Load Distribution at Arch Crowns of Curved Debris Dams

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Load Distribution at Arch Crowns of Curved Debris Dams*

Masanori SUYAMA

Arch dams differ from gravity masonry dams principally in that they transmit the water load to the sides rather than to the bottom of the canyon. The stability of arch dams depends, therefore, on the crushing strength of the material, rather than on the weight as in gravity dams. In general, it is surmised that a suitable site for an arch dam then must have canyon walls capable of resisting the arch thrust and sufficiently close together to insure arch action, but shapes of the natural valley are very variable.

In cace of designing of arch dams by the "Crown-Cantilever Method" supposing that the load carried by horizontal arch element is uniformly distributed over the entire surface of the each arches, it is said that the method will be suitable for a changeless valley or a "U-shaped" canyon. But, up to date, the precise limit of application is not exactly known yet.

In this paper, to find out the relation between the shape of the valley and the partition of the load by this method, supporsing that various valley sections, some tables were furnished.

1. Analysis of arch elements.

The arch rings, used in this paper, are assumed to be symmetrically water loaded, considered as fixed with relation to abutments, and to have a constant vertical thickness from abutment to abutment and from upstream to downstream face of dam. The theory of the strength of materials is applicable only to thin beams of small curvature, whose ratio, e/r, of height to the radius of curvature is small, if we wish to remain within the limits of application of the two basic hypothesis, that is to say, Hooke's Law, which states that stresses are proportional to strains, and Bernouilli's Hypothesis which assumes that plane sections remain plane after deformation. The symmetrical arch with symmetrical loads requires the analysis of only one half of the arch, the analysis of the other half being identical. For convenience, the right side of the arch is the one analyzed.

From the assumed conditions, it is obvious that the arch rings are statical indeterminateness of order three, but the analysis of stresses in them may be approached through the epuations of Bresse.

The displacements, u and v, of the point, A, under investigation on an arch ring are given by the equations of Bresse, Group I:

$$u=-r\int\frac{M(y-r\cdot\cos\alpha)}{EI}\,d\alpha-r\int\frac{N\cdot\cos\alpha}{ES}\,d\alpha-r\int\frac{T\cdot\sin\alpha}{GS}\,d\alpha-r\int\frac{M_t(y-r\cdot\cos\alpha)}{EI}\,d\alpha$$

$$-r\int\frac{N_t\cos\alpha}{ES}\,d\alpha-r\int\frac{T_t\sin\alpha}{GS}\,d\alpha+\lambda\Delta tr\int\cos\alpha d\alpha \tag{1}$$

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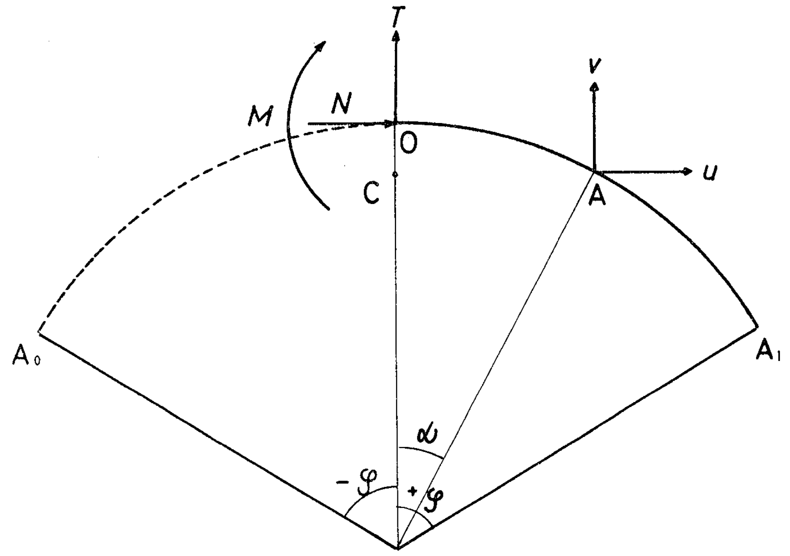
Fig. 1. Direction of positive loads, forces, moments, and movements of arch.



$$v=r\int\frac{M(x-r\cdot\sin\alpha)}{EI}d\alpha+r\int\frac{N\cdot\sin\alpha}{ES}d\alpha-r\int\frac{T\cdot\cos\alpha}{GS}d\alpha+r\int\frac{M_t(x-r\cdot\sin\alpha)}{EI}d\alpha$$

$$+r\int\frac{N_t\sin\alpha}{ES}d\alpha-r\int\frac{T_t\cos\alpha}{GS}d\alpha-\lambda\Delta tr\int\sin d\alpha \qquad (2)$$

in which we have the following values:

$$\begin{cases} M=-Qr\cdot\cos\alpha \\ N=R+Q\cdot\cos\alpha \\ T=Q\cdot\sin\alpha \end{cases}$$

$$\begin{cases} M_t=-Q_tr\cdot\cos\alpha \\ N_t=Q_t\cos\alpha \\ T_t=Q_t\sin\alpha \end{cases}$$

where M, N, T: moment, thrust, and shear, respectively.
Q: hyperstatic pressure.
R: reaction of the supports.
r: radius to center line of arch.
E: modulus of elasticity in direct stress.
G: modulus of elasticity in shear stress.
I: moment of inertia of a radial arch cross-section.
S: area of a radial arch section.
λ: coefficient of thermal expansion.
Δt: variation in the mean temperature.
x, y: co-ordinates of the point A.
α: variable of integration.

and initial subscript t represents a quantity due to temperature change.

If we take, as the origin of co-ordinates, the point O instead of elastic centre C, then M and M_t are rewritten

$$M=-Q(r\cdot\cos\alpha-r\cdot\sin\varphi/\varphi)$$
$$M_t=-Q_t(r\cdot\cos\alpha-r\cdot\sin\varphi/\varphi)$$

The integration is carried out from A_0 to A, that is from $-\varphi$ to α.

The calculations, which are lengthy but not complicated, lead to the following results:

$$u=(Q-Q_t)\Big[\frac{r^3}{EI}\Big(\frac{\alpha}{2}+\frac{\sin 2\alpha}{4}-\sin\alpha\cos\alpha+\frac{\alpha\sin\varphi\cos\alpha}{\varphi}-\frac{\sin\varphi\sin\alpha}{\varphi}\Big)$$

$$+\frac{r}{ES}\Big(\frac{\alpha}{2}+\frac{\sin 2\alpha}{4}\Big)+\frac{r}{GS}\Big(\frac{\alpha}{2}-\frac{\sin 2\alpha}{4}\Big)\Big]-\frac{r^2\cdot\sin\alpha}{ES}+\lambda\varDelta tr\cdot\sin\alpha \tag{3}$$

$$v=(Q-Q_t)\Big[\frac{r}{EI}\Big(\frac{1}{2}-\frac{\cos 2\alpha}{4}-\frac{\cos 2\varphi}{4}-\frac{\alpha\sin\varphi\sin\alpha}{\varphi}-\frac{\sin\varphi\cos\alpha}{\varphi}$$

$$+\frac{\sin\varphi\cos\varphi}{\varphi}\Big)+\frac{r}{ES}\Big(\frac{\cos 2\alpha}{4}-\frac{\cos 2\varphi}{4}\Big)-\frac{r}{GS}\Big(\frac{\cos 2\alpha}{4}-\frac{\cos 2\varphi}{4}\Big)\Big]$$

$$+\frac{r^2}{ES}(\cos\varphi-\cos\alpha)+\lambda\varDelta tr(\cos\alpha-\cos\varphi) \tag{4}$$

In this paper, horizontal arch elements are assumed to be unit high and have a horizontal thickness e.

Taking into account that $S=e$, $I=e^3/12$; $\beta=r/e$; $S/I=12\beta^2/r^2$ we write Q in the form

$$Q=\frac{-R}{C_1\beta^2+C_2}=\frac{-qr}{C_1\beta^2+C_2}$$

with

$$C_1=\frac{12}{\sin\varphi}\Big(\frac{\varphi}{2}+\frac{\sin 2\varphi}{4}-\frac{\sin^2\varphi}{\varphi}\Big)$$

$$C_2=\frac{1}{\sin\varphi}\Big[\frac{\varphi}{2}\Big(1+\frac{E}{G}\Big)+\frac{\sin 2\varphi}{4}\Big(1-\frac{E}{G}\Big)\Big]$$

Also we write Q_t, due to temperature change, in the form

$$Q=\frac{ES\lambda\varDelta t}{C_1\beta^2+C_2}$$

At the crown, for $\alpha=0$, $E/G=3$, v represents the deflection which we shall write in the form

$$f=(Q-Q_t)\Big[\frac{r^3}{EI}\Big(\frac{1}{4}-\frac{\cos 2\varphi}{4}-\frac{\sin\varphi}{\varphi}-\frac{\sin\varphi\cos\varphi}{\varphi}\Big)-\frac{r}{ES}\Big(\frac{1}{2}-\frac{\cos 2\varphi}{2}\Big)\Big]$$

$$-\frac{r^2}{ES}(1-\cos\varphi)+\lambda\varDelta tr(1-\cos\varphi) \tag{5}$$

which is introduced particularly in the method of crown cantilever.

2. Analysis of cantilever elements.

Having application in the ordinary theory of beams, we represent the equation for the deflection curve of the beam in the following differential equation

$$\frac{d^2y}{dx^2}=-\frac{E_z}{EI_z} \tag{6}$$

in which the quantity EI_z is called the flexural rigidity around the Z axis. It is seen that the deflection curve is obtained by integrating Eq. (6).

Here, if the cantilever beam is considered as a beam of rectangular cross section which has a depth of e and a length of unit distance, then geometrical moment of inertia at point A is obtained

$$I_z=\int_S y^2 ds=2\int_0^{e/2} 1\cdot y^2 dy=\frac{e^3}{12} \tag{7}$$

Substituting $e=B-mx$, we can represent this expression in the following form

$$I_z=\frac{1}{12}(B-mx)^3 \tag{8}$$

where I_z is represented by a dimension of order four in length.

For load conditions, we will consider to be represented by system of unit radial loads. The unit radial loads are used in applying radial forces carried by crown cantilever. The unit loads are triangular in shape and vary the unit pressure at one arch elevation to zero pressure at the sample arches directly above and below, see Fig. (2). With these loads it is possible to apply any horizontal force that varies as a straight line between successive elevations of sample arches. Shears and bending moments are computed for each unit load, and radial deflections due to each load are determined by the ordinary theory of flexure of beam with contributions from transverse shears included.

Calculations of deflection equations will be carried out in the following manner for each type of triangular loadings.

a) Deflection due to triangular load type I acting on abutment.

Reaction of the supporting point: $R_A=\varepsilon/2$.

Shearing force:

$0\leqq x<\varepsilon$: $Q=(\varepsilon-x)^2/2\varepsilon$.

$\varepsilon\leqq x\leqq l$: $Q=0$.

Bending moment :

$0\leqq x\leqq\varepsilon$: $M_z=-(\varepsilon-x)^3/6\varepsilon$.

$\varepsilon\leqq x\leqq l$: $M_z=0$.

Fig. 2. Typical unit cantilever loads.

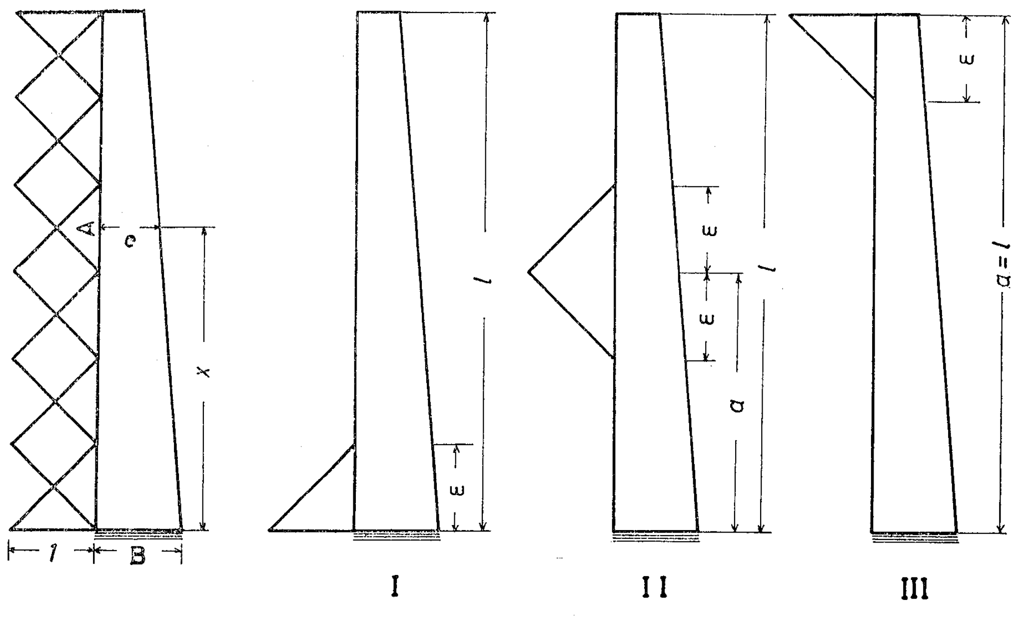
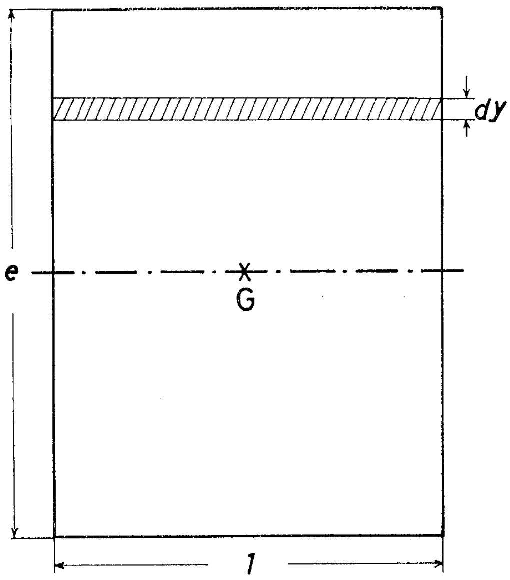


Fig. 3. Horizontal cross-section of cantilever.



Deflection:

$$0 \leq x \leq \varepsilon : \frac{d^2y}{dx^2} = \frac{2}{E\varepsilon} \cdot \frac{(\varepsilon - x)^3}{(B - mx)^3} \tag{9}$$

Using the boundary conditions $x=0$, $dy/dx=0$; $x=0$, $y=0$, we obtain by integration the Eq. (9).

$$y = \frac{1}{Em^5\varepsilon}\Bigg[6(B-m\varepsilon)(2B-m\varepsilon-mx)\log_e\left(\frac{B}{B-mx}\right) - \frac{(B-m\varepsilon)^3}{B-mx} - \frac{m^4\varepsilon^3 x}{B^2}$$

$$-\frac{m^3\varepsilon^2(3x+\varepsilon)}{B} + 6(B-m\varepsilon)(B-mx) - 5B(B+mx) + 3m\varepsilon(B+m\varepsilon) + m^2x(x+9\varepsilon)\Bigg] \tag{10}$$

$$\varepsilon \leq x \leq l : \frac{d^2y}{dx^2} = 0 \tag{11}$$

Using the boundary conditions at A, we obtain by integration the Eq. (11).

$$y = \frac{1}{Em^5\varepsilon}\Bigg[6(B-m\varepsilon)(2B-m\varepsilon-mx)\log_e\left(\frac{B}{B-m\varepsilon}\right) - \frac{m^4\varepsilon^3 x}{B^2} - \frac{m^3\varepsilon^2(3x+\varepsilon)}{B}$$

$$+6m^2\varepsilon x - 12m\varepsilon(B-m\varepsilon)\Bigg] \tag{12}$$

b) Deflection due to triangular load type II acting between on abutment and on crown.

Reaction of the supporting point: $R_A = \varepsilon$.

Shearing force:

$$0 \leq x \leq a-\varepsilon : Q = \varepsilon$$

$$a-\varepsilon \leq x \leq a : Q = \frac{\varepsilon}{2} + \frac{(a-x)(x-a+2\varepsilon)}{2\varepsilon}$$

$$a \leq x \leq a+\varepsilon : Q = \frac{(\varepsilon+a-x)^2}{2\varepsilon}$$

$$a+\varepsilon \leq x \leq l : Q = 0$$

Bending moment:

$$0 \leqq x \leqq a-\varepsilon : M_z = -\varepsilon(a-x)$$

$$a-\varepsilon \leqq x \leqq a : M_z = -\frac{(a-x)^2(3\varepsilon-a+x)+\varepsilon^2(\varepsilon+3a-3x)}{6\varepsilon}$$

$$a \leqq x \leqq a+\varepsilon : M_z = -\frac{(\varepsilon+a-x)^3}{6\varepsilon}$$

$$a+\varepsilon \leqq x \leqq l : M_z = 0$$

Deflection:

$$0 \leqq x \leqq a-\varepsilon : \frac{d^2y}{dx^2} = \frac{12\varepsilon}{E} \cdot \frac{a-x}{(B-mx)^3} \tag{13}$$

Approximating the boundary conditions $dy/dx=0$, $y=0$ at the abutment, we obtain by differential equation (13).

$$y = \frac{6\varepsilon}{Em^3}\left[1+2\log_e\left(\frac{B}{B-mx}\right) - \frac{B-ma}{B-mx} - \frac{m(B+ma)}{B^2}x - \frac{ma}{B}\right] \tag{14}$$

$$a-\varepsilon \leqq x \leqq a : \frac{d^2y}{dx^2} = \frac{2}{E\varepsilon} \cdot \frac{(x-b)^3+6\varepsilon^2(a-x)}{(B-mx)^3} \tag{15}$$

Approximating the boundary conditions at A, we obtain by differential equation (15).

$$y = \frac{1}{Em^5\varepsilon}\left[6(B-mb)(2B-mb-mx)\log_e\left(\frac{B-mx}{B-mb}\right) + 12m^2\varepsilon^2\log_e\left(\frac{B}{B-mx}\right)\right.$$

$$+\frac{(B-mb)^3}{B-mx} - \frac{6m^2\varepsilon^2(B-ma)}{B-mx} - \frac{6m^3\varepsilon^2x(B+ma)}{B^2} + \frac{6m^2\varepsilon^2(B-ma)}{B}$$

$$\left. +2m^2bx - 5Bmb - m^2(x+2b)(x-2b) - (B-mb)(B+5mb-11mx)\right] \tag{16}$$

$$a \leqq x \leqq a+\varepsilon : \frac{d^2y}{dx^2} = \frac{2}{E\varepsilon} \cdot \frac{(c-x)^3}{(B-mx)^3} \tag{17}$$

Approximating the boundary conditions at point B, we obtain by differential equation (17).

$$y = \frac{1}{Em^5\varepsilon}\left[6mx(B-mb)\log_e(B-mx+m\varepsilon) + 12m^2\varepsilon^2\log_e B\right.$$

$$-6(B-mc)(2B-mc-mx)\log_e(B-mx) - 6(B-mb)(2B-mb)\log_e(B-mb)$$

$$+12(B-ma)(2B-ma-mx)\log_e(B-ma) + 10m(a-x)(B-ma) - 5(B-mx)^2$$

$$+15m^2\varepsilon^2 + 6m^2a^2 + m^2x^2 - 8m^2\varepsilon a + 10m\varepsilon(B-ma) + 6(B-mx)(B-mc)$$

$$\left. -2m^2cx + 5mx(B-mb) - 5Bmb - \frac{(B-mc)^3}{B-mx} - \frac{6m^3\varepsilon^2x(B+ma)}{B^2} - \frac{6m^3\varepsilon^2a}{B}\right] \tag{18}$$

$$a+\varepsilon \leqq x \leqq l : \frac{d^2y}{dx^2} = 0 \tag{19}$$

Approximating the boundary conditions at point C, we obtain by differential equation (19).

$$y = \frac{1}{Em^5\varepsilon}\left[6m(B-mc)x\cdot\log_e(B-mc) - 12m(B-ma)x\cdot\log_e(B-ma)\right.$$

$$+6m(B-mb)x\cdot\log_e(B-mb) + 12(B-ma)(2B-ma)\cdot\log_e(B-ma) + 12m^2\varepsilon^2$$

$$\log_e B - 6(B-mb)(2B-mb)\cdot\log_e(B-mb)$$

$$-12(B-mc)^2\log_e(B-mc)-6m(B-mc)(a+\varepsilon)\cdot\log_e(B-mc)$$
$$-\frac{6m^3\varepsilon^2(B+ma)}{B^2}x-\frac{6m^3\varepsilon^2 a}{B}-5mc(B-mc)+14m^2\varepsilon^2+5Bmc+5m^2b^2-10m^2a^2\Big] \tag{20}$$

c) Deflection due to triangular load type III acting on crown.
Reaction of the supporting point: $R_A=\varepsilon/2$.
Shearing force:

$$0\leqq x\leqq a-\varepsilon:\ Q=\frac{\varepsilon}{2}$$

$$a-\varepsilon\leqq x\leqq a:\ Q=\frac{(a-x)(2\varepsilon-a+x)}{2\varepsilon}$$

Bending moment:

$$0\leqq x\leqq a-\varepsilon:\ M_z=-\frac{\varepsilon(3a-\varepsilon-3x)}{6}$$

$$a-\varepsilon\leqq x\leqq a:\ M_z=-\frac{(a-x)^2\cdot(3\varepsilon-a+x)}{6\varepsilon}$$

Deflection:

$$0\leqq x\leqq a-\varepsilon:\ \frac{d^2y}{dx^2}=\frac{2\varepsilon}{E}\cdot\frac{3a-\varepsilon-3x}{(B-mx)^3} \tag{21}$$

Approximating the boundary conditions $dy/dx=0$, $y=0$ at the abutment, we obtain by differential equation (21).

$$y=\frac{\varepsilon}{Em^3}\left[3+6\cdot\log_e\left(\frac{B}{B-mx}\right)-\frac{3B-3ma+m\varepsilon}{B-mx}+\frac{m^2x(\varepsilon-3a)}{B^2}-\frac{m(3x+3a-\varepsilon)}{B}\right] \tag{22}$$

$$a-\varepsilon\leqq x\leqq a:\ \frac{d^2y}{dx^2}=\frac{2}{E\varepsilon}\cdot\frac{(a-x)^2(3\varepsilon-a+x)}{(B-mx)^3} \tag{23}$$

Approximating the boundary conditions at point A, we obtain by differential equation (23).

$$y=\frac{1}{Em^5\varepsilon}\Big[\frac{(B-ma)^3+3m\varepsilon(B-ma)^2}{B-mx}-\frac{3m^3\varepsilon^2x}{B}-\frac{m^4\varepsilon^2(3a-\varepsilon)}{B^2}x-\frac{m^2\varepsilon^2(3ma-m\varepsilon)}{B}$$
$$-m^2x^2-6(B-mb)(B-mx)+2m^2bx+5m(B-mb)x+5(B-mb)(B-2mb)$$
$$-m^2(a^2-2\varepsilon a-2\varepsilon^2)-6(B-mb)(2B-mb-mx)\cdot\log_e(B-mb)+\{12B^2+18Bm\varepsilon$$
$$+6m^2a(a+x)-6m^2\varepsilon(2a+x)-6Bm(3a+x)\}\log_e(B-mx)+6m^2\varepsilon^2\cdot\log_e B\Big] \tag{24}$$

3. Dimensions of curved debris dam.

In this paper we shall consider the curved debris dam with constant radius and symmetrical single-centered thin arch rings of uniform thickness.

Height above foundation	h
Center angle at top	120°
Upstream radius	r
Upstream slopes	straight
Downstream slopes	10 per cent
Arch thickness at top	0.07 h
Arch thickness at bottom	0.17 h

Width of river at crest	$2l$
Unit weight of water	$w=1.0\ \text{ton/m}^3$
Modulus of elasticity of concrete	
in direct stress	$E=2.1\times10^6\text{ton/m}^2$
in shear stress	$E/3$

4. Cross section of gorge.

Let us now denote the approximate cross sectionnal shapes of gorge by the equation of parabola,

$$y^2=k^2x^{\mathrm{N}}\ \left(k=\frac{l}{h^{\mathrm{N}/2}}\right) \tag{25}$$

where the value l/h is determined by the ratio r/e of radius r to thickness e of arch ring.

Since in practice $\beta>5$, we may now express the value of l/h for every 0.1 from 0.8 to 1.2.

While the value of N may be also expressed for every 0.25 from 0, i.e. rectangular section, to 2.0, i.e. triangular section. Accordingly combining the value of l/h with that of N, forty five kinds of gorges are estimated.

5. Distribution of load.

The method of cantilever arches resolves the curved dam into a double series of horizontal parallel arches fixed with the sides of the gorge and of vertical cantilevers embedded at the base.

On the other hand, the crown-cantilever method referred to as the adjustment of the crown, is derived from the preceding by the reduction of the system of cantilevers to a single central cantilever.

In this method, we resolve the effect of the pressure on the upstream face into two forces, one acting on the crown cantilever, and the other acting on the arch. The points on the crown cantilever will be subjected to the same deformations whether they are considered as being points on the crown cantilever or on the arches.

The calculation of the cantilever will be simplified by a consideration of elementary laws of triangular loading, such as those represented in the Fig. (2) by the typical unit cantilever loads.

Here, the number of calculation points on the crown cantilever selected for calculation is twenty.

As a result, we calculate A_{mn} the coefficients of influence of the displcements due to each law, when the calculation points are designated by m and the loading points by n, then y_{cm}, the radial displacement due to the law of loading P_{cn}, will be given by the expression.

$$y_{cm}=A_{m1}P_{c1}+A_{m2}P_{c2}+\cdots\cdots+A_{m20}P_{c20} \tag{26}$$

When we calculate B_m, the coefficients of influence of the deflections due to the uniform unit load, the radial deflections, y_{am}, are then written as follows.

$$y_{am}=B_mP_{am} \tag{27}$$

The adjustment at the crown is derived from a solution of the system of equations $y_{cm}=y_{am}$ and

$$A_{m1}P_{c1}+A_{m2}P_{c2}+\cdots\cdots+A_{m20}P_{c20}=B_m(P_m-P_{cm}) \tag{28}$$

$$m=1,\ 2,\ 3,\cdots\cdots,\ 20$$

where P_m represents the total water pressure on the point m.

Finally, if we convert the variables into the followings

$$A_{mn}=\frac{ha_{mn}}{E};\ B_m=\frac{hb_m}{E};\ P_{cm}=Ex_m \tag{29}$$

we get

$$a_{m1}x_1+a_{m2}x_2+\cdots\cdots+(a_{mn}+b_m)x_m+\cdots\cdots+a_{m20}x_{20}=b_m\cdot\frac{P_m}{E} \tag{30}$$

Since the constants a_{mn} and b_m are dimensionless having no connection with h and the right-hand side of Eq. (30) may be expressed by $f(m)wh/E$, if the solution of the simultaneous equations for $h=1$ is expressed by x'_n, we finally obtain the equation $x_n=hx'_n$. From this expression for the deflection, the divided loads are readily obtained.

Each value of load, as shown in next tables, is in unit of $\frac{10}{E}\cdot x'_n$.

For example:

When $h=20$ m, $l=36$ m, $N=0$, then $l/h=0.8$.

Therefore, from Table 1,

Horizontal water load at 50% of height
$=2.380952\times 0.21\times 20=10.0$ (ton/m²)

Cantilever load at 50% of height
$=0.700420\times 0.21\times 20=2.941764$ (ton/m²)

Arch load at 50% of height
$=10.0-2.941764=7.058236$ (ton/m²)

When $N>1.25$, this method seems unsuitable for determining load partition within desired accuracy. However, there remains the possibility of adopting the general methods of cantilever arches to solve this problem.

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Table 1. $l/h=0.8$

Height	Horizontal water load	Cantilever load								
		N = 0.00	N = 0.25	N = 0.50	N = 0.75	N = 1.00	N = 1.25	N = 1.50	N = 1.75	N = 2.00
1.00	0	−0.533434	−0.530463	−0.528234	−0.527938	−0.530479	−0.535806	−0.543164	−0.549588	−0.553228
0.95	0.238095	−0.439241	−0.433752	−0.427604	−0.421477	−0.415920	−0.411029	−0.407045	−0.402064	−0.395086
0.90	0.476190	−0.357404	−0.349573	−0.339499	−0.327252	−0.312944	−0.296828	−0.280279	−0.262657	−0.243972
0.85	0.714286	−0.282702	−0.272987	−0.259314	−0.241033	−0.217722	−0.189774	−0.159840	−0.128771	−0.097770
0.80	0.952381	−0.208004	−0.197209	−0.180742	−0.157045	−0.125113	−0.085417	−0.041996	0.002559	0.045629
0.75	1.190476	−0.128734	−0.114418	−0.096467	−0.068811	−0.229588	0.020690	0.076540	0.133357	0.186864
0.70	1.428571	−0.025345	−0.016393	0.000985	0.030216	0.074084	0.128013	0.197663	0.263620	0.323851
0.65	1.666667	0.099750	0.104976	0.119005	0.146192	0.190343	0.251375	0.321214	0.390531	0.450900
0.60	1.904762	0.258163	0.257369	0.264516	0.284647	0.322581	0.378821	0.444503	0.507879	0.558082
0.55	2.142857	0.456827	0.447571	0.443626	0.450163	0.472814	0.512999	0.561781	0.605453	0.630685
0.50	2.380952	0.700420	0.681135	0.661390	0.646277	0.641672	0.650253	0.664239	0.669194	0.649893
0.45	2.619048	0.994759	0.961943	0.921526	0.875361	0.828403	0.784837	0.740072	0.681610	0.594219
0.40	2.857143	1.338675	1.291957	1.226352	1.138840	1.031372	0.909661	0.775576	0.623801	0.443655
0.35	3.095238	1.730851	1.670328	1.576119	1.436912	1.248264	1.016937	0.756481	0.478409	0.185734
0.30	3.333333	2.165835	2.093337	1.969247	1.769448	1.477916	1.101227	0.672674	0.237085	−0.173115
0.25	3.571429	2.633568	2.553097	2.401329	2.135870	1.721836	1.163250	0.525024	−0.089244	−0.595238
0.20	3.809524	3.119884	3.037991	2.866073	2.537828	1.990353	1.221231	0.341813	−0.453317	−1.003983
0.15	4.047619	3.605219	3.531067	3.354252	2.981124	2.311700	1.330652	0.204942	−0.756563	−1.300391
0.10	4.285714	4.061983	4.006203	3.849151	3.473699	2.742281	1.615633	0.297424	−0.805585	−1.330993
0.25	4.523810	4.461045	4.434323	4.336141	4.057009	3.444822	2.405250	1.032275	−0.332576	−1.200945
bottom	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905

Table 2. $l/h = 0.9$

Height	Horizontal water load	Cantilever load								
		N = 0.00	N = 0.25	N = 0.50	N = 0.75	N = 1.00	N = 1.25	N = 1.50	N = 1.75	N = 2.00
1.00	0	−0.623311	−0.618854	−0.614393	−0.610358	−0.607410	−0.606100	−0.606211	−0.606438	−0.605032
0.95	0.238095	−0.512006	−0.505988	−0.498968	−0.490867	−0.481853	−0.472290	−0.462427	−0.452014	−0.440356
0.90	0.476190	−0.409892	−0.402522	−0.393039	−0.380771	−0.365305	−0.346767	−0.325930	−0.303766	−0.280811
0.85	0.714286	−0.312146	−0.303849	−0.292261	−0.276038	−0.254087	−0.226234	−0.193817	−0.159200	−0.124344
0.80	0.952381	−0.212244	−0.203702	−0.190713	−0.171196	−0.143274	−0.106389	−0.062460	−0.015402	0.031183
0.75	1.190476	−0.102767	−0.094945	−0.081685	−0.060135	−0.027541	0.017152	0.071467	0.129806	0.186700
0.70	1.428571	0.023940	0.029799	0.041713	0.063279	0.098187	0.148116	0.210123	0.276856	0.340760
0.65	1.666667	0.175240	0.177648	0.186083	0.204748	0.238253	0.289031	0.353855	0.423655	0.488591
0.60	1.904762	0.357839	0.355145	0.357461	0.369289	0.395997	0.440863	0.500682	0.564883	0.621249
0.55	2.142857	0.577317	0.567823	0.560945	0.560906	0.573415	0.602584	0.645684	0.691197	0.724666
0.50	2.380952	0.837781	0.819921	0.800485	0.782466	0.771090	0.771054	0.780768	0.788893	0.779437
0.45	2.619048	1.141457	1.114018	1.078617	1.035557	0.988109	0.940890	0.894375	0.839586	0.760988
0.40	2.857143	1.488451	1.450850	1.396411	1.320628	1.222401	1.104878	0.971929	0.821055	0.641866
0.35	3.095238	1.876035	1.828584	1.752898	1.636691	1.470738	1.254195	0.996313	0.708733	0.396091
0.30	3.333333	2.298572	2.242864	2.145306	1.980017	1.730113	1.380596	0.951238	0.481611	0.010100
0.25	3.571429	2.746636	2.685778	2.568250	2.353970	1.998759	1.479181	0.826748	0.132292	−0.501105
0.20	3.809524	3.207483	3.146339	3.014566	2.751144	2.281064	1.557986	0.635141	−0.311407	−1.073362
0.15	4.047619	3.664068	3.609262	3.474504	3.174864	2.595101	1.656395	0.441050	−0.757695	−1.592112
0.10	4.285714	4.093069	4.052168	3.932542	3.628721	2.983380	1.879746	0.421667	−0.988541	−1.850788
0.05	4.523810	4.470075	4.450425	4.374964	4.143017	3.586166	2.545765	1.029839	−0.647761	−1.885839
bottom	4.761905	4.761905	4.760905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905

Table 3. l/h = 1.0

Height	Horizontal water load	Cantilever load N = 0.00	N = 0.25	N = 0.50	N = 0.75	N = 1.00	N = 1.25	N = 1.50	N = 1.75	N = 2.00
1.00	0	−0.698048	−0.693216	−0.687821	−0.681881	−0.675533	−0.669451	−0.663624	−0.657822	−0.651086
0.95	0.238095	−0.570967	−0.565220	−0.558196	−0.549488	−0.538651	−0.526223	−0.512025	−0.496677	−0.480307
0.90	0.476190	−0.450329	−0.443882	−0.435377	−0.423918	−0.408477	−0.389148	−0.365819	−0.339983	−0.313027
0.85	0.714286	−0.331767	−0.324998	−0.315359	−0.301417	−0.282541	−0.255114	−0.222262	−0.185384	−0.147301
0.80	0.952381	−0.209420	−0.202894	−0.192732	−0.176927	−0.153058	−0.100057	−0.077897	−0.030080	0.018971
0.75	1.190476	−0.076698	−0.071176	−0.061414	−0.044828	−0.018216	0.020216	0.070534	0.128156	0.186896
0.70	1.428571	0.073107	0.076673	0.084761	0.100476	0.127803	0.169346	0.225312	0.290063	0.355519
0.65	1.666667	0.246322	0.246825	0.251607	0.264132	0.289087	0.329922	0.387133	0.454142	0.520901
0.60	1.904762	0.448570	0.444813	0.444311	0.450582	0.468776	0.503132	0.554609	0.615950	0.675181
0.55	2.142857	0.684334	0.675128	0.667080	0.663241	0.668730	0.688330	0.723645	0.767215	0.805416
0.50	2.380952	0.956701	0.940985	0.922964	0.904382	0.887812	0.882876	0.887077	0.895214	0.892785
0.45	2.619048	1.267006	1.244018	1.213612	1.174975	1.129901	1.081922	1.034195	0.981985	0.911782
0.40	2.857143	1.614692	1.584154	1.539244	1.474804	1.387850	1.278597	1.150718	1.004205	0.830754
0.35	3.095238	1.996676	1.959007	1.898141	1.802140	1.659638	1.463953	1.218626	0.933286	0.613962
0.30	3.333333	2.407440	2.363972	2.286895	2.154331	1.941279	1.628445	1.218182	0.739095	0.229979
0.25	3.571429	2.838254	2.791418	2.699727	2.527584	2.229074	1.763735	1.131658	0.397852	−0.332286
0.20	3.809524	3.277667	3.231134	3.129226	2.918679	2.523439	1.870256	0.956922	−0.085868	−1.034621
0.15	4.047619	3.710733	3.669396	3.565701	3.326106	2.834891	1.972982	0.737416	−0.640660	−1.767070
0.10	4.285714	4.117472	4.086840	3.994861	3.750051	3.193873	2.155387	0.630183	−1.045158	−2.287387
0.05	4.523810	4.477107	4.462440	4.403917	4.212668	3.717058	2.709902	1.104970	−0.861944	−2.543285
bottom	4.761905	4.760905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905

Table 4. $l/h = 1.1$

Height	Horizontal water load	Cantilever load								
		N = 0.00	N = 0.25	N = 0.50	N = 0.75	N = 1.00	N = 1.25	N = 1.50	N = 1.75	N = 2.00
1.00	0	−0.758349	−0.754328	−0.748011	−0.741326	−0.733405	−0.724457	−0.714135	−0.703165	−0.691347
0.95	0.238095	−0.617196	−0.612820	−0.605355	−0.596821	−0.585763	−0.571950	−0.554810	−0.535533	−0.514881
0.90	0.476190	−0.480147	−0.475638	−0.467181	−0.456879	−0.442618	−0.423656	−0.399153	−0.370815	−0.340501
0.85	0.714286	−0.343295	−0.338986	−0.329853	−0.318065	−0.300784	−0.276681	−0.244606	−0.206810	−0.166383
0.80	0.952381	−0.201431	−0.197781	−0.188494	−0.175779	−0.156027	−0.127249	−0.087926	−0.040867	0.009486
0.75	1.190476	−0.048745	−0.046347	−0.037509	−0.024937	−0.003764	0.028567	0.074000	0.129210	0.188302
0.70	1.428571	0.120597	0.121036	0.128077	0.139485	0.160369	0.197720	0.243464	0.304350	0.369479
0.65	1.666667	0.312010	0.309706	0.313805	0.322051	0.340113	0.372229	0.421376	0.483522	0.549783
0.60	1.904762	0.530196	0.549036	0.523936	0.526629	0.538328	0.563867	0.607984	0.663054	0.722389
0.55	2.142857	0.778757	0.768566	0.761914	0.756087	0.756674	0.768741	0.796359	0.835746	0.875643
0.50	2.380952	1.059990	1.044804	1.030063	1.012182	0.995522	0.984627	0.983832	0.990115	0.991928
0.45	2.619048	1.374593	1.353973	1.329369	1.295410	1.253812	1.207215	1.159574	1.109330	1.046193
0.40	2.857143	1.721569	1.695439	1.659465	1.605086	1.529208	1.430175	1.310235	1.170441	1.005130
0.35	3.095238	2.097704	2.066506	2.018192	1.939051	1.817926	1.644918	1.418122	1.143371	0.826987
0.30	3.333333	2.497688	2.462539	2.401831	2.294139	2.115526	1.841574	1.462168	0.992485	0.466570
0.25	3.571429	2.913485	2.876296	2.804541	2.665962	2.417269	2.010023	1.419932	0.681515	−0.111819
0.20	3.809524	3.334791	3.298364	3.218997	3.050284	2.721074	2.145584	1.278089	0.192797	−0.902744
0.15	4.047619	3.748407	3.716406	3.635869	3.443867	3.032134	2.261746	1.059361	−0.430297	−1.818730
0.10	4.285714	4.137015	4.113512	4.042060	3.844629	3.371270	2.420851	0.892296	−0.982652	−2.603559
0.05	4.523810	4.482703	4.471509	4.425696	4.268296	3.833101	2.881635	1.240873	−0.964799	−3.108756
bottom	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905

Table 5. $l/h = 1.2$

Height	Horizontal water load	Cantilever load								
		N = 0.00	N = 0.25	N = 0.50	N = 0.75	N = 1.00	N = 1.25	N = 1.50	N = 1.75	N = 2.00
1.00	0	−0.804821	−0.801061	−0.797506	−0.788951	−0.780455	−0.768949	−0.756926	−0.742106	−0.725868
0.95	0.238095	−0.651247	−0.647341	−0.643116	−0.633433	−0.622731	−0.607516	−0.590144	−0.568284	−0.544119
0.90	0.476190	−0.499814	−0.495958	−0.480062	−0.480533	−0.467643	−0.448567	−0.425481	−0.396003	−0.363218
0.85	0.714286	−0.347050	−0.343525	−0.338489	−0.327146	−0.312284	−0.289455	−0.260576	−0.223229	−0.181478
0.80	0.952381	−0.188355	−0.185537	−0.180639	−0.169129	−0.152809	−0.126721	−0.092427	−0.047485	0.003003
0.75	1.190476	−0.018645	−0.017001	−0.012843	−0.001934	0.014935	0.043236	0.081889	0.133344	0.191449
0.70	1.428571	0.167128	0.167052	0.169673	0.178899	0.194888	0.209028	0.264588	0.320289	0.383540
0.65	1.666667	0.357683	0.371187	0.371285	0.377375	0.390413	0.416749	0.456688	0.512666	0.576620
0.60	1.904762	0.604625	0.599312	0.595731	0.596828	0.604055	0.623938	0.657575	0.707448	0.764793
0.55	2.142857	0.863165	0.854351	0.845813	0.839627	0.837248	0.844948	0.864458	0.898417	0.937688
0.50	2.380952	1.150892	1.138101	1.123272	1.107097	1.090227	1.077890	1.071971	1.075367	1.079162
0.45	2.619048	1.468048	1.450994	1.428594	1.399364	1.361885	1.318953	1.271562	1.222920	1.165437
0.40	2.857143	1.813359	1.792052	1.761009	1.715425	1.649888	1.562401	1.451069	1.319356	1.163305
0.35	3.095238	2.183595	2.158451	2.118109	2.052871	1.950471	1.800232	1.593821	1.334845	1.028122
0.30	3.333333	2.573714	2.545663	2.496064	2.408286	2.259052	2.022809	1.678826	1.231237	0.705353
0.25	3.571429	2.976327	2.946899	2.889149	2.777023	2.570411	2.219340	1.681383	0.964002	0.138904
0.20	3.809524	3.382137	3.353524	3.290305	3.154331	2.880979	2.382013	1.580462	0.496929	−0.699593
0.15	4.047619	3.779412	3.754431	3.690697	3.535972	3.192324	2.515805	1.380979	−0.156788	−1.757795
0.10	4.285714	4.152987	4.134736	4.078350	3.918415	3.518002	2.664678	1.180166	−0.822982	−2.787742
0.05	4.523810	4.487249	4.478585	4.442273	4.312488	3.933376	3.050352	1.415519	−0.965240	−3.549030
bottom	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905	4.761905

要　旨

アーチダムが重力ダムに較べて特に異なる点は，水圧荷重を渓谷の底部よりむしろ側壁へ伝達する事にある。従って，アーチダムの安定度は，重力ダムにおける重量よりも材料の抗圧強度のいかんによるものである。アーチダムに適する地点は，その渓谷の側壁が充分アーチ作用に抵抗し得るような峡谷であることが望ましい。

従来，中央片持ばり法によりアーチダムを設計する場合には，水平アーチ要素の分担荷重がアーチの全長にわたって一様に分布していると仮定するのであるから，渓谷の横断面が変化の少ない，狭いU字型の場合に適すると言われているが，その適用範囲は明らかでない。

本報では，アーチダムはアーチ要素と片持ばり要素から成るものと考え，先ず両要素のたわみ一般式を誘導した。次に渓谷の断面形の変化が荷重の分担状態に及ぼす影響を知るために，各種断面形を想定し，それぞれに対する分担荷重を計算し，その数値表を5種類の l/h の値につき作製した。

なお，N＞1.0 の場合は，中央片持ばり法ではダムの高さの半分以下の部分で正確な結果を示さないと思われる。これを確かめるためには，一般の荷重分割法による検討が必要であるが，N＜1.0 の場合には砂防アーチダムの設計計算に本表を用いて有効であろう。