of the center of the vortex completed with the illustration of using the formula for the motion of a viscous vortex pair.

Chapter 4 is concerned with illustration of a vortex pair in an inviscid fluid. For example we consider the Rankine vortex with uniform vorticity at leading order in the same way as a vortex pair for a viscous fluid. By using the matched asymptotic expansions up to fifth order in an inviscid fluid, the solutions of the motion of an anti-parallel vortex pair are obtained and has good agreement with the result of Yang and Kubota (1992). Therefore resulting solution serves to test the general formula of the traveling speed of a counter-rotating vortex that we obtained is Chapter 3.