Sparsity and Information Processing

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Sparsity and Information Processing

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Abstract Recently, many information processing methods utilizing the sparsity of the information source is studied. We have reported some results on this line of research. Here we pick up two results from our own works. One is an image reconstruction method for radio interferometry and the other is a motor command computation method for a two-joint arm.

Keywords: Sparsity, interferometry, motor control

1 Introduction

Since the proposal of LASSO [1] and Compressed Sensing [2], sparsity based methods are getting popular. We have proposed some related methods in different fields which include astronomy [3, 4], diffraction imaging [5], and neuroscience [6, 7]. Here, we show two of them.

2 Interferometry

First result is the image reconstruction method for radio interferometry in astronomy [4]. In radio astronomy, the VLBI (Very Long Vaseline Interferometry) is an important technology since it has a very high angular resolution. In VLBI, a signal from an astronomical radio source is recorded with multiple radio telescopes located in a remote locations on the earth. By combining the measurements of telescopes, an observation vector $\boldsymbol{y} \in \Re^M$ is computed. This vector corresponds to the Fourier transform of the image vector $\boldsymbol{x} \in \Re^N$. The relation between $\boldsymbol{y}$ and $\boldsymbol{x}$ is denoted as follows

$$\boldsymbol{y} = A\boldsymbol{x} + \boldsymbol{n}.$$

Here, $\boldsymbol{n}$ is the noise vector and A is a truncated Fourier matrix. The image reconstruction problem is to compute $\boldsymbol{x}$ from $\boldsymbol{y}$ knowing the matrix A. The problem is not straight-forward since $M < N$ generally holds and $\boldsymbol{x}$ cannot be computed by the inverse Fourier transformation. The image reconstruction is an ill-posed problem.

When the astronomical object is compact, we can expect $\boldsymbol{x}$ to be sparse, that is, a lot of components of $\boldsymbol{x}$ are 0. Thus, we can use LASSO to compute $\boldsymbol{x}$ from $\boldsymbol{y}$. More specifically, $\boldsymbol{x}$ is computed by solving the following optimization problem.

$$\hat{\boldsymbol{x}}_\lambda = \arg\min_{\boldsymbol{x}} \left[\|\boldsymbol{y} - A\boldsymbol{x}\|_2^2 + \lambda \|\boldsymbol{x}\|_1 \right] \qquad \text{subject to} \qquad x_i \geq 0 \quad (i = 1, \cdots, N).$$

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We have demonstrated the algorithm with simulated and real observed data and the proposed image reconstruction method will be used for the Event-Horizon Telescope [8].

3 Motor Control

The previous work was a method to solve an inverse problem. We have also proposed a method to generate the motor command for a two-joint arm [6, 7].

We have prepared an anatomical plausible two-joint arm model and have assumed the motor command representation in the brain is sparse. This framework enables us to reconstruct smooth movements of the arm for reaching tasks. It is also possible to store some motor commands and design new movements by combining stored motor commands (Fig. 1).

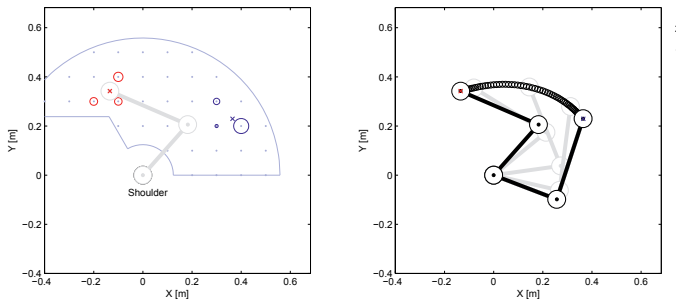

Figure 1: Left: Design of the motor control. The motor commands from a grid to another grid were stored and for a new movements, they are combined. Right. A smooth trajectory of the movement of the arm is generated by the proposed method.

References

[1] R. Tibshirani. Regression shrinkage and selection via the lasso. *J. R. Stat. Soc., Ser. B*, 58(1):267–288, 1996.

[2] D. Donoho. Compressed sensing. *IEEE trans. Inform. Th.*, 52(4):1289–1306, 2006.

[3] S. Ikeda, H. Odaka, M. Uemura, T. Takahashi, S. Watanabe, and S. Takeda. Bin mode estimation methods for Compton camera imaging. *Nucl. Inst. and Methods in Phys. Res. A*, 760:46–56, 2014.

[4] M. Honma, K. Akiyama, M. Uemura, and S. Ikeda. Super-resolution imaging with radio interferometry using sparse modeling. *Publ. Astron. Soc. Jpn*, 66(5):95, 2014.

[5] S. Ikeda and H. Kono. Phase retrieval from single biomolecule diffraction pattern. *Optics Express*, 20(4):3375–3387, 2012.

[6] Y. Sakaguchi and S. Ikeda. Motor planning and sparse motor command representation. *Neurocomputing*, 70(10-12):1748–1752, June 2007.

[7] S. Ikeda and Y. Sakaguchi. Motor planning as an optimization of command representation. In *Proc. 48th IEEE Conf. on Decision and Control*, pages 4499–4504, December 2009.

[8] Event Horizon Telescope official web page: `http://eventhorizontelescope.org`