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the 210 MM MAGNETIC OBSERVATION GROUP

https://doi.org/10.5109/1546807

出版情報：九州大学大学院理学研究院紀要 : Series D, Earth and planetary sciences. 31 (1), pp.19-28, 2000-12-25. Faculty of Science, Kyushu University

Mem. Fac. Sci., Kyushu Univ., Ser. D, Earth & Planet. Sci., Vol. XXXI, No. 1,
pp. 19-28, text-figs. 1-2, December 25, 2000

Seismic Waves in a horizontally layered half-space with a general point source

Hiroshi TAKENAKA* and Tsutomu SASATANI**

Abstract

SASATANI (1985) proposed a simple approach of the reflctivity method for the computation of seismic response of a horizontally stratifed half-space due to a point shear dislocation or explosion source. In the present paper we describe a formulation for introducing a more general source, i.e., single forces and moment tensors into his approach. Our numerical code based on this formulation has been already applied to many problems, by which the validation of the formulation has been implicitly but fully confirmed.

1. Introduction

The reflectivity method for calculating synthetic seismograms in horizontally layered media has been widely used. In the reflectivity method the response of the model in the space-time domain is transformed into the wavenumber-frequency domain by the Fourier transform or Fourier-Hankel transform, corresponding to plane-wave or cylindrical-wave expansion. Each wavenumber-frequency component of the response is generally computed by a matrix method (Fuch and Müller, 1971; Kind, 1978; Kennett, 1983; Kohketsu, 1985) that can automatically include contributions from all possible generalised rays in the whole or a part of region of the model. Many variants of the reflectivity method have been developed (Kohketsu, 1987).

SASATANI (1985) proposed a simple approach to evaluate seismic response of a horizontally layered medium due to a point "shear dislocation" or "explosion source". This approach is based on the three-dimensional seismic wavefield representation of AKI and RICHARDS (1980), the reflection and transmission matrices for elastic media of KENNETT and KERRY (1979) and the wavenumber integration scheme (the discrete wavenumber summation method) by BOUCHON (1981). Similar approaches have been developed by other people (e.g. YAO and HARKRIDER, 1983), but the three-dimensional elastic wavefield representation of AKI and RICHARDS (1980) was used in this approach for the first time among them. A theoretical merit of the representation of AKI and RICHARDS (1980) is that it gives the same layer matrix for the three-dimensional (cylindrical) waves as for the two-dimensional ones (plane waves). SASATANI also made a Fortran 77 code of his approach to show some numerical examples for a point shear dislocation source in SASATANI (1985), and used it for modelling some actual seismograms in his subsequent papers (e.g. SASATANI, 1990)

One of the authors of the present paper (H.T.) then extended his approach to obtain seismic

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Manuscript received September 1, 2000 ; accepted October 24, 2000.
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response due to a more general source, "single force" and / or "moment tensor", and developed a new code in March 1989. This code has been already used by many people for analysing actual seismograms (e.g. OHSHIMA et al., 1989; PITARKA et al., 1996; TAKENAKA et al., 1997; FUJII, 1998; KATO et al., 2000) and for checking different numerical codes (e.g. HAYASHIDA et al., 1999). However, unfortunately its theoretical bases and formulae have never been reported anywhere. The purpose of the present paper is to describe them. In this paper we focus on the description related to the newly exploited part of them.

2. The response of a stratified half-space

We consider a cylindrical coordinate system (r, θ, z) taken positive z downward, and assume time dependence of $\exp(-i\omega t)$, where t is time and ω is angular frequency. The elastic displacement $\boldsymbol{U}$ in an isotropic homogeneous body can be represented in the three scalar potentials ϕ, ψ and χ, as

$$\boldsymbol{U} = \nabla\phi + \nabla\times\nabla\times[0,0,\psi]^T + \nabla\times[0,0,\chi]^T, \tag{1}$$

where T denotes a transpose. the cylindrical-coordinate form of eq. (1) is

$$\begin{aligned} U_r &= \frac{\partial\phi}{\partial r} + \frac{\partial^2\psi}{\partial r\partial z} + \frac{1}{r}\frac{\partial\chi}{\partial\theta}, \\ U_\theta &= \frac{1}{r}\frac{\partial\phi}{\partial\theta} + \frac{1}{r}\frac{\partial^2\psi}{\partial z\partial\theta} - \frac{\partial\chi}{\partial r}, \\ U_z &= \frac{\partial\phi}{\partial z} + \frac{\partial^2\psi}{\partial z^2} + k_\beta^2\,\psi. \end{aligned} \tag{2}$$

The potentials ϕ, ψ and χ correspond to P-, SV – and SH-waves, respectively, and satisfy the following Helmholtz equations:

$$\nabla^2\phi + k_\alpha^2 = 0, \qquad \nabla^2\psi + k_\beta^2 = 0, \qquad \nabla^2\chi + k_\beta^2 = 0, \tag{3}$$

where

$$k_\alpha = \frac{\omega}{\alpha}, \qquad k_\beta = \frac{\omega}{\beta}. \tag{4}$$

α and β are P-wave and S-wave velocities, respectively. It follows by the method of separation of variables that general solutions can be obtained by a superposition of the basic solutions (e.g. AKI and RECHARDS, 1980)

$$\begin{aligned} \phi\,(r,\theta,z,\omega) &= J_m(kr)\exp(im\theta)\,[A\exp(-\gamma z) + B\exp(\gamma z)], \\ \psi\,(r,\theta,z,\omega) &= J_m(kr)\exp(im\theta)\,[C\exp(-\nu z) + D\exp(\nu z)], \\ \chi\,(r,\theta,z,\omega) &= J_m(kr)\exp(im\theta)\,[E\exp(-\nu z) + F\exp(\nu z)], \end{aligned} \tag{5}$$

with

$$\begin{aligned} \gamma &= \sqrt{k^2 - k_\alpha^2} \qquad (\Re(\gamma) \geq 0,\ \Im(\gamma) \leq 0)\ , \\ \nu &= \sqrt{k^2 - k_\beta^2} \qquad (\Re(\nu) \geq 0,\ \Im(\nu) \leq 0)\ , \end{aligned} \tag{6}$$

$J_m(kr)$ is the mth-order Bessel function, m is an integer, and A, B, C, D, E, F are constants. In this paper we employ the complex function $\exp(im\theta)$ for the azimuthal dependence in accordance with AKI and RICHARDS (1980) instead of the real functions $\cos m\theta$ and $\sin m\theta$ used by SASATANI (1985), which makes the formulation and computational scheme simpler.

We now consider a medium with stratification perpendicular to the z axis which passes through the point source. Substituting eq. (5) into eq. (2), the displacement can then take the following explicit form:

$$
\begin{aligned}
U_r(r,\theta,z,\omega) &= \frac{1}{4\pi\rho\omega^2}\sum_{m=-\infty}^{\infty}\exp(im\theta)\int_0^\infty \left[r_1^m\cdot\frac{\partial J_m(kr)}{\partial(kr)}+l_1^m\cdot\frac{im}{kr}J_m(kr)\right]k\,dk,\\
U_\theta(r,\theta,z,\omega) &= \frac{1}{4\pi\rho\omega^2}\sum_{m=-\infty}^{\infty}\exp(im\theta)\int_0^\infty \left[r_1^m\cdot\frac{im}{kr}J_m(kr)-l_1^m\cdot\frac{\partial J_m(kr)}{\partial(kr)}\right]k\,dk,\\
U_z(r,\theta,z,\omega) &= \frac{1}{4\pi\rho\omega^2}\sum_{m=-\infty}^{\infty}\exp(im\theta)\int_0^\infty \left[(-r_2^m)\cdot J_m(kr)\right]k\,dk,
\end{aligned}
\tag{7}
$$

where ρ is density. The z-dependence of the displacement is described by three scalar functions r_1^m, r_2^m and l_1^m. In the right hand side of eq.(7) the factor $1/(4\pi\rho\omega^2)$ has been taken outside because it appears commonly in the source representations as shown later. For an isotropic medium, the displacement scalars r_1^m, r_2^m and l_1^m depend on the azimuthal order only through the nature of the source. In the case of any single force $|m| \leq 1$, while in the case of any moment tensor source $|m| \leq 2$ (e.g. AKI and RICHARDS, 1980; KENNETT, 1983).

We will consider the calculation of displacements at the free surface ($z=0$) and set

$$
\boldsymbol{r}_0=\begin{cases}[r_1^m(k,z=0,\omega),r_2^m(k,z=0,\omega)]^T & (P-SV)\\ [l_1^m(k,z=0,\omega)]^T & (SH)\end{cases}
\tag{8}
$$

We assume the source is at a depth z_S and the medium beneath the level z_L is uniform. Following KENNETT (1983), we characterise the reflection properties of e.g. the region between $z = z_S$ and z_L by $\boldsymbol{R}_D^{SL}$, where the suffix D indicates an incident downward plane wave at the level $z = z_S$. For P-SV waves $\boldsymbol{R}_D^{SL}$ is a 2×2 matrix of the corresponding reflection coefficients and a scalar in the SH wave case. A similar notation is used for transmission (e.g. $\boldsymbol{T}_U^{0S}$). If we allow for reflections from the free surface to be included, we write, e.g. $\boldsymbol{R}_U^{fS}$. The reflection matrix just at the free surface is denoted by $\tilde{\boldsymbol{R}}$. In terms of this notation the surface-displacement vector in the wavenumber-frequency domain $\boldsymbol{r}_0$ takes the following form (KENNETT, 1983; SASATANI, 1985):

$$
\boldsymbol{r}_0=\tilde{\boldsymbol{W}}(\boldsymbol{I}-\boldsymbol{R}_D^{0S}\tilde{\boldsymbol{R}})^{-1}\boldsymbol{T}_U^{0S}(\boldsymbol{I}-\boldsymbol{R}_D^{SL}\boldsymbol{R}_U^{fS})^{-1}(\Sigma_U+\boldsymbol{R}_D^{SL}\Sigma_D),
\tag{9}
$$

where $\boldsymbol{I}$ is the identity matrix, $\tilde{\boldsymbol{W}}$ is the surface amplification matrix which takes amplitude of an upgoing wave into the displacement appropriate to the free surface, and $\boldsymbol{\Sigma}_U$ and $\boldsymbol{\Sigma}_D$ represent the upward and downward radiation parts of the source jump vector $\boldsymbol{\Sigma}$, respectively, due to the buried source, as

$$
\Sigma=[\Sigma_D,-\Sigma_U]^T.
\tag{10}
$$

This excitation term for an isotropic medium is the only one to depend on azimuthal order m, while the other terms in the right hand side of eq. (9) are independent of azimuthal order. The negative sign of $\boldsymbol{\Sigma}_U$ in eq. (10) is taken to facilitate physical interpretation (KENNETT, 1983). The sign for definition of $\boldsymbol{\Sigma}_U$ is negative in KENNETT (1983) and SASATANI (1985) as in eq. (10), while positive in KENNETT and KERRY (1979), KENNETT (1980) and TAKEO (1985). In the next section we derive the elements of the jump vector for a general point source (i.e. single forces and moment tensors).

3. Single force and moment tensor sources

The jump vector $\boldsymbol{\Sigma}$ is a discontinuity in the wavevector at the source level z_S due to the introduction of the source, which is given by

$$\boldsymbol{\Sigma} = F^{-1}[\boldsymbol{b}(k,m,z_S+,\omega) - \boldsymbol{b}(k,m,z_S-,\omega)], \tag{11}$$

where F is the layer matrix (eqs. (7.49) and (7.55) in AKI and RICHARDS (1980); eqs. (11) and (13) in SASATANI (1985)) for the source layer converting the wavevectors to the motion-stress vectors. The motion-stress vector $\boldsymbol{b}$ is a vector defined as

$$\boldsymbol{b}(k,m,z,\omega) = \begin{cases} [r_1^m,\ r_2^m,\ r_3^m,\ r_4^m]^T & (P-SV) \\ [l_1^m,\ l_2^m]^T & (SH) \end{cases}, \tag{12}$$

where r_3^m , r_4^m and l_2^m are the traction scalars describing the z-dependence of tractions, i.e. the counterparts of displacement scalars r_1^m , r_2^m and l_1^m corresponding to the tractions (AKI and RICHARDS, 1980).

In order to obtain the motion-stress vectors infinitesimally just above and below the source level, $\boldsymbol{b}$ $(k,\ m,\ z_S-,\omega)$ and $\boldsymbol{b}(k,\ m,\ z_S+,\omega)$, respectively, we employ the displacement potentials describing P-, SV -, and SH-waves separately. We first derive the displacement potentials for single point forces. All equations shown in the following subsections (Sec. 3.1 to Sec. 3.3) are originally derived by us.

3.1 Displacement potentials for single forces

From the formulae of the elastodynamic Green tensors for a homogeneous whole space (eq. (4.23) in AKI and RICHARDS (1980)), the Sommerfeld integral (eq. (6.7) in AKI and RICHARDS (1980)) and eq. (2), we derive the following representation of the displacement potentials for single forces in the Cartesian coordinate directions with a unit strength:

For P-wave

$$\begin{aligned} \phi^x &= C\cdot\cos\theta \int_0^\infty kF_\alpha J_1(kr)dk, \\ \phi^y &= C\cdot\sin\theta \int_0^\infty kF_\alpha J_1(kr)dk, \\ \phi^z &= C\cdot \int_0^\infty \mathrm{sgn}(z-h)\ \gamma\ F_\alpha J_0(kr)dk, \end{aligned} \tag{13}$$

where h is the source depth, the superscripts of ϕ denote the direction of the force,

$$\begin{aligned} C &= \frac{1}{4\pi\rho\omega^2}, \\ F_\alpha &= \frac{k\exp(-\gamma\,|\,z-h\,|\,)}{\gamma} \quad ; \end{aligned} \tag{14}$$

for SV-wave

$$\begin{aligned} \psi^x &= C\cdot\cos\theta \int_0^\infty \mathrm{sgn}(z-h)\frac{\nu}{k}F_\beta J_1(kr)dk, \\ \psi^y &= C\cdot\sin\theta \int_0^\infty \mathrm{sgn}(z-h)\frac{\nu}{k}F_\beta J_1(kr)dk, \\ \psi^z &= C\cdot \int_0^\infty F_\beta J_0(kr)dk, \end{aligned} \tag{15}$$

where

$$F_\beta = \frac{k\exp(-\nu \mid z-h \mid)}{\nu} \quad ; \tag{16}$$

for *SH*-wave

$$\begin{aligned}
\chi^x &= C \cdot \sin\theta \int_0^\infty \frac{k_\beta^2}{k} F_\beta J_1(kr)dk, \\
\chi^y &= -C \cdot \cos\theta \int_0^\infty \frac{k_\beta^2}{k} F_\beta J_1(kr)dk, \\
\chi^z &= 0.
\end{aligned} \tag{17}$$

If we take the Cartesian coordinate directions $[x,y,z]$ as [North, East, Down], the angle θ is identical to the usual station azimuth measured clockwise from the north.

3.2 Displacement potentials for moment tensors

The representation of the displacement potentials for moment tensor sources with a unit moment is obtained by differentiating those for single forces (eqs. (13) to (17)) with respect to the source coordinates as follows:
For *P*-wave

$$\begin{aligned}
\phi^{xx} &= C \cdot \left(-\frac{1}{2}\int_0^\infty k^2 F_\alpha J_0(kr)\,dk + \frac{1}{2}\cos 2\theta \int_0^\infty k^2 F_\alpha J_2(kr)\,dk\right), \\
\phi^{xy} &= C \cdot \frac{1}{2}\sin 2\theta \int_0^\infty k^2 F_\alpha J_2(kr)\,dk, \\
\phi^{xz} &= C \cdot \cos\theta \int_0^\infty \mathrm{sgn}(z-h)\, k\gamma F_\alpha J_1(kr)dk, \\
\phi^{yx} &= \phi^{xy}, \\
\phi^{yy} &= C \cdot \left(-\frac{1}{2}\int_0^\infty k^2 F_\alpha J_0(kr)\,dk - \frac{1}{2}\cos 2\theta \int_0^\infty k^2 F_\alpha J_2(kr)\,dk\right), \\
\phi^{yz} &= C \cdot \sin\theta \int_0^\infty \mathrm{sgn}(z-h)\, k\gamma F_\alpha J_1(kr)dk, \\
\phi^{zx} &= \phi^{xz}, \\
\phi^{zy} &= \phi^{yz}, \\
\phi^{zz} &= C \cdot \int_0^\infty \gamma^2 F_\alpha J_0(kr)dk \quad ,
\end{aligned} \tag{18}$$

where the superscripts of ϕ correspond to the moment tensor elements; for *SV*-wave

$$\begin{aligned}
\psi^{xx} &= C \cdot \left(-\frac{1}{2}\int_0^\infty \mathrm{sgn}(z-h)\, \nu F_\beta J_0(kr)\,dk + \frac{1}{2}\cos 2\theta \int_0^\infty \mathrm{sgn}(z-h)\, \nu F_\beta J_2(kr)\,dk\right), \\
\psi^{xy} &= C \cdot \frac{1}{2}\sin 2\theta \int_0^\infty \mathrm{sgn}(z-h)\, \nu F_\beta J_2(kr)\,dk, \\
\psi^{xz} &= C \cdot \cos\theta \int_0^\infty \frac{\nu^2}{k} F_\beta J_1(kr)dk, \\
\psi^{yx} &= \psi^{xy}, \\
\psi^{yy} &= C \cdot \left(-\frac{1}{2}\int_0^\infty \mathrm{sgn}(z-h)\, \nu F_\beta J_0(kr)\,dk - \frac{1}{2}\cos 2\theta \int_0^\infty \mathrm{sgn}(z-h)\, \nu F_\beta J_2(kr)\,dk\right), \\
\psi^{yz} &= C \cdot \sin\theta \int_0^\infty \frac{\nu^2}{k} F_\beta J_1(kr)dk,
\end{aligned}$$

$$\psi^{zx} = C\cdot\cos\theta\int_0^\infty kF_\beta J_1(kr)dk,$$

$$\psi^{zy} = C\cdot\sin\theta\int_0^\infty kF_\beta J_1(kr)dk,$$

$$\psi^{zz} = C\cdot\int_0^\infty \mathrm{sgn}(z-h)\,\nu F_\beta J_0(kr)\,dk \quad ; \tag{19}$$

for SH-wave

$$\chi^{xx} = C\cdot\frac{1}{2}\sin 2\theta\int_0^\infty k_\beta^2 F_\beta J_2(kr)dk,$$

$$\chi^{xy} = C\cdot\left(-\frac{1}{2}\int_0^\infty k_\beta^2 F_\beta J_0(kr)dk-\frac{1}{2}\cos 2\theta\int_0^\infty k_\beta^2 F_\beta J_2(kr)dk\right),$$

$$\chi^{xz} = C\cdot\sin\theta\int_0^\infty \mathrm{sgn}(z-h)\,k_\beta^2\frac{\nu}{k}F_\beta J_1(kr)dk,$$

$$\chi^{yx} = C\cdot\left(\frac{1}{2}\int_0^\infty k_\beta^2 F_\beta J_0(kr)dk-\frac{1}{2}\cos 2\theta\int_0^\infty k_\beta^2 F_\beta J_2(kr)dk\right),$$

$$\chi^{yy} = -\chi^{xx},$$

$$\chi^{yz} = -C\cdot\cos\theta\int_0^\infty \mathrm{sgn}(z-h)\,k_\beta^2\frac{\nu}{k}F_\beta J_1(kr)dk,$$

$$\chi^{zx} = \chi^{zy} = \chi^{zz} = 0. \tag{20}$$

3.3 Jump vector elements

The elements of the jump vectors

$$\Sigma_D=\begin{cases}[P_D^m, SV_D^m]^T & (P-SV)\\ [SH_D^m] & (SH)\end{cases}, \qquad \Sigma_U=\begin{cases}[P_U^m, SV_U^m]^T & (P-SV)\\ [SH_U^m] & (SH)\end{cases} \tag{21}$$

are derived using eqs. (11) to (20), and eqs. (2)and (7) together with the similar equations corresponding to the tractions, for each of azimuthal order as follows:
For m=0

$$P_D^0 = k_\alpha\left\{f_z-\left[\frac{k^2}{2\gamma}(M_{xx}+M_{yy})-\gamma M_{zz}\right]\right\},$$

$$P_U^0 = k_\alpha\left\{-f_z-\left[\frac{k^2}{2\gamma}(M_{xx}+M_{yy})-\gamma M_{zz}\right]\right\},$$

$$SV_D^0 = k_\beta k\left\{-\frac{1}{\nu}f_z+\left[\frac{1}{2}(M_{xx}+M_{yy})-M_{zz}\right]\right\},$$

$$SV_U^0 = k_\beta k\left\{\frac{1}{\nu}f_z+\left[\frac{1}{2}(M_{xx}+M_{yy})-M_{zz}\right]\right\},$$

$$SH_D^0 = SH_U^0 = -\frac{k_\beta^2 k}{2\nu}(M_{xy}-M_{yx}) \quad ; \tag{22}$$

for $m=\pm 1$

$$P_D^{\pm 1} = \frac{k_\alpha k}{2}\left[\frac{1}{\gamma}(\pm f_x-\mathrm{i}f_y)+(\pm M_{xz}-\mathrm{i}M_{yz})+(\pm M_{zx}-\mathrm{i}M_{zy})\right],$$

$$P_U^{\pm 1} = \frac{k_\alpha k}{2}\left[\frac{1}{\gamma}(\pm f_x-\mathrm{i}f_y)-(\pm M_{xz}-\mathrm{i}M_{yz})-(\pm M_{zx}-\mathrm{i}M_{zy})\right],$$

$$SV_D^{\pm 1} = -\frac{k_\beta}{2}\left[(\pm f_x-\mathrm{i}f_y)+\nu(\pm M_{xz}-\mathrm{i}M_{yz})+\frac{k^2}{\nu}(\pm M_{zx}-\mathrm{i}M_{zy})\right],$$

$$SV_U^{\pm 1} = -\frac{k_\beta}{2}\left[(\pm f_x-\mathrm{i}f_y)-\nu(\pm M_{xz}-\mathrm{i}M_{yz})-\frac{k^2}{\nu}(\pm M_{zx}-\mathrm{i}M_{zy})\right],$$

$$SH_D^{\pm 1} = -\frac{k_\beta^2}{2}\left[\frac{1}{\nu}(\mathrm{i}f_x \pm f_y) + (\mathrm{i}M_{xz} \pm M_{yz})\right],$$

$$SH_U^{\pm 1} = -\frac{k_\beta^2}{2}\left[\frac{1}{\nu}(\mathrm{i}f_x \pm f_y) - (\mathrm{i}M_{xz} \pm M_{yz})\right] \quad ; \tag{23}$$

for $m = \pm 2$

$$P_D^{\pm 2} = P_U^{\pm 2} = -\frac{k_\alpha k^2}{4\gamma}[-(M_{xx}-M_{yy}) \pm \mathrm{i}(M_{xy}+M_{yx})],$$

$$SV_D^{\pm 2} = SV_U^{\pm 2} = \frac{k_\beta k}{4}[-(M_{xx}-M_{yy}) \pm \mathrm{i}(M_{xy}+M_{yx})],$$

$$SH_D^{\pm 2} = SH_U^{\pm 2} = -\frac{k_\beta^2 k}{4\nu}[\pm \mathrm{i}(M_{xx}-M_{yy}) + (M_{xy}+M_{yx})]. \tag{24}$$

4. Numerical examples

The calculation of the seismograms proceeds in two stages, we first construct the response of the half-space at frequency ω, horizontal wavenumber k and angular order m and then generate the surface displacements themselves by integrating this response numerically with a scheme of BOUCHON (1979). To avoid the influence of the singularities of the integrand in the wavenumber integrals and to suppress the spatial and temopral wraparounds (aliasing phenomena) caused by the discretisations of wavenumber and frequency, we give to the frequency an imaginary part ω_I as

$$\omega_I = \pi / T, \tag{25}$$

where T is the total time window to be calculated. The resulting attenuation coupled with the geometrical spreading is sufficient to prevent aliasing in the solution. The displacements in the frequency domain are transformed into the time domain using the fast Fourier transform. The attenuation effect is then removed by multiplying them by the factor $\exp(\omega_I t)$ (Bouchon, 1979). In actual calculations we can use the following usual units of the input quantities: 's' for time; 'km' for distance and depth; 'km/s' for wave velocities; 'g/cm^3' for density. This combination of units gives displacement in 'cm' considering the units of single force and moment tensor are '10^{10} N (= 10^{15} dyne)' and '10^{13} Nm (= 10^{20} dyne cm)', respectively.

As mentioned in Sec.1, the new Fortran code based on the extended approach described in the present paper was made in 1989. It has been already used by many people, and the accuracy has been fully checked as well. Therefore we here only show two examples which cannot be calculated by the code of SASATANI (1985). One example is seismic response due to excitation by a vertically downward single force, the other is by a tension crack with a vertical opening. For the first example, f_z=1.0 × 10^{10} N and the other source components are set to be zero, while for the second example M_{xx}= 1.0 × 10^{13} Nm, M_{yy}= 1.0 × 10^{13} Nm, M_{zz}= 2.88 × 10^{13} Nm and the others are zero. In both cases the solutions have no azimuthal dependence, and should have no transverse component because of force system with symmetry around the z-axis.

The used source time function $S(t)$ is

$$S(t) = \begin{cases} \frac{1}{T_0}\left\{1 + \cos\left[\frac{2\pi}{T_0}\left(t - \frac{T_0}{2}\right)\right]\right\} & \text{for} \quad 0 \leq t < T_0 \\ 0 & \text{for} \quad t < 0,\ T_0 \leq t \end{cases}, \tag{26}$$

where T_0 is the pulse width. Here we set T_0 = 1.0 s. This function $S(t)$ is a normalised version of the function $M(t)$ in SASATANI (1985) so that it has a unit area. Unfortunately the subroutines for some source time functions in the code including the functions $S(t)$ and $M(t)$ had had bugs before April 11, 1994, which had not affected waveform of synthetic seismogram but always a larger amplitude by about 20 % compared with the correct one. The first author of the present paper (H.T.) found this bug, and revised these subroutines on April 11, 1994.

We employ the same structure model by SASATANI (1985) which is a half-space overlain by a lower velocity layer with the thickness of 2 km. The velocities and densities are α = 3.5 km/s, β = 2.0 km/s, ρ = 2.4 g/cm^3 for the upper layer; and α = 6.0 km/s, β =3.5 km/s, ρ = 2.7 g/cm^3 for the lower half-space. The source depth is 1 km, and the epicentral distances are set to be 0.5 km and 3 km. The incident angle at the free surface of the former is then less than the critical angle for the free surface, while the latter is beyond it. The displacements at the free surface for the first and second examples are shown in Fig. 1 and Fig. 2, respectively. In these figures, the peak amplitude of each trace is also shown. We can confirm that the computational results have in fact no transverse component in both examples. The first arriving phase at the two epicentral distances is the direct P wave. In Fig. 1(b) note the z-component of this phase has a small positive onset although its amplitude may be too small to be clearly seen.

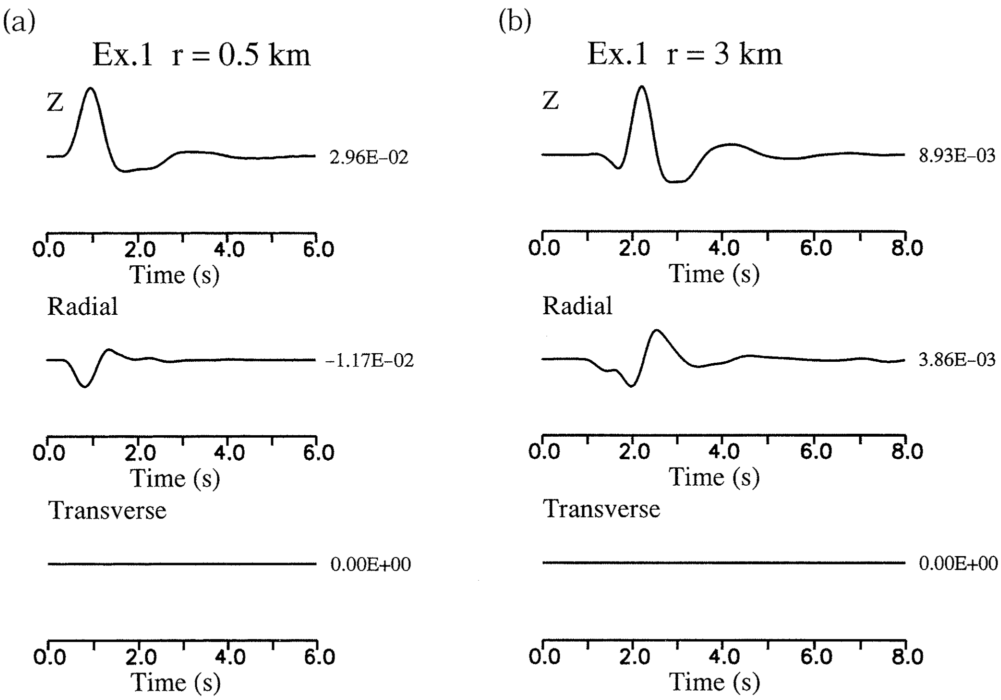

Fig. 1. Surface displacements for the first example (vertical single force source) at (a) r = 0.5 km and (b) r =3 km. Numbers at the right of each trace is its peak amplitude value in cm.

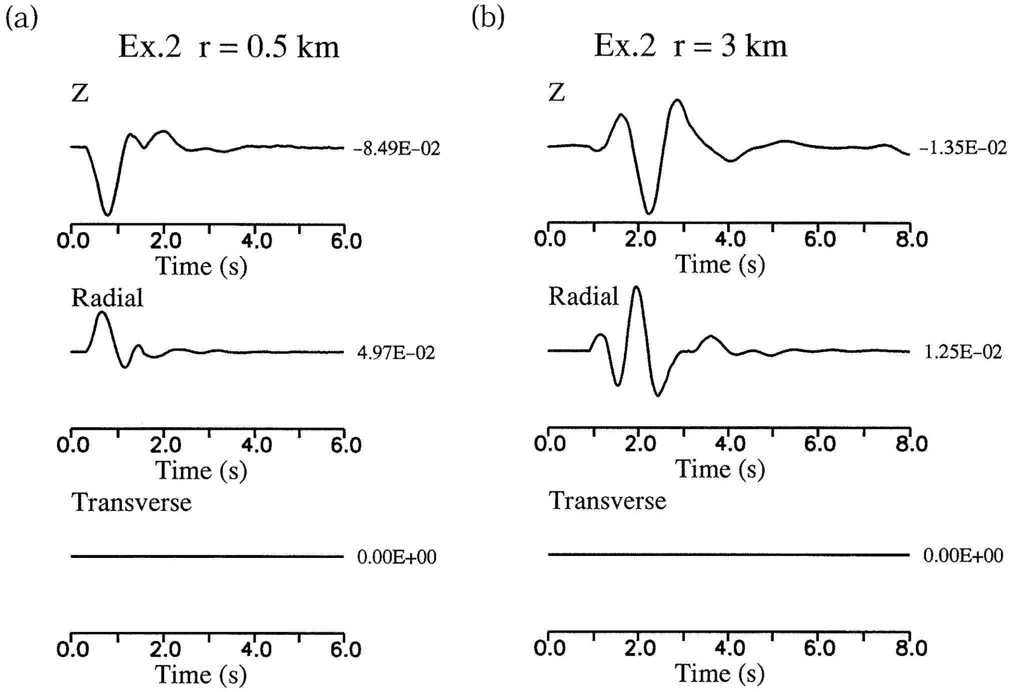

Fig. 2. Surface displacements for the second example (tension crack source) at (a) r =0.5 km and (b) r = 3 km. Numbers at the right of each trace is its peak amplitude value in cm.

5. Conclusion

We have described the theoretical bases and formulae exploited for a simple approach of the reflectivity method which calculates synthetic seismograms for horizontally layered media with general sources represented by single forces and moment tensors. We have also showed numerical examples using the Fortran code we had developed.

Acknowledgements

Drs. Daisuke Suetsugu and Arben Pitarka helped the first auther of this paper (H.T.) find the bugs in the code mentioned above. In the calculation of exact Bessel functions we used the codes DJ0 and DJ1 of the program package NUMPAC (Nagoya University Computation Center, 1982) . We are also grateful to an anonymous reviewer for constructive comments improving the manuscript.

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