Introducing Viewpoints of Mechanics into Basic Growth Analysis-(XIII) : Comparing Growth Mechanics between Logistic Functions and Basic Growth Functions-

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Introducing Viewpoints of Mechanics into Basic Growth Analysis – (XIII) Comparing Growth Mechanics between Logistic Functions and Basic Growth Functions –

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This study was conducted to compare growth mechanics between logistic functions and basic growth functions. The results obtained were as follows. Differential equation for basic growth function showed that the square of growth rate was described using the product of weight and growth acceleration. This form was similar to Newton's law of motion where differential of momentum is described using the product of mass of an object and acceleration. However, differential equation for logistic function took a form that was more complex, due to the existence of additional function, than that for basic growth function. Therefore, only the product of weight and growth acceleration was not sufficient to describe the square of growth rate in the differential equation for logistic function. This insufficiency was compensated by the additional function. This is one of the reasons why logistic functions are more complex than basic growth functions showing increases only. It was suggested from the present study that growth mechanics of logistic functions was more complex than that of basic growth functions.

INTRODUCTION

Basic growth analysis is one of the tools for analyzing growth process of an animal (Brody, 1945) and a plant (Blackman, 1919; Watson, 1952; Radford, 1967; Hunt, 1990). Basic growth function, which is used in basic growth analysis, shows an exponential increase that is applied to a certain period of growth. If longer period of growth process is analyzed, then other functions such as logistic functions are used (Hirata, 2004). Logistic functions give sigmoid curves that may be fitter for growth curves of longer period compared with basic growth functions. Shimojo (2007) and Shimojo *et al.* (2006, 2007a, 2007b, 2008, 2009a, 2009b, 2009c) suggested, by introducing viewpoints of mechanics into basic growth analysis, basic growth mechanics for an animal or a plant on an analogy with Newton's law of motion. In basic growth mechanics, the square of absolute growth rate is described using the product of weight and growth acceleration. This is similar to Newton's law of motion, where differential of momentum is described using the product of mass of an object and acceleration (Kawabe, 2006). It seems to be of interest to investigate what occurs if mechanical viewpoints are applied to logistic functions that are more complex in form than basic growth functions.

The present study was designed to investigate the comparison of growth mechanics between logistic functions and basic growth functions.

COMPARING GROWTH MECHANICS BETWEEN LOGISTIC FUNCTIONS AND BASIC GROWTH FUNCTIONS

(A) Introducing growth mechanics into logistic functions

Logistic function is given as follows (Hirata, 2004),

$$W(t) = \frac{a}{1 + b \cdot \exp(-r \cdot t)} \;, \tag{1}$$

where W = weight, t = time, a and b = constants, r = rate constant.

Introducing viewpoints of mechanics into logistic function (1) gives

$$\mathrm{GR} = \frac{dW(t)}{dt} = \frac{a \cdot b \cdot r \cdot \exp(-r \cdot t)}{(1 + b \cdot \exp(-r \cdot t))^2} \;, \tag{2}$$

$$\mathrm{GA} = \frac{d^2W(t)}{dt^2} = \frac{a \cdot b \cdot r^2 \cdot \exp(-r \cdot t) \cdot (b \cdot \exp(-r \cdot t) - 1)}{(1 + b \cdot \exp(-r \cdot t))^3}, \tag{3}$$

where GR = growth rate, GA = growth acceleration.

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Relating functions (1), (2) and (3) gives the following two functions,

$$\frac{dW(t)}{dt}\bigg/W(t)$$

$$=\frac{b\cdot r\cdot\exp(-r\cdot t)}{1+b\cdot\exp(-r\cdot t)}\ , \tag{4}$$

$$\frac{d^2W(t)}{dt^2}\bigg/\frac{dW(t)}{dt}$$

$$=\frac{r\cdot(b\cdot\exp(-r\cdot t)-1)}{1+b\cdot\exp(-r\cdot t)}\ . \tag{5}$$

Function (5) is transformed as follows,

$$\frac{d^2W(t)}{dt^2}\bigg/\frac{dW(t)}{dt}=\frac{r\cdot(b\cdot\exp(-r\cdot t)-1)}{1+b\cdot\exp(-r\cdot t)}$$

$$=\frac{dW(t)}{dt}\bigg/W(t)-W(t)\cdot\frac{r}{a}\ . \tag{6}$$

Thus,

$$\left(\frac{dW(t)}{dt}\right)^2=W(t)\cdot\frac{d^2W(t)}{dt^2}+(W(t))^2\cdot\frac{dW(t)}{dt}\cdot\frac{r}{a}\ . \tag{7}$$

The latter half of the right–hand side of function (7) is transformed as follows,

$$(W(t))^2\cdot\frac{dW(t)}{dt}\cdot\frac{r}{a}$$

$$=\left(\frac{a}{1+b\cdot\exp(-r\cdot t)}\right)^2\cdot\frac{a\cdot b\cdot r\cdot\exp(-r\cdot t)}{(1+b\cdot\exp(-r\cdot t))^2}\cdot\frac{r}{a}$$

$$=W(t)\cdot\frac{d^2W(t)}{dt^2}\cdot\frac{1}{b\cdot\exp(-r\cdot t)-1}\ . \tag{8}$$

Inserting function (8) into the right–hand side of function (7) gives

$$W(t)\cdot\frac{d^2W(t)}{dt^2}+W(t)\cdot\frac{d^2W(t)}{dt^2}\cdot\frac{1}{b\cdot\exp(-r\cdot t)-1}$$

$$=W(t)\cdot\frac{d^2W(t)}{dt^2}\cdot\left(\frac{b\cdot\exp(-r\cdot t)}{b\cdot\exp(-r\cdot t)-1}\right)\ . \tag{9}$$

Thus, inserting function (9) into function (7) gives

$$\left(\frac{dW(t)}{dt}\right)^2=W(t)\cdot\frac{d^2W(t)}{dt^2}\cdot\left(\frac{b\cdot\exp(-r\cdot t)}{b\cdot\exp(-r\cdot t)-1}\right)\ . \tag{10}$$

Function (10) takes the form of differential equation that is derived from logistic function (1), suggesting growth mechanics for logistic functions.

(B) Introducing growth mechanics into basic growth functions

Basic growth function is given by

$$W(t)=W_0\cdot\exp(r\cdot t), \tag{11}$$

where W = weight, t = time, W_0 = W at t = 0, r = relative growth rate.
Introducing viewpoints of mechanics into basic growth function (11) gives

$$\mathrm{GR}=\frac{dW(t)}{dt}=r\cdot W_0\cdot\exp(r\cdot t), \tag{12}$$

$$\mathrm{GA}=\frac{d^2W(t)}{dt^2}=r^2\cdot W_0\cdot\exp(r\cdot t). \tag{13}$$

$$\frac{dW(t)}{dt}\bigg/W(t)=\frac{d^2W(t)}{dt^2}\bigg/\frac{dW(t)}{dt}=r\ . \tag{14}$$

Thus,

$$\left(\frac{dW(t)}{dt}\right)^2=W(t)\cdot\frac{d^2W(t)}{dt^2}\ . \tag{15}$$

Function (15) is differential equation that is derived from basic growth function (11), suggesting growth mechanics for basic growth functions.

(C) Comparing growth mechanics between logistic functions and basic growth functions

Logistic function and basic growth function are taken up here again,

$$W(t)=\frac{a}{1+b\cdot\exp(-r\cdot t)}\ , \tag{1}$$

$$W(t)=W_0\cdot\exp(r\cdot t). \tag{11}$$

As shown in sections (A) and (B), differential equation derived from logistic function and that from basic growth function are given by

$$\left(\frac{dW(t)}{dt}\right)^2=W(t)\cdot\frac{d^2W(t)}{dt^2}\cdot\left(\frac{b\cdot\exp(-r\cdot t)}{b\cdot\exp(-r\cdot t)-1}\right), \tag{10}$$

$$\left(\frac{dW(t)}{dt}\right)^2=W(t)\cdot\frac{d^2W(t)}{dt^2}\ . \tag{15}$$

Shimojo (2007) and Shimojo *et al.* (2006, 2007b) suggest that the form of differential equation (15) for basic growth function is similar to that of Newton's law of motion given by

$$\frac{dp}{dt} = m \cdot \frac{d^2r}{dt^2} \ , \tag{16}$$

where p = momentum, m = mass of an object, r = position, t = time.

This hypothetic similarity suggests that basic growth function is analyzed using growth mechanics, where the square of growth rate is described using the product of weight and growth acceleration as shown in function (15).

However, differential equation for logistic function (10) takes the form that is more complex, due to the existence of additional function (17), than that for basic growth function (15),

$$\frac{b \cdot \exp(-r \cdot t)}{b \cdot \exp(-r \cdot t) - 1} \ . \tag{17}$$

This difference shows that only the product of weight and growth acceleration is not sufficient to describe the square of growth rate in growth mechanics for logistic functions. This insufficiency is compensated by the additional function (17). This is one of the reasons why logistic functions are more complex than basic growth functions showing increases only.

(G) Conclusions

It is suggested from the present study that growth mechanics of logistic functions is more complex than that of basic growth functions.

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