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Various studies to generate and control solitons in nematic liquid crystals

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1. Solitary waves

Zabusky and Kruskal [25] coined the term soliton to describe this particle-like behaviour of the solitary waves. A mathematical definition of a soliton was developed using two prominent features of the propagation of nonlinear waves. These features being that a soliton has a profile which is asymptotically constant at infinity; and solitons interact without changing their form or identity. The only effect of solitons colliding may be a phase change.

In the 1960's, theoretical developments sparked interest in finding physical applications of solitary waves in mediums which included nonlinear optical media. The first exploration of solitary waves in nonlinear optics began with Franken [7] who undertook experiments into optical second harmonics. It was later found that solitary waves or optical beams existed in two forms. These being temporal and spatial solitary waves. Each form are defined by a different nonlinear effect(s). These being:

- **Temporal solitary waves**. These waves occur in a medium which is dispersive, such as, silica glass fibres [10], with a temporal broadening of the beam. The mechanism that drives dispersion is a phase velocity difference between the frequency components of the beam. As the pulse evolves the beam frequency broadens since different components have different phase velocities, resulting in the frequencies moving at varying speeds. Finally, the beam disperses.
- **Spatial solitary wave.** These waves are signified by the broadening of the beam due to diffraction. The differences in the spatial phase velocity of each of the components causes the beam to diffract.

Both temporal and spatial solitary waves or beams can only occur if there is a counterbalance to the beams dispersion/diffraction by the propagation medium. Here the broadening of the wave or beam (in both cases) is balanced by the localised change in the refractive index of the medium. That is, the nonlinearity of the medium modifies the refractive index. This leads to light induced lensing [21] or self focusing [3] so that the combination of self-focusing and beam broadening allows the propagation (or evolution) of temporal and spatial solitary waves in nonlinear media. Nonlinear self-focusing is, therefore, a change in the refractive index of a medium by the beam. When this occurs the beam has formed its own waveguide and is said to be self-trapped [21].

In our discussion, spatial solitary waves will be considered based upon the following observations:

- the large nonlinearities associated with the media that supports spatial solitary waves which produce different physical effects;
- the equations that govern the evolution of spatial solitary waves are highly dependent on the properties of the propagation medium.
- keen interest has been aroused through the proposed use of spatial solitary waves or laser light in technology, especially in electronic switches and gates.

2. Liquid crystal: medium of propagation

Although there are numerous optical media that allow the formation of solitary waves, such as lead glass [19], [20], thermal liquids [6], ion gases [23], photo refractive crystals [22], [13] and Bose-Einstein condensates [15], [9], none of these media possess the same nonlinear refractive index as liquid crystals. A liquid crystal medium has a response that extends way beyond the local change in the refractive index to include the surrounding area of the media. The term used in this respect is 'nonlocal effect'.

Recently, liquid crystals have received intense acknowledgement due to their unique physical and optical properties. Physically, the liquid crystal molecules can flow like a liquid, but also maintains some degree of crystalline structure. Apart from this, liquid crystals can integrate seamlessly with current opto-electronic devices (due to their chemical stability), allowing for an easy transition to its use in all-optical communications. The remarkable benefit that liquid crystals have over other optical media is its large nonlinearity. This nonlinearity extends to in all of its physical states.

Further, liquid crystals are a thermotropic medium, which means that the three phases of a liquid crystal are temperature dependent. These three distinct phases vary from a highly ordered molecular state (cold temperature) to a highly disordered molecular state (hot temperature). The anisotropic, or smectic phase, is a highly structured phase for which the molecules maintain positional and directional ordering in layers, similar to crystal lattices. The next phase is the nematic phase, for which the medium develops the fluidity of a liquid and loses positional ordering within the crystal. However, directional ordering is maintained in the form of a preferred average direction. The final phase is the isotropic phase, for which all order within the crystal is lost [11], [12].

3. Nematic liquid crystal (NLC)

Based on the above explanations spatial solitary waves in nonlinear soft media are deemed to have a nonlocal response. In particular, soft matter like nematic liquid crystals, has nonlinear response which is deemed to be nonlocal. As a consequence, nematic liquid crystals, will be a subject of discussion in the talk given.

It is noted that a nematic liquid crystal (NLC) or the nematic phase of the liquid crystal has molecules that are rod-like structures [11], [12], [17] with no positional order, but a tendency to naturally align due to intermolecular forces, with the long axes of the molecules being close to parallel and in a common direction. These molecules are able to move and are randomly positioned as if in a liquid. However, a long-ranged directional ordering is preserved in space. This, orientational average molecule ordering and having a degree of angular/rotational ordering make NLC an attractive and flexible propagation medium to be used in optical technologies.

4. Nematicon: a solitary wave in NLC

It is noted that if an optical beam enters an optically dense medium, such as an NLC, the beam itself linearly diffracts. This beam can induce a local change in the refractive index of the medium, which results in a self-focusing of the optical beam to a point. Due to these properties in the medium, the formation of a solitary wave exists. Hence, when a coherent optical beam enters the NLC medium there is a balance between two optical effects, that is, linear diffraction and self-focusing. Imposing the nonlinear compensation of diffraction results in a a self-trapped beam solitary wave termed a 'nematicon' [2]. This The stability of a nematicon is due to the nonlocality of the NLC which can be demonstrated by looking at a local medium such as a Kerr medium. Nonlinear beam propagation in local Kerr media is governed by the nonlinear Schrödinger (NLS) equation

$$i\frac{\partial u}{\partial z} \; + \; \frac{1}{2}\nabla^2 u \; + \; |u|^2 u \; = \; 0.$$

In one space dimension we have that

$$\nabla^2 u = u_{xx}$$

and hence the NLS equation is exactly integrable [5] where the solitary wave solutions are stable. However, in two space dimensions,

$$\nabla^2 u = u_{xx} \; + \; u_{yy}$$

solitary wave solutions of the NLS equation show catastrophic collapse. That is, they develop into waves with infinite amplitude and zero width in finite z.

In contrast to the above Kerr affect, as a consequence of the elastic forces where the NLC's response is nonlocal, its affect extends beyond the optical beam. Thus, the refractive index perturbation is wider than the actual beam itself. That is, the NLC has an affects on the nematic molecules outside of the optical beam [1]. The result is that there is stabilisation of the nematicon [17]. Further, an increase in the power of the beam prevents the usual collapse associated with solitary waves in $2D$ local media therefore sustaining the nematicon.

5. Freédericksz threshold

The self-focusing of an optical beam in NLC is enabled provided a minimum power requirement is met. This is known as the Freédericksz threshold, Eth [17], [14]. If an optical beams power is below the Freédericksz threshold then no NLC molecular rotation will occur.

At the beginning, when NLC was first being considered as a propagation medium [4], difficulties in obtaining a self-focusing response were encountered. It was discovered that the use of large optical beam powers $O(1)$W induced a change in the refractive index, but at the same time introduced thermal effects due to localised heating of the NLC. Cumbersome, external cooling of the NLC was then attempted to counteract the localised heating. A dye introduced into the NLC, reduced the power level of the optical beam that was needed to obtain self-focusing. It was observed that there was an increase in the nonlinear response [24]. However, thermal effects were still present which altered the properties of the nematic. An idea emerged to use the pre-existing properties of the NLC to an advantage, which allowed for smaller optical beam powers to be used $O(1)$mW. This meant, at last, the the removal of the thermal effects. Noting that self-focusing of the optical beam requires the nematic molecules to be rotated, a low-frequency static external electric field across the NLC meant that nematic molecules could pre-tilted [2]. This electric field is perpendicular to the direction of the optical beam propagation. Consequently, the balance required to self-trap/form a nematicon is struck at low optical powers. It was found that the Freédericksz threshold is reduced to zero [16] when the NLC molecular pre-tilt angle is $\pi/4$ with the beam direction of propagation.

6. Various studies to generate and control solitons in nematic liquid crystals

The following discussions, work and talk are based on collaborations with Professors Smyth (Edinburgh University, Scotland), Minzoni (UNAM, Mexico) and Assanto (Roma 3, Italy) with contributions by Dr Sciberras (Wollongong University).

Great interest exists in using soliton solutions in non-linear optics. Importantly, the utilisation of spatial solitons (nematicons) in nematics liquid crystals is currently being studied as the backbone of photonic devices to produce all optical switching devices to improve, for instance, the speed of data transmission.

Based on a system of two non-linear partial differential equations where one is the NLS for the optical beam and the other for the nematic response, a mathematical treatise of experimental work on the generation and control of optical solitons in the nonlinear nonlocal nematic media will be discussed.

Using asymptotic methods along with the nematicon being is largely independent of functional form of its profile, it is shown how the optical beam evolves. It will be revealed that a nematicon sheds radiation whereby the velocity and position decoupled from its width and amplitude oscillations. It will be shown that having an additional localized voltage to form various suitable regimes causes the director or nematic to have different orientations with the cell. Some geometries that will be studied are the rectangular cross section, circular and elliptical regions. Further, due to the additional geometrics caused by an external applied voltage to the nematic upon twisting the molecules, the resulting polarized self propagating beams distorts and refracts. The resulting mathematical analysis is quick and efficient and is shown to give excellent agreement to both experimental work and numerical simulations.

Also, the collision of two or more nematicons with relevance to the elasticity of the NLC will be discussed. This is modelled by considering a system of two non-linear partial differential equations with the assumption that the nematicons have a sech profile and cross-sectionally circular shape in an averaged Lagrangian are considered. Comparisons are made by with numerical methods. When the nematicons have an initial separation and angular momentum, it is found that the nematicons form a bounded state, called a nematic on dipole [8]. Due to different diffractions and coupling coefficients, the resulting bounded state exhibits walk-off. There is some disagreement between the analytic and the numerical solutions due to the change in nematic cross-section shape from circular to elliptical. Further, calculation of momentum loss due to shed radiation is less than the numerical.

References

[1] Alberucci, A., Assanto, G., Buccoliero, D., Desyatnikov, A. Marchant, T. and Smyth, N., Modulation analysis of boundary induced motion of optical solitary waves in a nematic liquid crystal. Phys. Rev. A., vol. 79, p. 043816, 2009

[2] Assanto,G., Peccianti, M. and Conti,C., Nematicons: Optical spatial solitons in nematic liquid crystals. Optics & Photonics News, vol. 14, pp. 4448, 2003

[3] Beeckman, J., Neyts, K., Hutsebaut, X., Cambournac, C. and Haelterman, M., Time dependence of soliton formation in planar cells of nematic liquid crystals. IEEE J. Quantum Electron., vol. 41, pp. 735740, 2005

[4] Braun, E., Faucheux, L. and Libchaber, A., Strong self-focusing in nematic liquid crystals. Phys. Rev. A., vol. 48, pp. 611622, 1993

[5] Cao, L. , Zhu, Y., Lu, D, Hu, W, and Guo, Q. 'Propagation of nonlocal optical solitons in lossy media with exponential-decay response.' Optics Communications, 281:50045008, 2008

[6] Dreischuh, A., Neshev, D., Peterson, D., Bang, O. and Krolikowski, W., Observation of attraction between dark solitons. Phys. Rev. Lett., vol. 96, p. 043901, 2006

[7] Franken, P., Hill, A., Peters, C. and Weinreich, G., Generation of optical harmonics. Phys. Rev. Lett., vol. 7, pp. 118119, 1961

[8] Garcia Reimbert, C., Minzoni, A.A., Skuse, B.D., Smyth, N.F., Marchant, T.R. and Worthy, A.L., Mathematical Modelling of Nematicons and their Interactions. 2008 IEEE/LEOS, pp 119-120

[9] Griesmaier, A., Werner, J., Hensler, S., Stuhler, J. and Pfau, T., Bose-einstein condensation of chromium. Phys. Rev. Lett., vol. 94, p. 160401, 2005

[10] Mollenauer, L., Stolen, R. and Gordon, J., Experimental observation of picosecond pulse narrowing and solitons in optical fibers. Phys. Rev. Lett., vol. 45, pp. 10951098, 1980

[11] Khoo, I., Liquid Crystals: Physical Properties and Nonlinear Optical Phenomena. Wiley, 2nd ed., 1995

[12] Khoo, I., Nonlinear optics of liquid crystalline materials. Phys. Rep., vol. 471, pp. 221267, 2009

[13] Krolikowski, W., Saffman, M., Luther-Davies, B. and Denz, C., Anomalous interaction of spatial solitons in photorefractive media. Phys. Rev. Lett., vol. 80, pp. 32403243, 1998

[14] McLaughlin, D., Muraki, D. and Shelley,M., Self-focussed optical structures in a nematic liquid crystal. Physica D, vol. 97, no. 4, pp. 471497, 1996

[15] Parola, A., Salasnich, L. and Reatto, L., Structure and stability of bosonic clouds: Alkali-metal atoms with negative scattering length. Phys. Rev. A., vol. 57, pp. R3180R3183, 1998

[16] Peccianti, M., Brzdakiewicz, K. and Assanto, G., Nonlocal spatial soliton interactions in nematic liquid crystals. Opt. Lett., vol. 27, pp. 14601462, 2002

[17] Peccianti , M. and Assanto, G., Nematicons. Phys. Reports, vol. 516, pp. 147208, 2012

[18] Rotschild, C., Alfassi, B., Cohen, O. and Segev, M., Long-range interactions between optical solitons. Nat. Phys., vol. 2, pp. 769774, 2006

[19] Rotschild, C., Cohen, O., Manela, O., Segev, M. and Carmon, T., Solitons in nonlinear media with an infinite range of nonlocality: first observation of coherent elliptic solitons and of vortex-ring solitons. Phys. Rev. Lett., vol. 95, p. 213904, 2005

[20] Rotschild, C., Alfassi, B., Cohen, O. and Segev, M., Long-range interactions between optical solitons. Nat. Phys., vol. 2, pp. 769774, 2006

[21] Segev, M. Optical spatial solitons. Opt. & Quantum Electron., vol. 30, pp. 503533, 1998

[22] Segev, M., Crosignani, B., Yariv, A. and Fisher, B., Spatial solitons in photorefractive media. Phys. Rev. Lett., vol. 68, pp. 923926, 1992

[23] Suter, D. and Blasberg,T., Stabilization of transverse solitary waves by a nonlocal response of the nonlinear medium. Phys. Rev. A., vol. 48, pp. 45834587, 1993

[24] Warenghem, M., Henninot, J. and Abbate, G., Nonlinearly induced self wave guiding structure in dye doped nematic liquid crystals confined in capillaries. Opt. Express, vol. 2, pp. 483490, 1998

[25] Zabusky, N. and Kruskal, M., Interaction of solitons in collisionless plasma and the recurrence of initial states. Phys. Rev. Lett., vol. 15, pp. 240243, 1965