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Modeling Agricultural Tractor Motion on Sloping Ground Considering Longitudinal and Lateral Gradients

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INTRODUCTION

With the popularization of agricultural machinery, the use of tractors has greatly increased because of their compatibility with a variety of agricultural implements. However, non–neglectful safety problem still troubles and threatens the operators’ lives. Research on tractor accidents has shown that tractor rollovers are responsible for more than half of tractor–related accidents. To understand a tractor’s performance on uneven terrain, a three–dimensional (3D) mathematical model describing the general bouncing and pitching motions of a tractor would be very useful. In this study, we design a rigid–body tractor system with the tires modeled as spring–damper units. To predict the tractor’s motion on undulating surfaces, a half sine wave bump is adopted for the front and rear wheels on both the longitudinal and transverse slopes. The tractor’s dimensional parameters and physical properties are taken from the work of Takeda et al. (2010). The passing over process is divided into four detailed phases and the attitude of the tractor is obtained by capturing the motion of the vehicle’s center of gravity (COG). The displacement, pitch angle, and accelerations of the COG are numerically analyzed by applying the Runge–Kutta method. We introduce in this study three coordinates for the vehicle, slope and horizontal ground–based observations. The results are presented as factors of forward velocity, bump height, and slope gradient. Predictions show that there is a decrease in stability and an increase in danger at higher velocities, higher bump heights, and steeper inclines. The specific cases indicating dangerous situations for the tractor are pointed out according to the results.

Key words: tractor dynamics, mathematical model, longitudinal and lateral gradients, affecting factor

MATERIALS AND METHODS

As a general case, the ground on which a tractor travels is not horizontal but an inclined plane. Figure 1 below
shows the model of a tractor traveling over an obstacle on slopes in a three-dimensional (3D) situation. The sloping ground is defined along both the lateral and longitudinal dimensions. The vehicle coordinates \( o-xyz \), slope coordinates \( O-XYZ \), and flat ground coordinates \( e-\xi,\eta,\zeta \), are used for motion solution and reference observation. Road excitation stems from a half-sine profile function applied to tire \( i \), \( N \cdot m \). The supporting force applied to each tire involves spring damping characteristics, and is therefore defined as

\[
F_i = k_i z_i + c_i \dot{z}_i \tag{4}
\]

where, \( k_i \) is the spring constant of tire \( i \), \( N \cdot m^{-1} \); \( c_i \) is the viscous damping coefficient of tire \( i \), \( N \cdot m^{-1} \); \( z_i \) is the vertical deformation of tire \( i \), m.

Introducing sloping ground conditions consequently causes friction and the possibility of slide. Before the occurrence of slide, the friction due to the longitudinal and lateral gradients (\( \alpha \) and \( \beta \)) can be presented as follows:

\[
f_{\alpha} = mg \sin \alpha \tag{5}
\]

\[
f_{\beta} = mg \sin \beta \tag{6}
\]

Furthermore, noting that the tractor’s lateral slide may start at the front or rear wheels, it is important to monitor the variations in the corresponding resultant frictions. They can be obtained from the following two equations:

\[
f_{r} = f_{r} + f_{f} \tag{7}
\]

\[
l_f f_{r} - l_r f_{f} = 0 \tag{8}
\]

where, \( f_{r} \) is resultant lateral friction of the front/rear tires, N.

Considering that the tractor bounce and supporting forces are assumed to be vertical to the slope surface, tire deformation is then defined in the slope coordinates as

\[
z_{s} = z_{c} + z_{r} + z_{p} + z_{c} + f_i(t) \tag{9}
\]

where, \( z_{c} \) is the tire deformation caused by tractor pitch, \( m \); \( z_{r} \) is the tire deformation caused by tractor roll, \( m \); \( z_{p} \) is the static tire deformation, \( m \); \( f_i(t) \) is an obstacle profile function applied to tire \( i \), \( m \); \( t \) is time, s.

The final term of Equation (9) represents the extent of the initial deformation of the tires on flat ground. When the tractor travels on slopes, these values change to some extent. As a con-sequence, a stable tractor on slopes shows an attitude relative to that on flat ground.

For space-fixed observation, it is necessary to transform the rotational results in the vehicle coordinates to those in the space-fixed (slope or flat-ground fixed) coordinates. Following the sequence of the roll, pitch, and yaw, the transformation matrix to the slope coordinates generates from

\[
A = A_{r} A_{p} A_{v} \tag{10}
\]
where, $A$ is transformation matrix; $A_{xyz}$ is a step transformation matrix according to the rotation order of the vehicle coordinates.

The angular velocities in the slope coordinate then derive from

$$
\frac{d(A^\prime)}{dt} A = \left( \begin{array}{ccc}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0 \\
\end{array} \right)
$$

(11)

where, $\omega_{xyz}$ is the component of the tractor’s angular velocity about the $X/Y/Z$ axis, rad s$^{-1}$.

The corresponding angle values $\Theta$, $\Phi$, and $\Psi$ afterwards are yielded by integrating $\omega_x$, $\omega_y$, and $\omega_z$. Recall that we assume that tractor yaw does not occur. Thus, we can ignore the global yaw angle after coordinate transformation because of the negligible values of $\omega_z$. To monitor the tractor status if it approaches the overturning threshold, the coordinates fixed on the flat ground are used for rotational angle observation.

To investigate the detailed obstacle surmounting process of the tractor, we adopt the partitioning method proposed by Yamamoto and Shimada (1957). The four passage periods are as-signed as Fig. 2 shows. Periods 1 and 3 refer to the obstacle contact durations of the front and rear tires, respectively, whereas periods 2 and 4 involve no obstacle surmounting by the tractor. Thus, two periods of external excitation vibration and damped-free vibration arise, as shown in Fig. 2.

### RESULTS AND DISCUSSION

We developed a computer program to solve the differential equations. To investigate tractor behavior under the influences of forward velocity, obstacle height, and slope gradient, we defined a reference case with the parameters $0.5$ m s$^{-1}$, $0.08$ m, and $10^\circ$ (for both longitudinal and lateral slopes), respectively, for each term. Because the obstacle is set transversely under both the right and left side tires for tractor front and rear wheels, the lateral gradient in this case has minor dynamic effect on tractor behavior when considering obstacle-passing process. Therefore the cases we focused include factors of tractor velocity, obstacle height, and longitudinal gradient. These parameter values are listed in Table 1.

Since the responses of a vehicle are dominated by external forces, it is necessary to monitor the variations of the ground supporting forces and friction. Fig. 3 shows the result of supporting force $F_{zi}$ for the reference case.

We find that the front-right tire is more susceptible, when encountered with an obstacle on a slope, to lose contact with the ground (by approaching zero) than the other tires. Thus, $F_{z1}$ is selected as the major parameter describing the tire-ground contact condition.

Figure 4 shows the effect of tractor velocity on vehicle motion responses. As the other parameters are the same as in the reference case, velocity values of $0.5$ m s$^{-1}$, $0.8$ m s$^{-1}$, and $1.1$ m s$^{-1}$ are chosen for comparison. From the results, we see that tractor bounce displacement and pitch angle tend to fluctuate rapidly as tractor velocity increases. In particular, drops arise when the tractor nearly travels over the obstacle peak in period 3 at a...
velocity of 1.1 m s$^{-1}$ (Figs. 4(a) and (c)). Apparent growth in tractor bounce and pitch acceleration are simultaneously observed with the increase in tractor velocity, as shown in Figs. 4(b) and (d).

Figure 5 shows the effect of tractor velocity on $F_{z_1}$. When the velocity is 0.8 m s$^{-1}$, the front-right tire lifts from the ground twice (in periods 1 and 3), whereas both the occurrence and duration of the off-ground status increase for a 1.1 m s$^{-1}$ velocity. Since the loss of contact of the uphill tire is traditionally defined as the onset of rollover, cases where the tractor’s velocity is 0.8 m s$^{-1}$ and 1.1 m s$^{-1}$ are thus considered dangerous in terms of safety.

The lateral frictions are shown in Fig. 6, where the longitudinal friction is found not to be approaching its limit value in these cases. According to Equations (6) through (8), the total resultant lateral friction is divided into friction applied to the tractor’s front and rear tires. The resolved components of friction are found to reach their respective limit values for tractor velocities of

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**Fig. 3.** Ground supporting forces on tires for the reference case.

**Fig. 4.** Effect of velocity on: (a) tractor bounce displacement; (b) tractor bounce acceleration; (c) tractor pitch angle; (d) tractor pitch acceleration, considering velocities of 0.5 m s$^{-1}$, 0.8 m s$^{-1}$, and 1.1 m s$^{-1}$.

**Fig. 5.** Variation of $F_{z_1}$ at tractor velocities of 0.5 m s$^{-1}$, 0.8 m s$^{-1}$, and 1.1 m s$^{-1}$.

**Fig. 6.** Lateral resultant frictions applied to the front and rear tires at a velocity of: (a) 0.8 m s$^{-1}$; (b) 1.1 m s$^{-1}$.
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0.8 m s\(^{-1}\) and 1.1 m s\(^{-1}\). In Fig. 6 (a), the tractor slideslips twice at the front tires while the rear tires remain stable. When the velocity is 1.1 m s\(^{-1}\), however, the front tire sideslip increases to three times while rear–sideslip rises to twice. Therefore, a 1.1 m s\(^{-1}\) tractor velocity is also extremely risky with regard to sideslip.

Obstacle height effects on tractor motion responses are shown in Fig. 7. We can see that as the obstacle height increases, the tractor bounce displacement and pitch angle exhibit identical gradual growth tendencies. Nevertheless, the fluctuations near the peaks of periods 1 and 3 tend to be apparently intensive when the tractor travels over a 0.20 m high obstacle. Furthermore, the acceleration amplitudes are significantly enlarged with the increment in obstacle height, indicating the instability of the vehicle.

Gradients of 10\(^{\circ}\), 20\(^{\circ}\), and 25\(^{\circ}\) are defined as the longitudinal slope parameters. With regard to rollover, Fig. 8 (a) indicates two and three occurrences of the onset of lateral rollover, considering 20\(^{\circ}\) and 25\(^{\circ}\) slopes, respectively. Referring to longitudinal tractor slide, on the other hand, Fig. 8 (b) implies a relative high margin in safety even for a slope of up to 25\(^{\circ}\). In the case of sideslip, both the 20\(^{\circ}\) and 25\(^{\circ}\) slopes lead to two front sideslips, while no rear sideslips are observed.

CONCLUSIONS

In this study, we formulated a 3D mathematical model of tractor motion over a half–sine obstacle on undulating ground. Our model is applicable to large tractor rotational motion because we introduced unsimplified nonlinear geometric relationships while taking into account the conditions governing the contact between the tractor tire and the ground. The sloping ground on which the tractor moved was defined in both the longitudinal and lateral dimensions. The corresponding frictions indicating the tractor’s sliding status and the ground supporting forces dominating tractor behavior can be calculated.

Our case study predicted the influence of tractor velocity, obstacle height, and slope gradient on vehicle motion and force variation. According to the results, higher tractor velocities lead to intensive motion fluctuations and, therefore, larger acceleration amplitudes that increase the danger of rollover and sideslip. In particular, a velocity of 1.1 m s\(^{-1}\) was determined to be extremely risky. Similarly, increasing obstacle height and longitudi-

Fig. 7. Effect of obstacle height on: (a) tractor bounce displacement; (b) tractor bounce acceleration; (c) tractor pitch angle; (d) tractor pitch acceleration, considering obstacle heights of 0.08 m, 0.14 m, and 0.20 m.

Fig. 8. Effect of longitudinal gradient on: (a) \(F_z\); (b) friction, considering gradients of 10\(^{\circ}\), 20\(^{\circ}\), and 25\(^{\circ}\).
nal slope gradient results in apparent instability of tractor, with risks of rollover and slide. An obstacle 0.20 m high with a longitudinal slope of 25° was judged risky. Thus, we concluded that values of velocity, obstacle height, and longitudinal slope gradients over 1.1 m s$^{-1}$, 0.20 m, and 25°, respectively, must be avoided.

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