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Modeling Agricultural Tractor Motion on Sloping Ground Considering Longitu–dinal and Lateral Gradients

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Tractor overturning or rollover is a serious accident owing to its high fatality rate. Reviews of farm tractor rollover incidents indicate that overturns account for over 50% of all tractor-related deaths. To understand a tractor's performance on uneven terrain, a three-dimensional (3D) mathematical model describing the general bouncing and pitching motions of a tractor would be very useful. In this study, we design a rigid-body tractor system with the tires modeled as spring-damper units. To predict the tractor's motion on undulating surfaces, a half sine wave bump is adopted for the front and rear wheels on both the longitudinal and transverse slopes. The tractor's dimensional parameters and physical properties are taken from the work of Takeda *et al.* (2010). The passing over process is divided into four detailed phases and the attitude of the tractor is obtained by capturing the motion of the vehicle's center of gravity (COG). The displacement, pitch angle, and accelerations of the COG are numerically analyzed by applying the Runge-Kutta method. We introduce in this study three coordinates for the vehicle, slope and horizontal ground-based observations. The results are presented as factors of forward velocity, bump height, and slope gradient. Predictions show that there is a decrease in stability and an increase in danger at higher velocities, higher bump heights, and steep inclines. The specific cases indicating dangerous situations for the tractor are pointed out according to the results.

Key words: tractor dynamics, mathematical model, longitudinal and lateral gradients, affecting factor

INTRODUCTION

With the popularization of agricultural machinery, the use of tractors has greatly increased because of their compatibility with a variety of agricultural implements. However, non-neglectful safety problem still troubles and threatens the operators' lives. Research on trac-tor accidents has shown that tractor rollovers are responsible for more than half of trac-tor-related accidents (Abubakar *et al.*, 2010). In Japan, 256 fatal accidents while operating farm machinery were reported in 2012, of which 106 (41%) were associated with tractors. Further-more, 72 of these deaths were attributed to tractor rollovers, accounting for 68% of trac-tor-related fatalities (Ministry of Agriculture, Forestry and Fisheries of Japan, 2014).

Considering the high percentage of fatalities due to tractor rollovers, a fundamental under-standing of the mechanism and factors that cause instability in a tractor should be prioritized, despite the fact that the Roll Over Protective Structure (ROPS) provides a passive means of safety. The development of a mathematical tractor model would be very useful in order to pre-dict the dynamic behavior of tractors in a parameterized environment.

Treating tractor tires as rigid bodies (without elastic characteristics) is an assumption made in some models in the field in order to render applicable to them the Law of Conservation of En-ergy and the Lagrange equations (Yang et al., 1991; Ahmadi, 2011, 2013). Moreover, the geo-metric relationships of tractor rotation are often simplified to be linear, which consequently constrains the rotation motions to be small (Homori et al., 2003; Takeda et al., 2010a, 2010b). In consideration of the fact that tractor rollovers frequently occur on uneven sloping ground, it is useful to model tractor motion on slopes instead of horizontal ground, which is traditionally assumed for vibration analysis (Rabbani et al., 2011). Furthermore, instead of transversal in-clines (Ahmadi, 2011, 2013; Li et al., 2014a, 2014b, 2015; Li et al., 2015a, 2015b), slopes con-sidering both longitudinal and lateral gradients refer to a more general condition under which a practical tractor is operated. The objective of our study, therefore, is to model tractor motion that is applicable to large rotations on sloping ground involving longitudinal and lateral gradi-ents when considering the spring damping characteristics of tires. We present tractor stability from the perspective of ground supporting forces and friction acting on the vehicle by taking into account tractor velocity, obstacle height, and slope gradient.

MATERIALS AND METHODS

As a general case, the ground on which a tractor travels is not horizontal but an inclined plane. Figure 1 below

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shows the model of a tractor traveling over an obstacle on slopes in a three–dimensional (3D) situation. The sloping ground is defined along both the lateral and lon– gitudinal dimensions. The vehicle coordinates o-xyz, slope coordinates O-XYZ, and flat ground coordinates $e-\xi_1\xi_2\xi_3$ are used for motion solution and reference observation. Road excitation stems from a half–sine obstacle set under the front tires in the tractor's path. Subsequently, trac–tor behavior is presented by motion descriptions of the center of gravity (COG) of the tractor and external force variations applied to the tires. The key assumptions for developing the model are as follows:



Fig. 1. Three–dimensional model of tractor driving over an obstacle on sloping ground.

- The tractor has a rigid body travelling at a constant speed.
- The tires are modeled as spring damping units.
- The COG is located on the tractor's longitudinal midplane.
- There is no yaw motion of the tractor.
- Tire-ground contact occurs at points.

Since the tractor climbs up the longitudinal slope, the tractor bounce displacement caused by the obstacle is more meaningful if described in the slope coordinates:

$$m\ddot{z}_{g} = \sum F_{i} - \operatorname{mg} \cos\alpha \, \cos\beta \tag{1}$$

where, *m* is mass of the tractor, kg; z_{g} is the vertical displacement of the COG, m; F_{i} is the ground supporting force of the tire *i*, N; α is the longitudinal gradient, rad; β is the lateral gradi–ent, rad; g is the gravitational acceleration, 9.81 m s⁻².

Tractor rotation can be primarily calculated from the vehicle coordinates. Although tractor roll is not the main concern in this study, it does have a minor effect on vehicle attitude. The existence of the slope redistributes the ground supporting forces and therefore creates an initial orientation of the tractor that differs from the nonsloping ground. Considering the nonlinear geometric features, the pitch and roll motions are given as follows:

$$I_x \dot{\phi} = \sum M_x \tag{3}$$

where, $I_{x/y}$ is the tractor's moment of inertia about the x/y axis, kg m²; θ is the pitch angle, rad; ϕ is the roll angle, rad; $M_{x/y}$ is the moment of force about the x/y axis, N m.

The supporting force applied to each tire involves spring damping characteristics, and is therefore defined as

$$F_i = -k_i z_i - c_i \dot{z}_i \tag{4}$$

where, k_i is the spring constant of tire *i*, N m⁻¹; c_i is the viscous damping coefficient of tire *i*, N s m⁻¹; z_i is the vertical deformation of tire *i*, m.

Introducing sloping ground conditions consequently causes friction and the possibility of slide. Before the occurrence of slide, the friction due to the longitudinal and lateral gradients (α and β) can be presented as follows:

$$f_{\alpha} = mg \sin \alpha \tag{5}$$

$$f_{\beta} = mg \sin\beta \tag{6}$$

Furthermore, noting that the tractor's lateral slide may start at the front or rear wheels, it is important to monitor the variations in the corresponding resultant frictions. They can be ob-tained from the following two equations:

$$f_{\beta} = f_f + f_r \tag{7}$$
$$l_t f_f - l_r f_r = 0 \tag{8}$$

where, f_{jjr} is resultant lateral friction of the front/rear tires, N.

Considering that the tractor bounce and supporting forces are assumed to be vertical to the slope surface, tire deformation is then defined in the slope coordinates as

$$z_{i} = z_{G} + z_{p} + z_{r} + z_{s} - f_{i}(t)$$
(9)

where, z_p is the tire deformation caused by tractor pitch, m; z_r is the tire deformation caused by tractor roll, m; z_s is the static tire deformation, m; $f_i(t)$ is an obstacle profile function applied to tire i, m; t is time, s.

The final term of Equation (9) represents the extent of the initial deformation of the tires on flat ground. When the tractor travels on slopes, these values change to some extent. As a con-sequence, a stable tractor on slopes shows an attitude relative to that on flat ground.

For space-fixed observation, it is necessary to transform the rotational results in the vehicle coordinates to those in the space-fixed (slope or flat-ground fixed) coordinates. Following the sequence of the roll, pitch, and yaw, the transformation matrix to the slope coordinates gener-ates from

$$A = A_z A_y A_x \tag{10}$$

$$I_{y}\ddot{\theta} = \sum M_{y} \tag{2}$$



Fig. 2. Schematic presentation of obstacle surmounting divisions.

where, A is transformation matrix; A_{XYYZ} is a step transformation matrix according to the ro-tation order of the vehicle coordinates.

The angular velocities in the slope coordinate then derive from

$$\frac{\mathrm{d}(\mathrm{A}^{\mathrm{T}})}{\mathrm{d}\mathrm{t}}\mathrm{A} = \begin{pmatrix} 0 & -\omega_{z} & \omega_{y} \\ \omega_{z} & 0 & -\omega_{x} \\ -\omega_{x} & \omega_{x} & 0 \end{pmatrix}$$
(11)

where, ω_{XYZ} is the component of the tractor's angular velocity about the *X/Y/Z* axis, rad s⁻¹.

The corresponding angle values Θ , Φ , and Ψ afterwards are yielded by integrating ω_x , ω_y , and ω_z . Recall that we assume that tractor yaw does not occur. Thus, we can ignore the global yaw angle after coordinate transformation because of the negligible values of ω_z . To monitor the tractor status if it approaches the overturning threshold, the coordinates fixed on the flat ground are used for rotational angle observation.

To investigate the detailed obstacle surmounting process of the tractor, we adopt the parti-tioning method proposed by Yamamoto and Shimada (1957). The four passage periods are as-signed as Fig. 2 shows. Periods 1 and 3 refer to the obstacle contact durations of the front and rear tires, respectively, whereas periods 2 and 4 involve no obstacle surmounting by the tractor. Thus, two periods of external excitation vibration and dampedfree vibration arise, as shown in Fig. 2.

RESULTS AND DISCUSSION

We developed a computer program to solve the differential equations. To investigate tractor behavior under the influences of forward velocity, obstacle height, and slope gradient, we de-fined a reference case with the parameters 0.5 m s^{-1} , 0.08 m, and 10° (for both longitudinal and lateral slopes), respectively, for each term. Because the obstacle is set transversely under both the right and left side tires for tractor front and rear wheels, the lateral gradient in this case has minor dynamic effect on tractor behavior when considering obstacle-passing process. Therefore the cases we focused include factors of tractor velocity, obstacle height, and longitudinal gradient. These parameter values are listed in Table 1.

Since the responses of a vehicle are dominated by external forces, it is necessary to monitor the variations of the ground supporting forces and friction. Fig. 3 shows the result of supporting force F_{zi} for the reference case. We find that the front-right tire is more susceptible, when en-countered with an obstacle on a slope, to lose contact with the ground (by approaching zero) than the other tires. Thus, F_{zi} is selected as the major parameter describing the tire-ground con-tact condition.

Figure 4 shows the effect of tractor velocity on vehicle motion responses. As the other pa-rameters are the same as in the reference case, velocity values of 0.5 m s^{-1} , 0.8 m s^{-1} , and 1.1 m s^{-1} are chosen for comparison. From the results, we see that tractor bounce displacement and pitch angle tend to fluctuate rapidly as tractor velocity increases. In particular, drops arise when the tractor nearly travels over the obstacle peak in period 3 at a

Parameters	Reference case	Velocity factor	Obstacle factor	Longitudinal slope factor
Tractor velocity/m s^{-1}	0.5	0.5, 0.8, 1.1	0.5	0.5
Obstacle height/m	0.08	0.08	0.08, 0.14, 0.20	0.08
Longitudinal gradient/°	10	10	10	10, 20, 25
Lateral gradient/°	10	10	10	10

Table 1. Parameters used in case study



Fig. 3. Ground supporting forces on tires for the reference case.



Fig. 4. Effect of velocity on: (a) tractor bounce displacement; (b) tractor bounce acceleration; (c) tractor pitch angle; (d) tractor pitch acceleration, considering velocities of 0.5 m s⁻¹, 0.8 m s⁻¹, and 1.1 m s⁻¹.

velocity of 1.1 m s^{-1} (Figs. 4(a) and (c)). Apparent growth in tractor bounce and pitch acceleration are simultaneously observed with the increase in tractor velocity, as shown in Figs. 4(b) and (d).

Figure 5 shows the effect of tractor velocity on F_{z1} . When the velocity is 0.8 m s⁻¹, the front–right tire lifts from the ground twice (in periods 1 and 3), whereas both the occurrence and duration of the off–ground status increase for a 1.1 m s⁻¹ velocity. Since the loss of contact of the uphill tire is traditionally defined as the onset of rollover, cases where the tractor's velocity is 0.8 m s⁻¹ and 1.1 m s⁻¹ are thus considered dangerous in terms of safety.

The lateral frictions are shown in Fig. 6, where the longitudinal friction is found not to be ap-proaching its limit value in these cases. According to Equations (6) through (8), the total resul-tant lateral friction is divided into friction applied to the tractor's front and rear tires. The resolved components of friction are found to reach their respective limit values for tractor ve-locities of



Fig. 5. Variation of F_{z_1} at tractor velocities of 0.5 m s⁻¹, 0.8 m s⁻¹, and 1.1 m s⁻¹.



Fig. 6. Lateral resultant frictions applied to the front and rear tires at a velocity of: (a) 0.8 m s^{-1} ; (b) 1.1 m s^{-1} .



Fig. 7. Effect of obstacle height on: (a) tractor bounce displacement; (b) tractor bounce acceleration; (c) tractor pitch angle; (d) tractor pitch acceleration, considering obstacle heights of 0.08 m, 0.14 m, and 0.20 m.

 0.8 m s^{-1} and 1.1 m s^{-1} . In Fig. 6 (a), the tractor sideslips twice at the front tires while the rear tires remain stable. When the velocity is 1.1 m s^{-1} , however, the front tire sideslip in-creases to three times while rear-sideslip rises to twice. Therefore, a 1.1 m s^{-1} tractor velocity is also extremely risky with regard to sideslip.

Obstacle height effects on tractor motion responses are shown in Fig. 7. We can see that as the obstacle height increases, the tractor bounce displacement and pitch angle exhibit identical gradual growth tendencies. Nevertheless, the fluctuations near the peaks of periods 1 and 3 tend to be apparently intensive when the tractor travels over a 0.20 m high obstacle. Furthermore, the acceleration amplitudes are significantly enlarged with the increment in obstacle height, indi-cating the insta-



Fig. 8. Effect of longitudinal gradient on: (a) F_{z1} ; (b) friction, considering gradients of 10°, 20°, and 25°.

bility of the vehicle.

Gradients of 10°, 20°, and 25° are defined as the longitudinal slope parameters. With regard to rollover, Fig. 8 (a) indicates two and three occurrences of the onset of lateral rollover, con–sidering 20° and 25° slopes, respectively. Referring to longitudinal tractor slide, on the other hand, Fig. 8 (b) implies a relative high margin in safety even for a slope of up to 25°. In the case of sideslip, both the 20° and 25° slopes lead to two front sideslips, while no rear sideslips are observed.

CONCLUSIONS

In this study, we formulated a 3D mathematical model of tractor motion over a half-sine ob-stacle on undulating ground. Our model is applicable to large tractor rotational motion because we introduced unsimplified nonlinear geometric relationships while taking into account the conditions governing the contact between the tractor tire and the ground. The sloping ground on which the tractor moved was defined in both the longitudinal and lateral dimensions. The cor-responding frictions indicating the tractor's sliding status and the ground supporting forces dominating tractor behavior can be calculated.

Our case study predicted the influence of tractor velocity, obstacle height, and slope gradient on vehicle motion and force variation. According to the results, higher tractor velocities lead to intensive motion fluctuations and, therefore, larger acceleration amplitudes that increase the danger of rollover and sideslip. In particular, a velocity of 1.1 m s⁻¹ was determined to be ex-tremely risky. Similarly, increasing obstacle height and longitudi-

nal slope gradient results in apparent instability of tractor, with risks of rollover and slide. An obstacle 0.20 m high with a longitudinal slope of 25° was judged risky. Thus, we concluded that values of velocity, obstacle height, and longitudinal slope gradients over 1.1 m s⁻¹, 0.20 m, and 25° , respectively, must be avoided.

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