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Subspace Predictive Control for General Dual-Rate Sampled-Data Systems

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Abstract: In this paper a novel predictive control design method is proposed. Using a modified subspace state-space identification algorithm, lifted state-space models for general dual-rate systems is identified from the input-output data. Based on the estimated lifted system matrices, we establish two predictors which realize the prediction of the output of general dual-rate systems. Then the predictors are applied to predictive control design subject to a linear quadratic cost function for general dual-rate systems.

Keywords: Multi-rate system, General dual-rate system, Lifted state-space model, Subspace state-space identification, Predictive control

1. Introduction

Owing to the need arising in practical industries, there has been a steady stream of research on the topic of general dual-rate sampled-data control, mostly based upon optimal control theory. Some of the most popular contributions include the references [1),2)]. On the other hand, the developments had little impact on the process industry. During the last decade, the process control research community and industry have witnessed the emergence of a new control technique called model predictive control (MPC). Some good reviews of developments in MPC are in survey papers[3),4)].

MPC designates a wide range of control algorithms which make an explicit use of a process model in a cost function minimization to obtain the control signal. From the process plant model, predictors can be obtained (for example, by solving Diophantine equations iteratively[3)]). The predictors are used to obtain predictions of the plant output which are used in the control design. Hence a model of the process plant is the crucial requirement for the predictive control design.

System identification techniques are the most popular methods to obtain a plant model based on the experimental data. Typical identification techniques include the classical least square (LS) method, the instrumental variables (IV) method, *etc*[6)]. In the decade, Subspace State-Space IDentification (4SID) methods, such as MOESP[7)], N4SID[8)], are attractive not only because of their numerical simplicity and stability, but also for their state-space representation that is very convenient for optimal estimation, filtering and prediction.

Recently, the researchers and engineers use the experimental data in conjunction with various 4SID techniques to form predictors for predictions of the output of the plant. The predictors are then used with the predictive control design to synthesize the so-called *subspace predictive control* (SPC) law[9)]. Although an SPC law is designed for a special class of dual-rate systems where the sampling frequency of the the plant input is M times that of the plant output[10)], it cannot be extended to the whole scope of dual-rate sampled-data systems. Because for such a system, the typical SPC law can be used directly without considering any constraints associated with most dual-rate sampled-data system[13)].

Therefore, we are motivated to perform SPC design for dual-rate systems. To explain why the SPC is used for dual-rate systems, the term 'model free' should be discussed. Model free denote that in stead of traditional explicit plant models, only the predictors obtained by subspace methods are used to predict the output (although model free is somewhat of a misnomer for that the predictors obtained by subspace methods can be considered as a high order plant model, the terminology is retained corresponding to previous literature). On the other hand, Dual-rate systems are a class of periodically time-varying (PTV) systems[11)], they are difficult to quantify by analytic modeling from first principles. Hereby, control problems with dual-rate sampled-data systems can receive the most benefit from model free attributes of the SPC.

It is well known that the SPC is only available to single-rate systems. However, as a class of PTV sys-

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tems, an linear time-invariant(LTI) single-rate isomorphism can be derived for general dual-rate system by lifting technique. The isomorphism is the so-called *lifted model*. Unfortunately, typical 4SID type algorithms cannot handle the causality constraints raised by the lifting technique. Therefore, a modified 4SID algorithm is proposed to identify the lifted state-space models from dual-rate sampled data. Based on the identified lifted system matrices, predictors are established to predict the dual-rate system output. With the predictors, the predictive control design is performed for general dual-rate systems.

The paper is organized as follows. Section 2 provides the problem formulation. The modified 4SID algorithm is provided in section 3. The predictive control design is provided in section 4. Section 5 provides numerical examples. Conclusions are stated in section 6.

2. Problem Statement

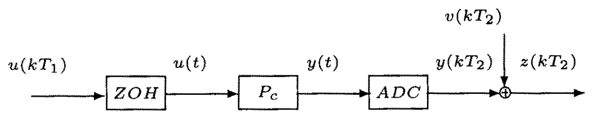

Fig. 1 A general dual-rate system.

Consider a general dual-rate system depicted in **Fig.1**. Here $u(kT_1)$ is the input; ZOH is the zero order holder with period T_1; P_c represents a linear time-invariant(LTI) continuous-time process with the following state-space representation:

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c u(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^r$, $y(t) \in R^m$, A_c, B_c, C and D are the matrices of appropriate dimensions; ADC represents a sampler with period T_2; The output $y(t)$ is sampled by a sampler ADC with period T_2; The sampled output is corrupted by a stochastic measurement noise $v(kT_2)$ which will be specified later; The measurement of the output is denoted by $z(kT_2)$.

All the samplers and zero order holders are synchronized at time $t = 0$. For the general dual-rate system, without loss of generality, it is assumed that the sampling periods satisfy $T_1 = pT_b$ and $T_2 = qT_b$(p and q are coprime integers), where $T_b \in R$ and $T_f = pqT_b$ are respectively the base period and the frame period[11].

It is well known that most typical identification algorithms and control laws are confined to single-rate sampled-data system. However, to the $m \times r$ dual-rate system in **Fig.1**, one can associate a $pm \times qr$ LTI system with the frame period T_f. And, such an LTI system defines the dual-rate system. Moreover, the LTI system preserves the algebraic and analytic properties of the dual-rate system. This is the idea of *lifting technique*[11]. The LTI system in the frame period T_f is called lifted system.

In order to obtain the lifted system in the frame period T_f, the dual-rate system input-output data are lifted. The lifted system input data can be obtained by the q-fold lifting operator L_q[11]. L_q maps $u(kT_1)$ to $\underline{u}(kT_f)$ (underline denotes lifting) as

$$\underline{u}(kT_f) = L_q u(kT_1) = \begin{bmatrix} u(kT_f) \\ u(kT_f + T_1) \\ \vdots \\ u(kT_f + (q-1)T_1) \end{bmatrix} \tag{2}$$

where $\underline{u}(kT_f) \in R^{rq}$. And L_q^{-1}[11] maps $\underline{u}$ back to u. The lifting operator and the inverse lifting operator have the identities[11]

$$L_q^{-1} L_q = I, \; L_q L_q^{-1} = I$$

Similarly, the real output, output measurement and noise of the lifted system are given as follows

$$\underline{y}(kT_f) = L_p y(kT_2) = \begin{bmatrix} y(kT_f) \\ y(kT_f + T_2) \\ \vdots \\ y(kT_f + (p-1)T_2) \end{bmatrix} \tag{3}$$

$$\underline{z}(kT_f) = L_p z(kT_2) = \begin{bmatrix} z(kT_f) \\ z(kT_f + T_2) \\ \vdots \\ z(kT_f + (p-1)T_2) \end{bmatrix} \tag{4}$$

$$\underline{v}(kT_f) = L_p v(kT_2) = \begin{bmatrix} v(kT_f) \\ v(kT_f + T_2) \\ \vdots \\ v(kT_f + (p-1)T_2) \end{bmatrix} \tag{5}$$

where $\underline{y}(kT_f) \in R^{mp}$, $\underline{z}(kT_f) \in R^{mp}$ and $\underline{v}(kT_f) \in R^{mp}$.

It is obvious that the system in **Fig.2** is equivalent to the dual-rate system in **Fig.1**. Then the part in the rectangle is the so-called *lifted system*.

Suppose the state-space representation of the lifted system is as follows

$$\begin{aligned} x((k+1)T_f) &= A_l x(kT_f) + B_l \underline{u}(kT_f) \\ \underline{y}(kT_f) &= C_l x(kT_f) + D_l \underline{u}(kT_f) \end{aligned} \tag{6}$$

where $A_l \in R^{n \times n}$, $B_l \in R^{n \times rq}$, $C_l \in R^{mp \times n}$ and $D_l \in R^{mp \times rq}$. Replacing the lifted output $\underline{y}(kT_f)$

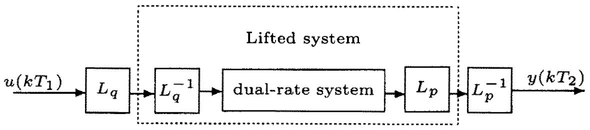

Fig. 2 Lifted system.

by the lifted noise corrupted output measurement $\underline{z}(kT_f)$ and omitting T_f, we have

$$\begin{aligned} x(k+1) &= A_l x(k) + B_l \underline{u}(k) \\ \underline{z}(k) &= C_l x(k) + D_l \underline{u}(k) + \underline{v}(k) \end{aligned} \tag{7}$$

Note that (A_l, B_l, C_l, D_l) in (6) can be derived from the continuous-time state-space model (1)[11].

Remark 1 *Notice that the output element $y(kT_f + (a-1)T_2)$ $(a = 1, 2, \dots, p)$ in $\underline{y}(kT_f)$ depends on the input element $u(kT_f + (b-1)T_1)$ $(a = 1, 2, \dots, q)$ in $\underline{u}(kT_f)$ when $kT_f + (a-1)T_2 \geq kT_f + (b-1)T_1$. This is the so-called causality constraints of the lifted state-space model. To ensure the causality constraints, the feedthrought term D_l should be a block lower triangular structure*[11].

Because of the causality constraints, the typical 4SID type algorithm cannot be used to identify the lifted state-space model (6) directly.

For a dual-rate system in **Fig.1**, the objective of this paper is two-fold:

- Identify the lifted state-space model (6) by using a modified 4SID algorithm.
- Based on the identified lifted model, establish predictors which can predict the output of the dual-rate system in **Fig.1**. Then by using the predictors, perform the predictive control design for the dual-rate system in **Fig.1**.

3. Identification of Lifted State-space Models

In this section, a modified 4SID algorithm is proposed to identify the lifted state-space model (6). In order to identify the lifted state-space model (6), we make the following assumption

Assumption 1 *1. The eigenvalues of A_l are strictly inside the unit circle.*

2. The lifted noise vector $\underline{v}(k)$ is a stationary, zero mean white noise.

3. The lifted input vector $\underline{u}(k)$ and the lifted noise vector $\underline{v}(j)$ are uncorrelated for $\forall k$ and $\forall j$.

4. The input signal is quasi-stationary[6] *and is persistently exciting of order $2i$, where i is both future and past horizons to be defined later.*

5. The equation (6) is observable and controllable.

Like typical 4SID type algorithms, the identification algorithm starts from defining the input and output block Hankel as follows

$$U_{0|i-1} := \begin{bmatrix} \underline{u}(0) & \underline{u}(1) & \cdots & \underline{u}(j-1) \\ \underline{u}(1) & \underline{u}(2) & \cdots & \underline{u}(j) \\ \cdots & \cdots & \ddots & \cdots \\ \underline{u}(i-1) & \underline{u}(i) & \cdots & \underline{u}(i+j-1) \end{bmatrix} \tag{8}$$

$$Z_{0|i-1} := \begin{bmatrix} \underline{z}(0) & \underline{z}(1) & \cdots & \underline{z}(j-1) \\ \underline{z}(1) & \underline{z}(2) & \cdots & \underline{z}(j) \\ \cdots & \cdots & \ddots & \cdots \\ \underline{z}(i-1) & \underline{z}(i) & \cdots & \underline{z}(i+j-1) \end{bmatrix} \tag{9}$$

$U_{i|2i-1}$ and $Z_{i|2i-1}$ can be defined in the similar way. i and j are user-defined indexes which are large enough. i should at least be larger than the maximum order of the lifted state-space model, i.e. $i > n$. j is typically equal to $N - 2i + 1$ where N is the data length of all available data samples. In any case, j should be larger than $2i - 1$.

By performing LQ decomposition and singular value decomposition (SVD), the following two state sequcences

$$X_i := \begin{bmatrix} x(i) & x(i+1) & \cdots & x(i+j-1) \end{bmatrix} \tag{10}$$

$$X_{i+1} := \begin{bmatrix} x(i+1) & x(i+2) & \cdots & x(i+j) \end{bmatrix} \tag{11}$$

can be estimated. For the details of the estimation of the state sequences, the readers are refered to the reference [12].

For the typical 4SID type algorithms, once the state sequences are determined, the system matrices can be estimated by solving the following LS problem:

$$\begin{bmatrix} \widehat{X}_{i+1} \\ Z_{i|i} \end{bmatrix} = \begin{bmatrix} A_l & B_l \\ C_l & D_l \end{bmatrix} \begin{bmatrix} \widehat{X}_i \\ U_{i|i} \end{bmatrix} \tag{12}$$

As mentioned in **Remark 1**, D_l should be block lower triangular structure with respect to the causality constraints of the lifted state-space model (6). However, solution of the LS problem (12) cannot ensure that D_l is a block lower triangular matrix. That is, a non-causal lifted state-space model is identified. A non-causal lifted model will lead to the erroneous prediction of output of the dual-rate system[12] in **Fig.1**. Therefore, before solve the LS problem (12), the feedthrough term D_l should be parameterized to be a block lower triangular matrix according to the causality constraints. The following proposition clarifies this problem. See the proof in the reference [13].

Proposition 1 *The causality constraints of the lifted state-space model (6) are ensured if and only* *if the subblock matrices* D_{ab} $(a = 1, 2, \ldots, p;\ b = 1, 2, \ldots, q)$ *in* D_l *satisfy*

$$D_{ab} = 0, \ \textit{for} \ \ (a-1)q < (b-1)p.$$

Remark 2 *For convenience of the readers, we illustrate a simple example here. If* $p = 3$ *and* $q = 5$, D_l *is parameterized as*

$$D_l = \begin{bmatrix} D_{11} & 0 & 0 & 0 & 0 \\ D_{21} & D_{22} & 0 & 0 & 0 \\ D_{31} & D_{32} & D_{33} & D_{34} & 0 \end{bmatrix} \tag{13}$$

Fianlly, the lifted system matrices (A_l, B_l, C_l, D_l) can be estimated by solving the LS problem (12) subject to the block lower triangular structure of D_l.

Taking the causality constraints into consideration, the modified 4SID algorithm for general dual-rate systems is summarized as follows

1. Construct input and output block Hankel matrices.
2. By performing LQ decomposition and SVD decomposition, estimate the state sequences X_i and X_{i+1}.
3. According to **Proposition 1**, the feedthrough term is parameterized as a block lower triangular matrix.
4. Finally, the matrices (A_l, B_l, C_l, D_l) of the lifted state-space model (6) are determined by solving the LS problem (12) subject to the parameterized D_l.

4. Predictive Control Design

The predictive control design strategy is characterized as following:

1. Finding a model to predict the output of the plant under control.
2. Then the control inputs are attained by optimizing a given criterion called cost function in order to keep the plant output as close as possible to the reference trajectory.

Figure 3 shows the predictive control design strategy. In this paper, the predictive control problem for a dual-rate plant is stated as: Given a set of open-loop measurements of the input $u(kT_1)$ and the outnut $y(kT_2)$ of the unknown dual-rate plant in Figure 2.1, find the lifted control input u_c such that the following cost function is minimized over the horizon i:

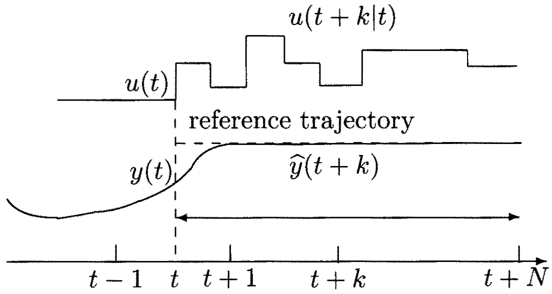

Fig. 3 Predictive Strategy.

$$J = \sum_{k=n}^{n+i-1} \left\{ [\underline{e}(k)^T \underline{e}(k) + \underline{u}(k)^T R_k \underline{u}(k) \right\} \tag{14}$$

where

$$\underline{e}(k) = \underline{y}(k) - \underline{r}(k)$$

matrices $R_k \in R^{pm \times pm}$ is the user-defined weighted matrices of the lifted future incremental input vectors and $\underline{r}(k)$ is attained by lifting the output reference.

4.1 Predictors

In the predictive control design, it is important to predict the output of the plant by using a certain model. In this paper, we propose two predictors which predict the output of a dual-rate plant

$$y_f := \begin{bmatrix} \underline{y}(n) \\ \underline{y}(n+2) \\ \vdots \\ \underline{y}(n+i-1) \end{bmatrix}$$

based on input-output data in the past

$$w_p := \begin{bmatrix} \underline{u}(n-i) \\ \underline{u}(n-i-1) \\ \vdots \\ \underline{u}(n-1) \\ \underline{y}(n-i) \\ \underline{y}(n-i-1) \\ \vdots \\ \underline{y}(n-1) \end{bmatrix}$$

and input data in the future

$$u_f := \begin{bmatrix} \underline{u}(n) \\ \underline{u}(n+2) \\ \vdots \\ \underline{u}(n+i-1) \end{bmatrix}$$

where n denotes the present time. To derive the predictors, the following lemma is introduced:

Lemma 1 *A high order formulation of the lifted state-space model are as following*

$$Y_{0|i} = \Gamma_i X_0 + H_i U_{0|i} \quad (15)$$
$$Y_{i+1|2i} = \Gamma_i X_i + H_i U_{i+1|2i} \quad (16)$$
$$X_i = A^i X_0 + \Delta_i U_{0|i} \quad (17)$$

where

$$\Gamma_i = \begin{bmatrix} C_l \\ C_l A_l \\ \vdots \\ C_l A_l^{i-1} \end{bmatrix} \quad (18)$$

is an extended observability matrix,

$$\Delta_i = \begin{bmatrix} A_l^{i-1} B_l & A_l^{i-2} B_l & \cdots & A_l B_l & B_l \end{bmatrix} \quad (19)$$

is a reverse extended controllability matrix,

$$H_i = \begin{bmatrix} D_l & 0 & 0 & \cdots & 0 \\ C_l B_l & D_l & 0 & \cdots & 0 \\ C_l A_l B_l & C_l B_l & D_l & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_l A_l^{i-2} B_l & C_l A_l^{i-3} B_l & C_l A_l^{i-4} B_l & \cdots & D_l \end{bmatrix} \quad (20)$$

is a block lower triangular Toeplitz matrix.

The three equation in **Lemma 1** directly follows from the lifted state-space equation (6) by iterative substitutions. **Theorem 1** is very useful in insights in the predictors for the output of a dual-rate plant.

Then the predictors which can predict the output of a dual-rate plant is given in the following theorem:

Theorem 1 *There exit two predictors which predict the output of a dual-rate plant based on w_p and u_f as follows*

$$y_f = E_w w_p + E_u u_f \quad (21)$$

where

$$E_w = \Gamma_i \left[\Delta_i - A_l^i \Gamma_i^\dagger H_i \;\; A_l^i \Gamma_i^\dagger \right] \quad (22)$$
$$E_u = H_i \quad (23)$$

Theorem 1 can be proved from Lemma 1. For the details, the readers are refered to reference [13].

4.2 Control input

In the previous content, we discussed the predictors which predict the output of the dual-rate plant under control. By using the predictors, the cost function (14) can be reformulated as

$$J = (E_w w_p + E_u u_f - r)^T (E_w w_p + E_u u_f - r) + u_f^T R u_f \quad (24)$$

where

$$r := \begin{bmatrix} \underline{r}(n) \\ \underline{r}(n+1) \\ \vdots \\ \underline{r}(n+i-1) \end{bmatrix}$$

Then the minimization problem of J can be solved by putting the derivative of J with respect to the input sequence u_f to be zero. Consequently, the control input u_c is determined as:

$$u_c = -\left(E_u^T E_u + R\right)^{-1} E_u^T (E_w w_p - r) \quad (25)$$

Then the predictive control design for a dual-rate plant is summarized as follows:

1. Identify the dual-rate plant under control by using the 4SID algorithm in **Section 3.**
2. According to **Lemma 1**, construct the predictors E_w and E_u from the estimated lifted system matrices.
3. Finally, by using equation (25), the control input is obtained.

5. Numerical Results

Consider a continuous time process P_c in **Fig.1** as the following

$$P(s) = \frac{1}{2s^2 + 3s + 1} \quad (26)$$

and $T_1 = 0.2s, T_2 = 0.3s$. Then we have the frame period $T_f = 0.6$ and the base period $T_b = 0.1$.

So the lifted input vector and the lifted output vector are given as follows

$$\underline{u}(0.6k) = \begin{bmatrix} u(0.6k) \\ u(0.6k+0.2) \\ u(0.6k+0.4) \end{bmatrix} \quad (27)$$

$$\underline{y}(0.6k) = \begin{bmatrix} y(0.6k) \\ y(0.6k+0.3) \end{bmatrix} \quad (28)$$

First, the lifted state-space model for such a dual-rate system is identified. The input $u(kT_1)$ is a zero mean white signal (variance 1). $v(kT_2)$ is a zero mean white noise sequence. To show the performance of the N4SID method in the presence of considerable noise, the N4SID algorithm ($i = 15, j = 5000$) was implemented for 20 realizations of the measurement noise of NSR (noise to signal ratio)=20%. NSR was defined as the ratio of σ_v/σ_y, where σ_v and σ_y are the standard deviations of the measurement noise and of the noise-free output, respectively. We plot the estimated

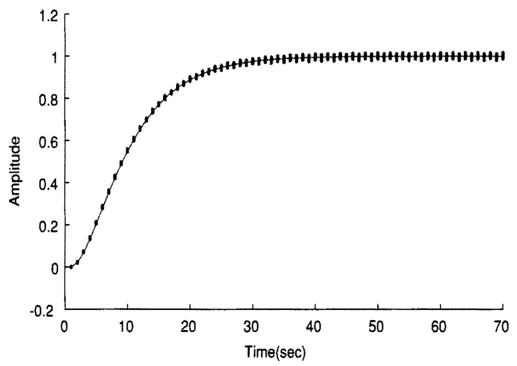

Fig. 4 The step response of the continuous-time process (solid line) and the estimated lifted step response of 20 realizations (dot).

lifted step response of 20 realizations together with the continuous-time step response in **Fig.4**. As expected, the points of the estimated lifted step response sit on that of the continuous-time process.

Then based on the identified lifted system matrices, we perform the predictive control for the dual-rate plant by using the proposed predictive control design method. **Figure 5** shows the control performance. The numerical result indicates that the plant output tracks the reference very well by using the propose predictive control design method.

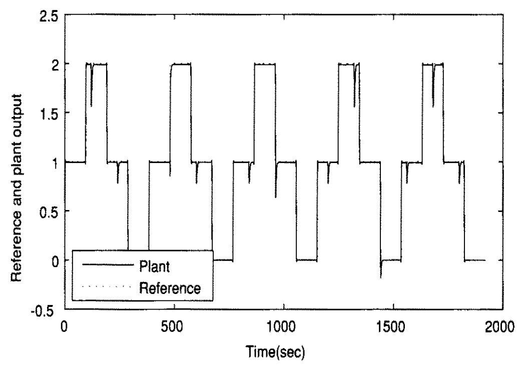

Fig. 5 Predictive control performance.

6. Conclusions

In this paper, we propose a modified 4SID algorithm to identify the lifted state-space models for general dual-rate systems. The numerical results indicates that the identified state-space models can capture the dynamics of the dual-rate systems under study very well. Then we establish two predictors from the identified lifted system matrices and perform predictive control design for the general dual-rate systems. To the best of our knowledge, it is the first time to establish predictors from the system matrices to perform predictive control design in the literature. The numerical result indicates that the output of the dual-rate plant under control tracks the reference quite well.

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