# Analysis of Profits/Price Chaoticity in Formalizing Collaboration among Agents and its Application to Control of Chaos

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# Analysis of Profits/Price Chaoticity in Formalizing Collaboration among Agents and its Application to Control of Chaos

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## 1 Introduction

Over the past two decades, industries have begun ordering complex assemblies or systems rather than traditional, simple components from their suppliers (called outsourcing) [1]-[3]. While maintaining a firm's brand integrity, industries retain low cost by purchasing labors and resources by using collaboration with outside firms. Researches on tactical outsourcing have mostly been concentrated on the questions related to contract writing such as price setting [1][2]. However, delegating (outsourcing) tasks to outside firms must be always optimal, and the problems to examine the collaboration based on the mathematical models depending on the situations reveal as another important factors [4]-[8]. This paper deals with the analysis of profits/price chaoticity in formalizing collaboration among agents and its application to the control of chaos [9]-[11].

Even though many conventional works related to outsourcing stress the contracting, but inherent in any act of outsourcing is the control loss, and a manufacture suffers from overseeing the delegated process. As a result, it is necessary manufactures frequently observe and update relationships with suppliers, even contracting on functionality and finished products. Especially, research by Caballe et al. shows economies experiencing a process of financial development due to outsourcing are more unstable and chaos appears via a border collision bifurcation [7]. Also, Curries et al. discuss the chaos in the core-peripheral model under high transportation costs where workers move to other regions to improve real wages and purchasing of labor is unstable [8]. However, it is not well known yet whether the unstability is in the competition to avoid the cost for upfront payment and warranty clauses.

At first, we consider a small open economy where a firm agent manufactures goods by using resources provided by outside agents as well as his own capital and labor [7][11]. Then, it is shown that an increase in wealth raises the investment and the wealth time series bears chaoticity. Secondly, the model for single firm and outside agent is extended to multi-agent systems where the relations of collaboration is represented by a partly-connected network having several layers of cites or nodes representing firm and outside agents [9]-[11]. We assume dynamic alternate routing (allocation) of flow to adjust (search) lower cost for labor price $p(t)$ (expected price $\pi(t)$ for $p(t)$ is called as pricing in the following) by firm agents [12]-[16]. A population of agents use linear cost functions depending on the demand rate $\lambda(t)$ of labor supply, and the flow placed on a intermediate node is allocated to the next intermediate nodes inversely proportional to the pricing on the destination nodes. Then the pricing $\pi(t)$ bears chaoticity despites the deterministic scheme of the model, and raises chaoticity in the wealth time series [12]-[17]. It is assumed that the generation of flow to the networks and the waiting time (cost) of nodes are deterministic rather than stochastic. More over different from the system with single node, the flows in network is not balked outside of the network, and still remain in the network [9]-[11]. We show the condition for inducing chaotic fluctuation in pricing based on the simulation studies. Simultaneously, by considering the deterministic decision on input and pricing, we utilize the control (suppression) scheme for the chaotic fluctuation base on the control of chaos [11]-[13][18]-[21]. Chaotic behavior in queueing system is a topic that is just beginning to be addressed in the literature. We also showed several works related to the chaoticity in queueing systems [9][10]. However, our model of the paper differs from both these works in that we emphasize rises of chaoticity induced by outside agents in the collaboration of productions (outsourcing), rather than interested merely in stability of chaos of the dynamic input-output pricing mechanism.

The paper is organized as follows. In Section 2, we show basic model for the analysis of chaoticity observed in collaboration between two single agents. Section 3 treats modeling of extension of collaboration into multi-agent system and the analysis of chaoticity. In Section 4, we show the way of control of chaotic behavior of wealth time series. Section 5 shows the simulation studies for chaoticity reveals in the collaboration of multi-agent system and the control of chaos.

# 2 Chaoticity observed in Collaboration between two single agents

## 2.1 Basic model for collaboration between firm and outside agent

Even though the models treated in the paper are not restricted to production systems, but for simplicity we assume that firm agents manufactures a good by using the equipment (called as capital in the following) and the labor force (called as labor in the following) provided by outside agents. At first, we treat the case where there exits a single firm agent and a single outside agent [7][11]. Since it is not necessary to distinguish outside agents, we simply define a single agent who provide labor for the production. In the definition of basic model for collaboration, we assume followings.

(1) production by firm agent

We consider a small open economy in discrete time, and we denote the time period between two time stamps $t$ and $t+1$ as the period $t$ rather than instantaneous time. Because agents are assumed to process information between two time stamps $t$ and $t+1$ and make decision for the period $t$. A firm agent manufactures goods by using his own capital $K_1(t)$ and labor $L_1(t)$ in time period $t$. Beside $K_1(t)$ and $L_1(t)$, a firm uses the capital $K_2(t)$ and labor $L_2(t)$ provided by outside agent. Under these conditions, the output of products (goods) $y(t)$ is usually represented by the so-called production functions. There are several types of production functions, but we use the production function of the Cobb-Douglus type described as follows.

$$y(t) = A[K_1(t) + K_2(t)]^{\rho}[L_1(t) + L_2(t)]^{1-\rho}. \tag{1}$$

where $A$ (a constant value) denotes the total factor productivity, and $\rho(0 < \rho < 1)$ is the elasticity of production (also a constant value).

(2) purchase of capital and labor

A firm agent obtain the wealth $W(t)$ at the end of production in period $t$, and then besides the profit he makes the total investment $I(t)$ in period $t$ for the production in period $t+1$ which is devoted to purchase both capital and labor from the outside agent. For the given level of investment, the optimal demand for the labor input $p(t)L_2(t)$ and for capital $K_2(t)$ in each period rise from the maximization of profit function subject to the budget constraint.

$$I(t) = K_2(t) + p(t)L_2(t). \tag{2}$$

where $p(t)$ is the market specific labor price for a unit of purchased labor. Now, we assume that the capital $K_1(t)$ and labor $L_1(t)$ prepared by the firm agent are determined at the beginning of whole production, and are not included in the investment in period $t$. We also assume that the capital $K_2(t)$ come from outside is purchase in a long range, and is not affected by the market, even though the capital $K_2(t)$ is still remains as variable to be determined to in the profit maximization.

(3) optimal production

Then, the maximization of $y(t)$ in equation (1) is reduced to the problem to determine the capital $K_2(t)$ and the labor $L_2(t)$. By substituting the equation (2) into equation (1) and taking the derivatives with respect to the variable $L_2(t)$, we have the next relation.

$$L_2(t) = [B + (1-\rho)I(t)]/p(t), K_2(t) = -B + I(t). \tag{3}$$

$$B = K_1(t)(1-\rho) - \rho p(t)L_1(t). \tag{4}$$

By substituting the value of $L_2(t)$ into the equation (1), we obtain the vale for $K_2(t)$ which maximizes the function $y(t)$, but the expression is omitted here. It must be noted that the optimal value of the variable $L_2(t)$ includes the price $p(t)$, and depends on it. However, for simplicity we assume that the price $p(t)$ is known as a prescribed value, even though in the latter sections of the paper dealing with multiple-agents systems we change the assumption into fluctuating $p(t)$.

By using the optimal solution to maximize $y(t)$, we have the expression for the optimal value of production as follows.

$$y(t) = A[K_1(t) - B + \rho I(t)]^{\rho}[D + E\rho I(t)]^{1-\rho}, \tag{5}$$

$$C=\rho(1+\alpha), D=L_1(t) + B/p(t), E=(1-\rho)/p(t). \tag{6}$$

We use another expression for the variable $I(t)$, and then the $y(t)$ will be further transformed into another form in the succeeding discussions.

(4) time dependency of profit and credits

Let $W(t)$ be the wealth of firm agent in period $t$. Since the wealth serves as a collateral for the loan with amount $L(t)$ where $L(t) \leq \alpha W(t)$ and $\mu$ is the credit multiplier reflecting the level of financial development of the domestic economy. Then, the firm agent can invest in the productive project where the largest amount he can borrow is given as.

$$I(t) = W(t) + \alpha W(t) = (1+\alpha)W(t), \tag{7}$$

where $r$ is the interest rate. Hence, in period $t+1$ the firm agent receives the corresponding profits and pay the cost of debt $rL(t)$. Therefore, the dynamics of the wealth of firm is given by

$$W(t+1) = q[y(t) - r\alpha W(t)], \tag{8}$$

where $q$ is the rate of profit with respect to the gross sales. By substituting equation (8) into the above equation, and also substituting $y(t)$ into equation (8), we have the following relation.

$$\begin{aligned} W(t+1) \quad &= \quad [qA[K_1(t) - B + CW(t)]^{\rho} \\ &\qquad [D + E'W(t)]^{1-\rho} - r\alpha W(t)]. \end{aligned} \tag{9}$$

$$E' = (1-\rho)(1+\alpha)/p(t). \tag{10}$$

Even though we use the above equations to evaluate the change of profit between periods $t$ and $t+1$, but if the interest rate $r$ is greater than the rate of profit obtained by the production , the firm agent feels no incentive to borrow up to the credit limit and use his own current wealth $W(t)$ for production. Then, the firm agent select productive activity rather than credit. These situations occur if the specific value $W^m$ of $W$ satisfies the relation $y(t) - r\alpha W(t) = rW(t)$. Then we obtain the value $W^m$ which satisfies,

$$q[A[K_1 - B + CW(t)]^{\rho}[D + E'W(t)]^{1-\rho} - r\alpha W(t)] = rW(t). \tag{11}$$

This is so because, even if an increase in wealth raises the investment, the amount of invested wealth depends negatively on the price $p(t)$.

By using the value $W^m$, including the the case where $W(t) \leq W^m$ is satisfied, the asymptotic behavior of wealth is thus determined by the iterate of the following functions.

$$W(t+1) = \begin{cases} q[y(t) - r\alpha W(t)]; 0 \leq W(t) \leq W^m; \\ rW(t); W(t) > W^m \end{cases} \tag{12}$$

## 2.2 Chaoticity result for $W(t)$

In the following, we show the chaoticity result for the wealth time series $W(t)$. Since the functional form for $W(t)$ is complicated, it is hard to show analytically the chaoticity for $W(t)$, and then we use the bifurcation diagram for $W(t)$ depending on the parameter $\alpha$ based on simulation studies.

Fig.1 shows an example for the time series $W(t)$ with $\rho = 1/3, A = 1.5, L_1 = K_1 = 100, r = 1.02, \alpha = 58$. Fig.2 shows the bifurcation diagram for $W(t)$ depending on the parameter $\alpha$. We also show the maximum Liapunov exponent $L_P$ for $W(t)$ with the same condition where the embedded dimension is two and the time delay is one. As expected, the wealth time series $W(t)$ converges to a single point, namely, the stable equilibrium, when the parameter $\alpha$ is less than $\alpha_B = 10$. A periodic-doubling bifurcation of the equilibrium occurs as $\alpha$ is increases over $\alpha_B$. For $\alpha$ just over $\alpha_B$, the time series alternates between a stable two-cycle. Then, we see the two cycle splits into a four-cycle, which then turn into the band of Lee-Yorke chaos when $\alpha > 56$. Further we see the large window of the stable three cycle when $\alpha > 61$.

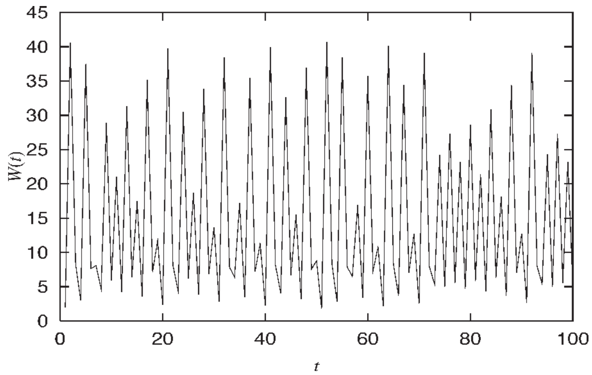


Figure 1: An example of time series $W(t)$ ($\rho = 1/3, A = 1.5, L_1 = K_1 = 100, r = 1.02, \alpha = 58$).

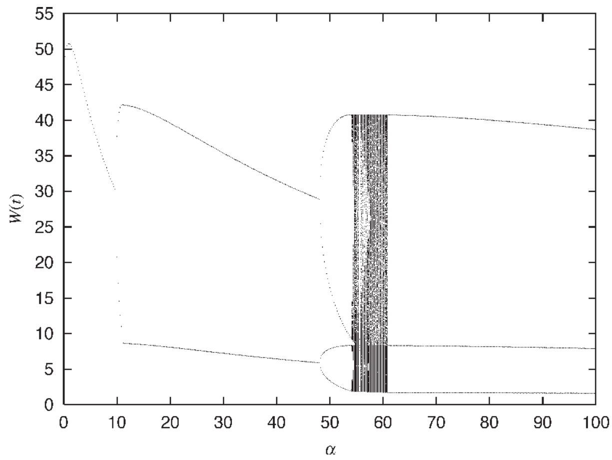


Figure 2: Bifurcation diagram for $W(t)$ depending on $\alpha$ ($\rho = 1/3, A = 1.5, L_1 = K_1 = 100, r = 1.02$).

Table 1: Maximum Liapunov exponent for $W(t)$ depending on $\alpha$ ($\rho = 1/3, A = 1.5, L_1 = K_1 = 100, r = 1.02$).

| $\alpha$ | 5 | 10 | 20 | 50 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|---|
| $L_P$ | -0.21 | -0.32 | -0.12 | 0.29 | 0.01 | -0.01 | -0.01 |

## 2.3 Chaoticity induced by price $p(t)$

Even though the wealth time series $W(t)$ is stable and has a single value if $\alpha < \alpha_B, \alpha_B = 10$ and with constant value of price $p(t)$, but $W(t)$ becomes to be chaotic if $p(t)$ is chaotic. Sources of chaoticity for $p(t)$ is discussed in Appendix A, and we only show an example of rise of chaoticity in $W(t)$. Fig.3 show an example of wealth time series $W(t)$ with $\alpha = 5$ before and after time point $t_A$ after which the time series $p(t)$ bears chaoticity. As is seen form Fig.3, the time series $W(t)$ becomes no more stable after $t_A$. Then, we must note that chaoticity of $W(t)$ is induced by increasing $\alpha$ greater than $\alpha_B$, as well as the rise in chaoticity in $p(t)$.

The issue of whether the time series $W(t)$ keeps stability is closely related to the labor price $p(t)$. Most of paper dealing with the outsourcing stress the contract writing and therefore fixed and long term price setting[1][2]. However, we can observe in real world, many firms change purchasing scheme in production system from ordering assemblies (outsourcing) again back to their own manufacturing (insourcing). Most of the reasons to change the purchasing scheme are related to the higher labor price $p(t)$ provided by outside agents. Therefore, we must also observe how the change of $p(t)$ affects the stability of $W(t)$. Even though, in the latter half of the paper we use the result our paper already appeared, but we emphasize the chaoticity of $W(t)$ induced by $p(t)$ [9][11].

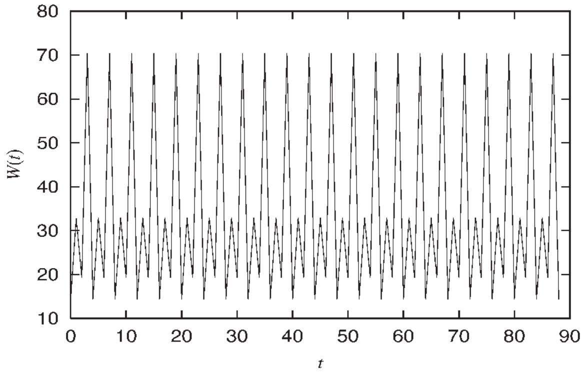


Figure 3: An example of rise of chaoticity in $W(t)$ with $\alpha = 5$ due to chaoticity in $p(t)$ ($\rho = 1/3, A = 1.5, L_1 = K_1 = 100, r = 1.02$).

# 3 Chaoticity analysis in collaboration among multiple agents

## 3.1 Collaboration network among multiple agents

As is seen, in the collaboration between two agents if the parameter $\alpha$ is restricted to be small such as $\alpha < \alpha_B$, the wealth time series $W(t)$ is stable. However, the stability is attained on the basis of stationarity (constant value) of labor price $p(t)$ throughout manufacturing. Note that particular cases may occur where the labor price is no more stable and determined by the competitive edge among firm and outside agents. If we assume there exist multiple firm agents and outside agents, then firms agent tend to search cheaper labor price across outside agents. Moreover, if firm agents seek cheaper labor price, then outside agents react to purchasing by changing (decreasing) the labor price. However, the demand from firm agents may be concentrated on a certain outside agent, then the labor price will be raised to adjust the imbalance in the supply-demand framework. Then, the optimal labor price is selected and updated through a competitive manner.

In the following, we extend the basic model into collaboration among multiple agents by the network structure to represent revealed preferences of firm agents. For simplicity, we assume that the relation (purchasing) among firm agents and outside agents is represented by two types of network.
Case S: Single layer
Case M: Multiple layers
(1) two types of network

We assume agents are represented by nodes in networks, and a layer consists of several nodes. Nodes in a single layer are partly connected to another nodes in the succeeding layers, however at a random fashion. Firm agents are placed on the input layer (consisting $N_{in}$ input nodes), and intermediate layers except for the input layer are composed of outside agents. Then, the flow generated at input layer (nodes) in network corresponds to the purchase of labor by firm agents bought from outside agents, and the flows leave the networks via output nodes (consisting of $N_{out}$ output nodes). This gives rise to an intricate hierarchy, but we derive it to following two types for connection of intermediate layers.
Case S: Single layer

There is only a single intermediate layer of nodes for outside agents besides input layer

corresponding to firm agents. Then, the intermediate nodes are at the same time output ones.
Case M: Multiple layers

The network is allowed to include multiple layers of nodes consisting of outside agents. As described by a schematic diagram, the flow is delivered from the left hand side of the network, and then is transmitted to another intermediate node from left to right, and finally leave the network from the left end (output nodes) of the network. In Case M, the intermediate nodes representing also outside agents play only for transmitting flows to the succeeding nodes and generate no flows.

(2) allocation of flow to next intermediate nodes

We assume dynamic alternate routing (allocation) of flow to adjust (search) lower cost for labor price $p(t)$ (expectation $\pi(t)$ for $p(t)$ is called pricing) by firm agents. According to the state of destination nodes (outside agents), flows are routed (re-allocated) in order to limit the excess labor cost due to higher labor prices proposed by outside agents. Then, the flow placed on a intermediate node is allocated to the next intermediate nodes (usually, we find multiple next intermediate nodes) inversely proportional to the pricing on the destination nodes.

(3) pricing

We consider a population of agents use linear cost functions depending on the demand rate $\lambda(t)$ of labor supply. After estimating the labor price expectation $\pi(t)$, firm agents send the message to purchase the labor from the outside agents. Firm agents allocate the flow (demands for labor supply) by considering pricing $\pi(t)$. Generally, the labor price $p(t)$ and its expectation $\pi(t)$ are not the same, then pricing $\pi(t)$ bears chaoticity despites the deterministic scheme of the model.

## 3.2 Chaoticity analysis in input pricing with adaptive customers

Prior to the discussion of chaoticity analysis of pricing time series in nodes in network structured agent system, we consider the dynamic behavior of an input-pricing mechanism for a service facility (a single node) in which heterogeneous self-optimizing customers base their future join/balk decisions on their previous experiences of pricing (or congestion) [12]-[15]. The model is derived from studies of stability of the equilibrium in a service facility. To formalize and analyze stability, the multiperiod model and the mean waiting time are introduced. A customer has an incentive to join (balk) the system if its service cost is lower (higher) than the admission price, namely, the sum of an admission fee and the expected delay cost. We assume it is impossible for customers to observe the current level of congestion (or next pricing) before deciding whether or not to join. The expected delay cost can therefore only be estimated based on the system parameters and perhaps on previous experiences.

Followings are assumed in our model [12]-[15].

(1) multiperiod model

We consider a service facility operating over a finite time interval from time $t$ to time $t+1$, which we refer to as the period. Conventional studies related to equilibrium (stability) analysis of service facilities with a fixed service rate introduce a type of multiperiod model. A period lasts long enough for the system to attain steady state. In the model each period consists of a time interval during which the arrival rate and therefore the price remain constant and the steady state is achieved throughout the period. Then, in the description of the variables with period $t$ in the following discussion such as $\lambda(t)$ mean the values of $\lambda(t)$ in period $t$.

(2) deterministic model for waiting

We prefer to describe the system in general terms keeping stochastic assumption at a minimum. Stability means (roughly) that the price and arrival rate tend to return to the equilibrium after a perturbation. With a linear delay cost, the expected delay cost in this setting is proportional to the expected steady-state waiting time in the system. Then, the customers base their join/balk decision in each period on the delay cost experienced in the previous period. Therefore, different form conventional stochastic model in the service facilities, we use the deterministic model for the analysis of equilibrium and stability.

These conditions are available to figure out the chaotic behavior of input pricing for a service facility, but at the same time models based on theses assumption may not cover the rest field of underlying problems. However, it is hard to include various assumptions having diversity into the same models, and we must be satisfied with the result under restricted assumptions.

For a given arrival rate $\lambda$ of customer, we allow a delay cost $W_\mu(\lambda)$, which is expected waiting time and $\mu$ is the measure of service capacity such as the service rate of a single server queue. Define the admission price $p$ by

$$p(t) = f + W_\mu(\lambda(t)). \tag{13}$$

where $f$ is a fixed admission fee.

A potential arriving customer seeking to maximize its net benefit, has an incentive to join the system if its service cost does not exceed the allowable price. Customers cannot observe the congestion in the system before deciding whether to join. Hence, they do not know the admission price, and use predicted price $\pi(t)$.

Let $\lambda(t)$ denote the arrival rate during period $t$ which induces a price $p(t) = f + W_\mu(\lambda(t))$. Observing the price $p(t)$, the customer then collectively form a prediction price $\pi(t+1)$ for the next period by the following exponentially smoothing equation.

$$\pi(t+1) = (1-\omega)\pi(t) + \omega p(t), \tag{14}$$

where $0 < \omega < 1$. In our framework for discussing collaboration among agents, even though $p(t)(\pi(t))$ is real (estimated) values for labor price in the period $t$, we substitute $\pi(t)$ into the time series $p(t)$ for evaluating the wealth time series $W(t)$.

Since each successive forecast $\pi(t)$ seeks to predict the price $p(t+1)$, we can view this dynamic pricing process as an equilibrium-seeking pricing algorithm governed by the first order nonlinear differential equation.

$$p(t) = f + W_\mu(\Lambda\bar{F}(\pi(t)), \tag{15}$$

$$\lambda = \Lambda\bar{F}(\pi) = \begin{cases} \Lambda \quad (0 \leq \pi \leq d); \\ \Lambda(a-\pi)/(a-d) \quad (d \leq \pi \leq a); \\ 0 \quad (\pi \geq a) \end{cases}, \tag{16}$$

where $\Lambda$ is the mean arrival rate of potential arrivals come from a Poisson process. For a price expectation $pi$, therefore customers enter the system according to a Poisson process with mean rate $\lambda$. In our framework, firm agents have an incentive to join the system (use the outside agent) if their service value exceed the admission price, otherwise they balk from the system (do not use the outside agent).

As expected, the price converges to a single point (the stable equilibrium) when the capacity $\mu$ is at least $\mu_+$ (large enough value of $\mu$ and is close to 1). A periodic-doubling bifurcation of the equilibrium occurs as $\mu$ is decreases below $\mu_+$. In the region, the prices alternate between a stable two-cycle, and then we see the two-cycle split into a four-cycle if the capacity is decreased further. Then, the band of Li-Yorke chaos begins as the service rate parameter is decreased and continues until $\mu$ drops below $\mu_-$.

The condition for $\mu$ inducing chaoticity in time series $p(t)$ is given by Stidam [16][17]. If the capacity $\mu$ is limited in the range $\mu_- \leq \mu \leq \mu_+$, then the time series $p(t)$ reveals as a chaotic time series in the Li-Yorke sense. The values $\mu_-, \mu_+$ giving boundary values are given by the two solution of the following second-order equation.

$$\mu^2 - [\frac{\Lambda a}{a-d} + \kappa]\mu + \beta' \frac{\Lambda}{a-d} = 0, \tag{17}$$

$$\kappa = -2(1-\omega)[\frac{(1-\beta')\Lambda}{\omega(2-\omega)(a-d)}]^{1/2}. \tag{18}$$

The reduction of the formula is shown in Appendix A. The function $W_\mu(\lambda)$ is usually represented by the formula of average waiting time in M/G/1 of the queueing theory (symbols correspond as M: Markovian arrival, G: General servicibility, 1: single server). The function $W_\mu(\lambda)$ is given by the following equation.

$$W_\mu(\lambda) = (\mu - \lambda\beta')[\mu(\mu-\lambda)]^{-1}, \beta' = (1-c_s^2)/2. \tag{19}$$

where $c_s$ is the variance coefficients of service time, and $mu$ is the capacity of the system (such as service rate). Even though, original queueing models include stochastic behaviors, but as earlier mentioned, we use only the average characteristics of queuing models.

## 3.3 Relations among flows at nodes

We now study the asymptotic behavior of the dynamical system about the fixed-point equilibrium. In particular, we seek to develop conditions for equilibrium stability, namely, asymptotic convergence of the algorithm to the unique equilibrium. We shall focus attentions on the steady-state properties of nodes in the network structured collaboration among agents.

In the following, to simplify the expressions, we omit the subscript $t$ meaning the period from variables. We denote several factors for the model as.

$\lambda^{(i)}$: rate of input flow to node $i$ (same as outgoing flow through node $i$).

$\mu^{(i)}$: marginal capacity $\mu$ for node $i$ to keep the equilibrium

Assume that $\lambda^{(j)}$ is the total flow going from the node $j$. To obtain the relation for the distribution of flow from a certain node $j$, we define the set of nodes denoted $i$ as $D(j)$ $(i \in D(j))$ located at the downstream of the node $j$. Then, the flow $\lambda^{(ji)}$ is allocated from the flow $\lambda^{(j)}$ to the node $i$, and the part of flow $\lambda^{(ij)}$ reach to node $i$. Namely,

$$\lambda^{(j)} = \sum_{i \in D(j)} \lambda^{(ji)}, j \in U(i). \tag{20}$$

$$\lambda^{(ji)} = \frac{F(\pi^{(i)})}{\sum_{k \in D(j)} F(\pi^{(k)})} \lambda^{(j)}, F(\pi^{(i)}) = (a - \pi^{(i)})/(a-d). \tag{21}$$

where the symbol $j \in U(i)$ means the set $j$ of nodes are placed at the upstream of node $i$. Here, we see that the flow delivered to the downstream nodes is allocated in reversely proportional to the value $\lambda^{(ji)} = F(\pi^{(i)})$ in equation (21). On the other hand, the upstream node $j$ seen from the node $i$ may be included in the range as $j \in U(i)$. Then, by summing up the flow from the upstream nodes at node $i$ and denoting it as $\lambda^{(i)}$, we have.

$$\lambda^{(i)} = \sum_{j \in U(i)} \lambda^{(ji)}. \tag{22}$$

If we assume flow is kept in the state of equilibrium at each node, it is easily found that the equation $\pi^{(i)} = p^{(i)}$ must be hold, add then we have.

$$a - (a-d)\lambda^{(i)} = (\mu^{(i)} - \beta'\lambda^{(i)})[\mu^{(i)}(\mu^{(i)} - \lambda^{(i)})]^{-1}. \tag{23}$$

Moreover, the condition under which the equilibrium is still kept and continued is given as.

$$\lambda^{(i)} = \mu^{(i)} - [\frac{(1-\beta')\omega}{(a-d)(2-\omega)}]^{1/2}. \tag{24}$$

These two relation are reduced to following single relation.

$$[\mu^{(i)}]^2 - [\frac{a-f}{a-d} + \kappa^{(i)}]\mu^{(i)} + \beta' \frac{1}{a-d} = 0. \tag{25}$$

$$\kappa^{(i)} = -2(1-\omega)[\frac{(1-\beta')}{\omega(2-\omega)(a-d)}]^{1/2}. \tag{26}$$

These equation are also used for the discussion of control of chaos, then we denote the equations as the FER (Flow-Equilibrium Relations) representing the relations among flows.

As is seen from above equations, we obtain $2N$ equations to be solved and at the same time we have two unknown variables $\mu^{(i)}, \lambda^{(i)}$ for each node and are aggregated into $2N$ variables. Therefore, the $2N$ nonlinear simultaneous equations show us the marginal values of capacity with which the chaoticity of labor price is induced.

Similar to the single node case, we obtain two marginal value of capacity such as $\mu_-^{(i)}, \mu_+^{(i)}$. By solving the simultaneous equations and obtaining the solutions for the marginal values of capacity. Then, the maximum (minimum) values among $\mu_-^{(i)}$ (among $\mu_+^{(i)}$) is defines as $\mu_+$ (as $\mu_+$), then the condition for the chaoticity for the labor price time series is given by $\mu_- \leq \mu \leq \mu_+$.

# 4 Control of chaos by imposing external force

## 4.1 Available region for $\alpha$ to control of chaos

As we discussed in previous sections, it is seen there are two regions for the parameter $\alpha$ related to the induction of chaoticity. If the parameter $\alpha$ is in sufficiently large (in the first region), for example, $\alpha > \alpha_B, \alpha_B = 10$ in Fig.1 and 2, the wealth time series $W(t)$ bears complicated chaoticity, and the control of chaos is seemed to be very difficult. On the other hand, in the second region where the parameter $\alpha$ is smaller than a threshold value, for example, $\alpha \leq \alpha_B$ in Fig.1 and 2, the wealth time series is stable and takes a single value.

However, when we assume network structure between the firm agents and the outside agents, there exists chaoticity found in the wealth $W(t)$ even in the first region where the parameter $\alpha$ is relatively small ($\alpha \leq \alpha_B$), and originally we observe no chaoticity in the wealth process. Because, as we shown due to the chaoticity induced by the completion among firms, the labor prices provided by the outside agents bear chaoticity. The fact suggests us that even in the usual manufacturing situations where firm agents utilize relatively smaller debt and avoid excess credit, the wealth times series $W(t)$ still bears chaoticity.

Then, we focus on the region where $\alpha$ is relatively small, but firm agents seek more cheaper labor price and the chaotic fluctuation of the labor price $p(t)$ also induce the chaoticity of wealth time series $W(t)$. However, we must also note that solution of FER is still available to find equilibrium (fixed point), because we can use the equations (20) through (26) by setting $\mu^{(i)} = \mu$. Then, the control scheme of the paper emphasizes only making small time-dependent perturbations so that the system dynamics is lead to the fixed point.

## 4.2 Estimation of dynamics by the GP

The presence of chaos in physical system has been demonstrated, however, it is often desired that chaos be avoided without costly alternation in the system. The well-known OGY method is based on the key observation that chaotic attractor typically embedded within an infinite number of unstable periodic orbits, and one obtain improved performance by making only small time-dependent perturbations in an accessible system parameters [19]. Desired perturbations are by a linear approximation for the map based on the embedding techniques. Several extensions of the OGY method are now available such as delayed feedback method proposed by Pyragas and related improvements [20][21].

The scheme for controlling chaos of the paper is, however, much simpler than conventional methods [12][13][18][19][22][23]. We imagine that the dynamical equations describing the behavior of agents are not known, but the numerical estimation (approximation) of the dynamics by using the GP is not difficult. Since the agents' behavior is simply described by the function in equation (19) representing the mean pricing function, and the control of input flow in equation (16) has piece-wise linear characteristics, the task to direct estimate (approximate) these equation is not complicated. Moreover, we assume that all agents decide their labor prices according to the same equations, and we only need to estimate two equations (16) and (19). As we already show, the pricing mechanism employed by the outside agents can be lead to equilibrium (fixed point) by solving the FER, even though the bifurcation process of pricing is somewhat complicated. Once the dynamical equations describing agents' behavior are estimated, it is not difficult to lead the system to a stable fixed point by changing the state of agents.

For simplicity, we omit here the details of the GP method for estimating (approximating) chaotic dynamics from the observations with sufficiently small approximation errors [12][13][18][19][22][23]. We apply the same control scheme for chaotic fluctuation used in the control of pricing, and then we assume that the dynamics of the system is estimated and already known in the following discussion.

## 4.3 Control of chaos for known system

Prior to the discussion of control of chaotic fluctuation of pricing in network structured system, we argue the control of chaos for system where the dynamics of the system is known [10]-[14][19]-[21].

Assuming that the dynamics of the system inducing chaotic fluctuation is described as the function using the state variable $x(t)$ as follows (in discretized equation).

$$x(t+1) = f_c(x(t)). \tag{27}$$

Then, we impose a small external force to the current state variable $x(t)$ (to distinguish the term "input" to the system, we use term "external force") so that the next state $x(t+1)$ at time $t+1$ is identical to the fixed point $x_f$.

$$\hat{x}(t+1) = f_c(x(t) + u(t)). \tag{28}$$

The scheme is similar to the OGY method, however, at this moment we know the dynamics $f_c(.)$ in a functional form obtained by using the GP. Under the chaotic fluctuation, the state variable is usually placed on the attractors which are very close to the fixed points or limit cycles, then the orbit of the state variable is easily transmitted to the stable orbit by imposing small external force.

### 4.4 Control of chaos in network structure

Different from ordinary dynamical systems, in the paper the dynamics (agents' behaviors) is represented as a set of equations through the network. Hence, the scheme of controlling chaos using known dynamics is still modified to adopt the control in the network. The applicable control scheme is restricted for the system of the paper where the multiple nodes are connected in a network structure, and the input flow does not balk from the system traveling via several nodes. Then, we can find following restriction to the control scheme through many simulation studies.

(1) simultaneous start to impose external force to each node

Since the behaviors of agents are related each other, then the orbit will leave the strip and continue wander chaotically if there was any node of the network without control. It is seen that uncontrolled chaoticity in any node will propagate through the network, then controlled chaotic fluctuation attained by imposed external force will again return to chaos. Hence, it is necessary that we simultaneously start to impose external force to each node.

(2) multiple iterations of imposing external force rather than once

Due to similar reason as the propagation of chaos through networks, a single shot (procedure) of control to impose external force can not suppress the chaotic fluctuation, because the chaoticity remained in another nodes are delivered to the underlying node through the network. Therefore, it is generally necessary to impose multiple iterations (for example, five iterations) of imposing external force to suppress the chaotic fluctuation.

(3) making perturbation in pricing by leading to fixed point

Based on the key idea that a chaotic attractor has embedded within a unstable orbit, then we seek to exploit the already known fixed point obtained by solving a set of equations using FER for attaining equilibrium in the system. Assuming that the capacity for each node is the same as $\mu$.

$$\mu^{(i)} = \mu. \tag{29}$$

Then, the FER includes $N$ unknown variables $\lambda^{(i)}$, since the unknown variables $\mu^{(i)}$ are already given. However, the procedure to solve FER is still available to find equilibrium, because we can use the equations (20) through (26) by setting $\mu^{(i)} = \mu$.

Additionally, if the system is in equilibrium, then the following relation must be hold on the downstream node $i$.

$$\lambda^{(i)} = \Lambda F(p^{(i)}), p^{(i)} = \frac{1}{\mu - \lambda^{(i)}}. \tag{30}$$

where, $p^{(i)} = \pi^{(i)}(t)$ is the pricing of node $i$. By obtaining the optimal value of flow $\hat{\lambda}^{(i)}$ satisfying above relations, then we have the optimal pricing of $\hat{p}^{(i)}$ node $i$ so that the pricings are moved to the equilibrium points.

$$\hat{p}^{(i)} = 1/(\mu - \hat{\lambda}^{(i)}). \tag{31}$$

Since the optimal flow satisfying the relations in above equations is obtained only by the successive approximation, then we iterate these steps (procedures) in the algorithm until sufficient approximation is attained.

## 5 Applications

### 5.1 Chaoticity of $W(t)$ due to $p_i(t)$

In the following, at first we show the relations between rise of chaoticity and parameters based on the simulation studies. The range of parameters is restricted as follows.

total number of nodes: $N = 100$
numbers of input/output nodes: $N_{in} = N_{out} = 2 \sim 10$
average numbers of branches on intermediate nodes: $N_b \geq N_{in}$
capacity at nodes: $\mu = 0.1 \sim 0.9$
parameters related to transformation functions $\Lambda F(\pi)$ : $\omega = 0.7$, $a = 5 \sim 20$, $d = 1$, $f = 1$

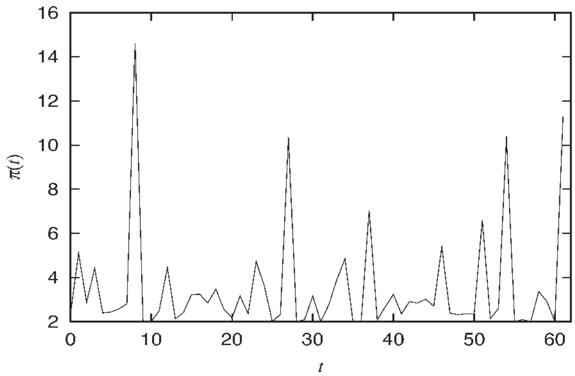


Figure 4: An example of chaotic change of $\pi^{(i)}(t)$ for $N_{in} = N_{out} = N_b = 5$ in Case S.

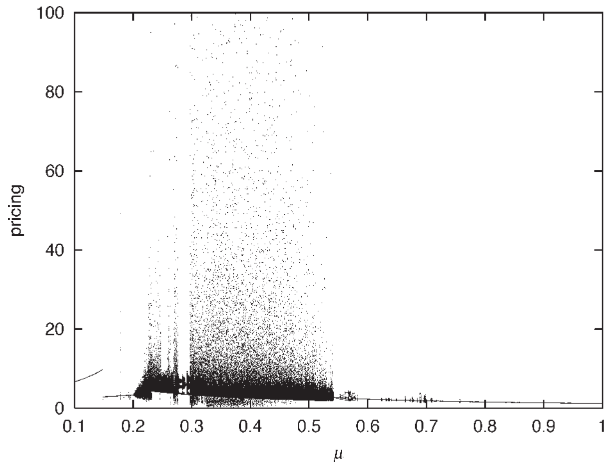


Figure 5: Bifurcation diagram of time series $\pi^{(i)}(t)$ for $N_{in} = N_{out} = N_b = 5$ in Case S.

Fig.4 shows an example for chaotic change of pricing $\pi^{(i)}(t)$ of a certain node $i$ for $N_{in} = N_{out} = N_b = 5$ in Case S. For simplicity, the diagram for chaotic fluctuation in wealth time series $W(t)$ due to the chaoticity of $p_i(t)$ is omitted here, since an example is already shown in Fig.3. Fig.5 shows the bifurcation diagram of $\pi^{(i)}(t)$ for a certain node $i$ for the case $N_{in} = 5$ in Case S over a range of $\mu$. We also show the maximum Liapunov exponent $L_P$ for $\pi^{(i)}(t)$ with the same condition where the embedded dimension is two and the time delay is one. As is seen from Fig.5, prices converge to a single point (stability equilibrium) at least $\mu \geq 0.97$. A periodic-doubling bifurcations of the equilibrium occurs as $\mu$ is decreased. The band of Li-York chaos beings, and

Table 2: Maximum Liapunov exponent for $\pi^{(i)}(t)$ depending on $\mu$ ($N_{in} = N_{out} = N_b = 5$).

| $\mu$ | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|
| $L_P$ | -0.11 | 0.33 | 0.42 | 0.29 | 0.01 | -0.01 | 0.01 | -0.01 |

Table 3: Estimation of values $\mu_-, \mu_+$ for inducing chaotic fluctuation in pricings.

| $N_{in} = 5$ | $c_s = 1$ | | $c_s = \sqrt{0.5}$ | | $c_s = 0$ | |
|---|---|---|---|---|---|---|
| $a$ | $\mu_-$ | $\mu_+$ | $\mu_-$ | $\mu_+$ | $\mu_-$ | $\mu_+$ |
| 5 | 0.0 | 0.97 | 0.13 | 0.90 | 0.18 | 0.88 |
| 10 | 0.0 | 0.90 | 0.05 | 0.89 | 0.06 | 0.89 |
| 20 | 0.0 | 0.90 | 0.05 | 0.89 | 0.06 | 0.89 |
| $N_{in} = 10$ | $c_s = 1$ | | $c_s = \sqrt{0.5}$ | | $c_s = 0$ | |
| $a$ | $\mu_-$ | $\mu_+$ | $\mu_-$ | $\mu_+$ | $\mu_-$ | $\mu_+$ |
| 5 | 0.0 | 0.98 | 0.15 | 0.90 | 0.20 | 0.89 |
| 10 | 0.0 | 0.92 | 0.10 | 0.87 | 0.08 | 0.90 |
| 20 | 0.0 | 0.92 | 0.09 | 0.88 | 0.09 | 0.89 |

continues until $\mu$ drops below $\mu_- = 0.2$. Since we have similar results for Case M, we omit these figures.

Then, we examine the condition for the occurrence of chaotic fluctuation in the pricing. Table 3 shows the values of $\mu_-$ and $\mu_+$ by changing the variable $c_s$. Even though we show the value $\mu_-$ as well, but in the network structured system only the $\mu_+$ is discussed, because at very small $\mu$, the flow in nodes induce overflows, and the value $\mu$ must be kept relatively large.

## 5.2 Applications of control of chaos

Then, we shown the result of simulation studies for suppressing the chaotic fluctuation in pricing by imposing small external force. Especially, we focus on the length of time period necessary to attain the suppression. Fig.6 shows the control of the chaotic fluctuation of pricing for a Case S network (where $N_{in} = 5, \mu = 0.35$) by imposing an external force, and the wealth time series is ultimately converged to the fixed point. External force necessary for the control of chaos is shown in the lower diagram in Fig.6. Fig.7 shows an example of control of chaotic fluctuation of wealth time series on a certain node for Case M (where $N_{in} = 5, \mu = 0.6$). In Fig.7, we see the chaotic fluctuation in the beginning of time, but by imposing the control, pricing is kept to a stable level.

To summarize the characteristics for controlling chaoticity, we show the average length of period $T_c$ necessary to impose external force for the control. By iterating simulation studies (50 times) using another network topologies and initial values for pricing, we have Table 4 presenting the average $T_c$ for case M by varying $N_{in} = N_{out}, \mu$. Even though we omit the result for other cases, the characteristic for the control of chaos is similar to the result in Table 2. As is seen from Table 2, we need about $T_c = 5.5$ for the control of chaos in the wealth time series $W(t)$.

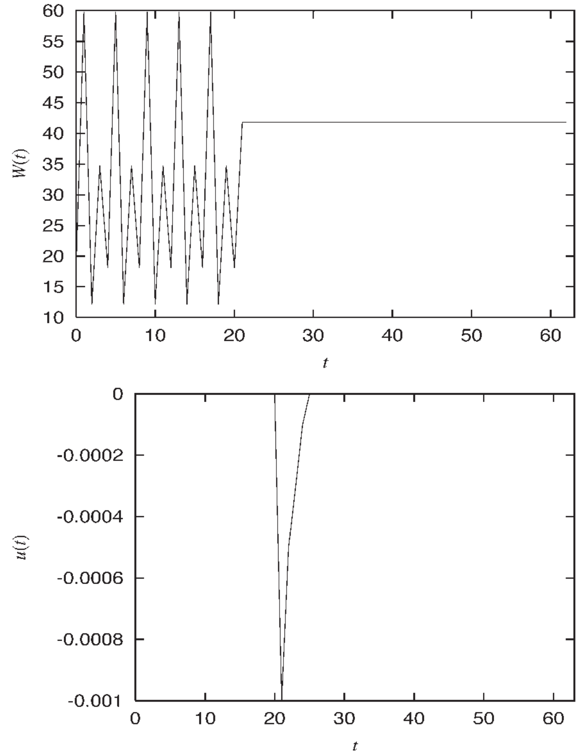


Figure 6: An example of control of chaos for a case of Case S($\mu = 0.35$).

# 6 Conclusion

This paper treated the analysis of profits/prices chaoticity in formalizing of collaboration among agents and its applications to control of chaos. By assuming firm agent produces goods by using support of another outside agent with several cost of labor usage, then the wealth of firm agent bears chaotic fluctuation depending on the rate of collaboration among agents. The model was extended to multi-agent systems under the collaboration in a network, and the chaoticity of labor price was found. Using the deterministic decision on input and pricing, we showed the control (suppression) scheme for the chaotic fluctuation base on the control of chaos.

For future works, we will examine the real examples of collaborations and usefulness of chaotic control.

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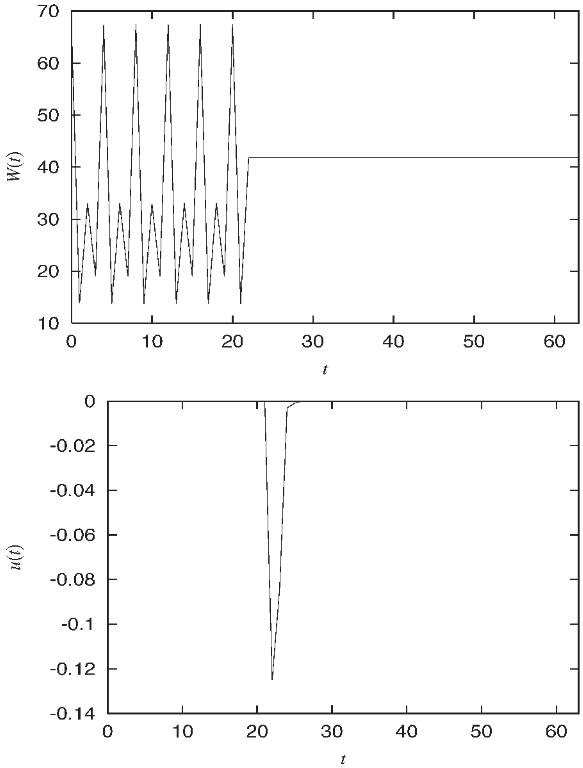


Figure 7: An example of control of chaos for a case of Case M ($N_{in} = 5, \mu = 0.6$).

Table 4: Average values of $T_c$ for controlling chaos in $W(t)$.

| $N_{in}$ | $\mu$ | $T_c$ |
|---|---|---|
| 2 | 0.55 | 5.8 |
| 3 | 0.50 | 6.1 |
| 5 | 0.45 | 6.3 |
| 8 | 0.45 | 6.5 |
| 10 | 0.45 | 6.8 |

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# Appendix A: Condition for chaoticity of $p(t)$

Since predicted price $\pi(t)$ is said to be equilibrium where no customer has incentive to deviate from the join/balk decision, an equilibrium arrival rate $\lambda = \lambda(t)$ satisfies the equilibrium equation $\pi(t) = p(t) = f + W_\mu(\pi(t))$, which now takes the form

$$a - (a-d)\lambda/\Lambda = f + (\mu - \beta'\lambda)[\mu(\mu-\lambda)]^{-1}. \tag{A1}$$

By transforming the equation, and by obtaining only desired solution (root) for $\lambda$ (denoted as $\lambda_\mu$), then we have.

$$\lambda_\mu = [-c_1 + (c_1^2 - 4c_0c_2)^{1/2}]/2c_0. \tag{A2}$$

$$c_0 = (a-d)\mu/\Lambda. \tag{A3}$$

$$c_1 = -\mu a - (a-d)\mu^2/\Lambda + \beta'. \tag{A4}$$

$$c_2 = \mu^2 a - \mu - f. \tag{A5}$$

Now we wish to examine the local stability result, which says that the equilibrium is stable or unstable. The dynamic pricing process given as first-order nonlinear difference equation (14) is rewritten as.

$$\Pi_\mu(\pi) = (1-\omega)\pi + \omega P_\mu, P_\mu = f + W_\mu[\lambda(\pi)]. \tag{A6}$$

It is shown by Sanderfur that if $\partial\Pi_\mu(\pi)/\partial\pi < 1(> 1)$ in equation (A6), then the system is stable (unstable) [16]. The case of equality at the bifurcation point between stability and unstability is given such as

$$\partial\Pi_\mu(\pi)/\partial\pi = 1. \tag{A7}$$

Then, the roots of equation (A7) (called bifurcation values) gives us the marginal value for the capacity $\mu$ inducing chaos and corresponding value for $\pi(t)$. We substitute the relation of equation (A7) into equation (A1), then we have the next relation.

$$\lambda_\mu = \mu - [\frac{(1-\beta')\Lambda\omega}{(a-d)(2-\omega)}]^{1/2}. \qquad (A8)$$

By substituting the relation in (A2) for $\lambda$, then we have the relations given in equations (17) and (18). From a simple observation, it is seen that if $\mu < \mu_-, \mu > \mu_+$ the system have a stable equilibrium point, and otherwise if $\mu_- \leq \mu \leq \mu_+$ then the system is unstable.

Then, we estimate the condition for the induction of chaotic fluctuation in $\pi(t)$ for several values of $c_s$ as follows ($a = 5, d = 1, f = 1$).
$\mu_- = 0, \mu_+ = 0.97$ for $c_s = 1$,
$\mu_- = 0.008, \mu_+ = 0.929$ for $c_s = \sqrt{0.5}$,
$\mu_- = 0.187, \mu_+ = 0.889$ for $c_s = 0$.

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