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Bertails-Descoubes, Florence
INRIA | Laboratoire Jean Kuntzmann | Grenoble University | CNRS

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Geometry and Mechanics of fibers: Some numerical models

Florence Bertails-Descoubes *

INRIA and Laboratoire Jean Kuntzmann (Grenoble University, CNRS)

(Joint work with Romain Casati, Gilles Daviet, and Alexandre Derouet-Jourdan)

Abstract In this talk I will give an overview of our work on the simulation of fibers and entangled materials, such as hair, with a specific interest for virtual prototyping and computer graphics applications. I will first introduce a family of high-order, reduced models for discretizing Kirchhoff's equations for thin elastic rods in a both faithful and robust way. Such models are particularly well-suited for simulating inextensible fibers subject to bending and twisting, and featuring an arbitrary curly resting geometry. Then I will show how such models can be coupled to frictional contact using the nonsmooth contact dynamics framework, and I will present a hybrid iterative solver suitable for robustly handling thousands packed fibers at reasonable frame rates. Finally, I will give some insights into the inverse modeling of fibers, consisting in taking an arbitrary curve geometry as input and inferring corresponding geometric and physical parameters of the simulator such that the input geometry corresponds to a stable configuration at equilibrium.

Keywords: Physics-based simulation, thin elastic rod, frictional contact, hair simulation, inverse physics-based design

Introduction Deformable slender structures such as hair fibers, cloth, ribbons, tree branches or leaves, are ubiquitous around us. They often feature an intricate natural shape, ranging from straight to curly, and are characterized by a complex motion involving strongly nonlinear deformations, such as buckling. Such complex shapes and motions greatly contribute to the visual richness of the real world. When multiple such structures are coupled together with contact and friction, the range of emerging phenomena is even more exacerbated, giving rise to stick-slip dynamical instabilities, entangling, or spontaneous collective behavior. Human hair, which is typically composed of 150,000 thin fibers, beautifully depicts such complex mechanical behaviors when fluttering in the wind.

*Florence.Descoubes@inria.fr

As the essence of Computer Graphics is to represent the visual appearance of the real world with the highest fidelity, it is important for practitioners to be able to capture all the relevant details of a given scene. In the case of a dynamical scene involving passive objects such as hair, cloth or natural phenomena, *physics-based simulation* has proven over the years to be a method of choice for capturing resulting visual effects. Unlike phenomenological methods which develop descriptive models for reproducing a given emerging phenomenon, physics-based methods provide generative models whose goal is to explain the physical causes of the phenomenon. From a set of initial conditions as well as a few physical parameters (e.g., the mass, stiffness, natural shape), a physics-based simulator may generate not just a single effect, but a wide range of emerging phenomena revealing the whole complexity of the underlying physics.

When designing a physics-based simulator for computer graphics, one has inevitably in mind the four following criteria:

Realism: Ingredients which are necessary to capture relevant visual effects should be identified, and translated numerically with as few quality loss as possible.

Robustness: The simulator should converge properly for a subsequent range of parameters.

Efficiency: The simulator should be fast enough for allowing complex scenes to be simulated in reasonable timings (in our case, a few days of computation for a given scene is an upper-bound).

Control: The simulator should provide the user with some handles to control the shape and motion of the object in an intuitive way.

Over the ten past years, we have been striving to develop some numerical models satisfying all four criteria at the same time. Our work has focused on the simulation of slender structures prone to contact and dry friction, and especially on the dynamics of thin fiber assemblies, with some direct applications in hair simulation and inverse physics-based design.

High-order reduced models for simulating dynamic fibers The first part of the talk will be devoted to the presentation of the numerical models that we have been developing for simulating the dynamics of flexible fibers, the so-called *super-helix* and *super-clothoid* models [1, 2, 4].

I will first introduce the mechanical equations for inextensible thin elastic rods, namely the *Kirchoff* equations, which take the form of second order partial differential equations subject to boundary conditions. Noting that curvatures and twist play a major role both in the geometric and dynamic description of this model, we have come up with a spatial rod discretization based on elements that are polynomial in such quantities.

Our first scheme relied on piecewise constant curvatures [1], and was then extended to piecewise linear curvatures [2, 4]. One major advantage of such curvature-based formulations is that the kinematics of the discretized rod remains, by construction, perfectly inextensible. Such intrisic inextensibility thus removes the burden of adding subsequent (stiff) inextensibility contraints when solving the dynamics. The price to pay, however, is that the geometry of the rod is not readily available but has to be computed iteratively from the curvatures.

In the piecewise constant case [1], each element turns out to be a perfect circular helix (hence the super-helix name for the model), leading to a cheap and exact evaluation of all the terms of the discrete dynamic equations. For higher orders however, one unfortunately loses such a closed-form formula and a both accurate and efficient spatial integration scheme has to be designed.

In the piecewise linear case (where each element takes the form of a 3D clothoid), we were able to build an accurate integration scheme which proved to be orders of magnitude faster compared to classical integration methods [4]. The key of our approach was to leverage the form of the solution as a power series expansion, while avoiding the pitfall of catastrophic cancellation through an adaptive integration strategy. With this tool in hand, we were able to demonstrate that the super-clothoid model could capture intricate shapes both robustly and efficiently, with better spatial accuracy and geometric fairness compared to state-of-the-art methods (see figure 1).

Figure 1: Many physical strands exhibit a smooth curled geometry with linear-like curvature profile, which is captured and deformed accurately thanks to our super-clothoid model [4]. From left to right and top to bottom, three examples of real strands whose shapes are synthesized and virtually deformed in real-time using a very low number of 3D clothoidal elements: a vine tendril (4 elements), a hair ringlet (2 elements), and a curled paper ribbon (1 single element). Left photograph courtesy of Jon Sullivan, pdphoto.org.

Robust frictional contact model for fiber assemblies The second part of the talk will be focused on the dynamic simulation of fiber assemblies, where individual fibers are coupled to each other through contact and friction.

I will first illustrate why capturing threshold effects in friction is key to realism (see figure 2), before introducing the nonsmooth *Signorini-Coulomb* friction model and its various formulations. We shall notably see what the numerical counter-parts are for each main formulation, and how each of them performs in terms of efficiency, robustness, and scalability [3].

Then I will explain how we managed to design a robust and scalable frictional contact solver by combining an iterative Gauss-Seidel strategy together with an extremely robust one-contact solver [5]. Our global solver proved to converge well in scenarios involving thousands fibers subject to tens thousands frictional contact points, and thus allowed us to enhance considerably the realism of hair simulations. Our method has been adopted by the special effects industry for simulating hair and fur accurately [10].

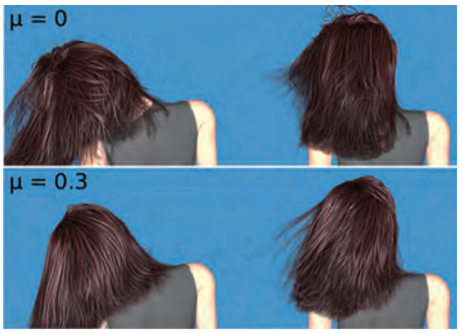

Figure 2: Simulation of a fast head movement without (top) and with (bottom) Coulomb friction, using our robust solver from [5]. In the latter case, hair remains much more coherent and depicts typical stick-slip effects.

From geometry to physics: Inverse design of fiber assemblies Finally, I will present some new important challenges regarding inverse physics-based design. While current simulators may succeed in reaching a good level of realism, they remain difficult to control in order to achieve a precise artistic goal. More precisely, to generate some desirable shapes and motions, one should be able to feed a simulator with the "right" parameters. Finding such parameters remains a very difficult task, which is often performed through a tedious trial and error process. To make this task fully automatic, we have started looking at *inverse* solutions in the case where a static shape is provided as input: the inverse model should be able to interpret automatically this shape as a stable equilibrium of the simulator, under gravity and other external forces such as contact and friction.

In the case of an isolated fiber, we have shown that inverting any of our super-model [1, 4] boils down to two decoupled problems that are both easy to solve [7, 8]: first, an equilibrium condition which appears to be linear in the natural shape of the fiber, thanks to the curvature-based parameterization of our fiber models; second, a sufficient stability condition that can be simply set by fixing a lower-bound for the ratio of stiffness over mass. Actually, the only remaining difficulty is to solve a merely geometric fitting problem – converting a curve as a piecewise helix or clothoid. In the case of helical fitting, we have already brought some efficient and robust solutions [7, 9].

In the presence of contact and friction, Coulomb sticking constraints have to be considered, which makes the overall inverse problem nonsmooth and ill-posed. We have shown that assuming known mass and stiffness and a simplified inverse model, it is possible to recover a plausible natural shape as well as frictional contact forces at play [6]. This work allowed us, for the first time, to animate in a plausible way a few hair geometries stemming from recent hair captures (see figure 3).

(a) (b) (c)

Figure 3: Real curly wig (a) captured from [11], inverted by our method in [6] and physically animated (b) and trimmed (c).

Conclusion Throughout this long-term work on the numerical modeling of fibers and frictional contact, we have learned that systematically concentrating the efforts on the upstream modeling and formulation of problems often pays off: even for very complex problems, the resulting numerics may be greatly simplified and thus solved more easily and robustly. Keeping in mind this key lesson, we are starting to investigate the case of 2D slender structures (namely plates and shells), for which many exciting challenges remain open.

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