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A Size Expansion Analysis using a Toy Model based on the Equivalence between Bondi K–Factor and Exponential Function

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This study was conducted to apply Bondi K–factor to the expansion analysis of the toy space model under the change in v (velocity) from $-c$ to c (speed of light in vacuum). Mathematical hypotheses (A) ~ (E) were suggested. (A) Bondi K–factor and its reciprocal were regarded as the toy space model in relative size and the toy energy density model in relative size, respectively. (B) If $v = -c$, then the size of the toy space model was 0 and the density of the toy energy model was ∞, a singular point that was under the control of the invariant. (C) The tiny toy space model was born from 0 or from the wave phenomenon, under something like a renormalization of infinities resolving the problem of the singular point. (D) The newborn tiny toy space model showed an early rapid expansion under the decelerated expansion that continued until the inflection point [$v = -c/2$, redshift = 0.732, the time of the middle between the birth ($v = -c$) and the present ($v = 0$) under a hypothetical condition], and then turned into the accelerated expansion. (E) If $v = c$, then the size of the toy space model was ∞ and the density of the toy energy model was 0, a singular point under the control of the invariant. The fate of the toy space model, an explosion or an everlasting expansion, would depend on the rate at which v would change with time. This study suggested a mathematical hypothesis that Bondi K–factor and its reciprocal gave a toy space model showing a relativistic expansion under the change in v from $-c$ to c.

Key words: accelerated expansion, decelerated expansion, early rapid expansion, inflection point, singular point

INTRODUCTION

The growth of the individual animal is related to the size expansion along space–time axes. The size expansion of the animal is different from that of the space, because the former is the matter but the latter is not. However, these two might look like each other due to a positive relationship between the expansion of the size and the increase in the internal energy. The size expansion of the individual animal is simulated using exponential function, and that of the space is roughly estimated using the redshift related to Bondi K–factor (Bondi, 1964). Bondi K–factor is an educational tool of the relativity theory [Sugiyama (2010), for example]. There are studies that show the equivalence between Bondi K–factor and exponential function [Lavenda (2000), for example]; to be exact, a quasi–equivalence because of the problem of whether or not taking the value of 0.

Shimojo (2014) suggested a mathematical hypothesis that Bondi K–factor gave a simplified space model showing an expansion with an inflection point. Since I could not understand the specialized papers and books on this issue at all, I read some books for the general public; Japanese books (Taniguchi, 2005; Sato, 2010; Doi and Matsubara, 2011; Futamase, 2013; Murayama and Takahashi, 2013; Suto, 2013), and the Japanese translation of English books (Guth, 1997; Kirshner, 2002; Farrell, 2005; Vilenkin, 2006; Gates, 2009; Greene, 2011; Panek, 2011; Krauss, 2012). These books still include many contents that are too difficult for me to understand. I am totally lacking in the specialized knowledge on this issue, but I am still fascinated by exponential function, Bondi K–factor, and the relationship between them.

The present study was designed to add a few pieces of information and modification to the previous report (Shimojo, 2014) on the expansion of simplified space model.

ANALYSIS OF THE EXPANSION OF THE TOY SPACE MODEL

Equivalence between Bondi K–factor and exponential function

The equivalence between Bondi K–factor and exponential function is given by

$$\sqrt{\frac{1+v/c}{1-v/c}} = \exp(\theta), \quad \frac{v}{c} = \frac{\exp(2\theta)-1}{\exp(2\theta)+1}, \quad (1)$$

where c = speed of light in vacuum, v = velocity of matter.

Properties of Bondi K–factor and its reciprocal

Expression (1) gives Bondi K–factor [$B(v)$], its reciprocal [$E(v)$], and related expressions,

$$B(v) = \sqrt{\frac{c+v}{c-v}}, \quad (2) \qquad E(v) = \sqrt{\frac{c-v}{c+v}}, \quad (3)$$

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$$B(v)\cdot E(v)=\sqrt{\frac{c+v}{c-v}}\cdot\sqrt{\frac{c-v}{c+v}}=1,\ (4)$$

$$B(v)=\frac{1}{E(v)}=\frac{1}{1+z},\ (5)$$

where $-c \leq v \leq c$, $0 \leq B(v) < \infty$, $\infty > E(v) \geq 0$, z = redshift.

The first derivative, the second derivative, and the inflection point of $B(v)$ are as follows,

$$\frac{dB(v)}{dv}=\frac{c}{\sqrt{(c+v)(c-v)^3}},\ (6)$$

$$\frac{d^2B(v)}{dv^2}=\frac{c(c+2v)}{\sqrt{(c+v)^3(c-v)^5}},\ (7)$$

$$\left(-\frac{c}{2},\sqrt{\frac{1}{3}}\right).\ (8)$$

The inflection point (8) is the point at which $B(v)$ turns from the decelerated increase into the accelerated increase.

The first derivative, the second derivative, and the inflection point of $E(v)$ are as follows,

$$\frac{dE(v)}{dv}=\frac{-c}{\sqrt{(c-v)(c+v)^3}},\ (9)$$

$$\frac{d^2E(v)}{dv^2}=\frac{c(c-2v)}{\sqrt{(c-v)^3(c+v)^5}},\ (10)$$

$$\left(\frac{c}{2},\sqrt{\frac{1}{3}}\right).\ (11)$$

The inflection point (11) is the point at which $E(v)$ turns from the decelerated decrease into the accelerated decrease.

Application of Bondi K–factor [$B(v)$] and its reciprocal [$E(v)$] to the analysi of the expansion of the toy space model

Expressions (2) ~ (5) seem to suggest mathematical hypotheses (I) and (II). (I) If $B(v)$ is regarded as a toy model of the space in relative size [(2)], then $E(v)$ is regarded as a toy model of the energy density in relative size [(3)] that is related to the expansion of the toy space model. The larger the value of $E(v)$ is, the larger the value of the redshift (z) is [(5)]. (II) The product of $B(v)$ and $E(v)$ with the domain of v ($-c \leq v \leq c$) is under the control of the invariant [(4)].

Expressions (6) ~ (11) seem to suggest mathematical examples (A) ~ (D) for $B(v)$ and $E(v)$.

(A) A value of v (12) gives the corresponding value (13) to the first derivative of $B(v)$,

$$v=-c\ (1-10^{-99}),\ (12)$$

$$\frac{dB(v)}{dv}=3.727\times10^{43}.\ (13)$$

(B) A change in v per unit time (14) gives the corresponding change per unit time (15) to $B(v)$,

$$v_1=(-c)\left(\frac{\exp(200)-1}{\exp(200)+1}\right)$$

$$\rightarrow v_2=(-c)\left(\frac{\exp(2)-1}{\exp(2)+1}\right),\ (14)$$

$$\frac{B(v_2)}{B(v_1)}=\exp(99)=9.889\times10^{42}.\ (15)$$

(C) The point of inflection (8) in $B(v)$ has the redshift of 0.732 that is given by (16) in $E(v)$,

$$(v,\ B(v))=\left(-\frac{c}{2},\sqrt{\frac{1}{3}}\right),\ (8)$$

$$(v,\ E(v))=\left(-\frac{c}{2},\sqrt{3}\right)$$

$$=\left(-\frac{c}{2},\ 1+0.732\right).\ (16)$$

(D) A value of v (17) gives the corresponding value (18) to the first derivative of $B(v)$,

$$v=c(1-10^{-99}),\ (17)$$

$$\frac{dB(v)}{dv}=7.454\times10^{142}.\ (18)$$

Expressions (2) ~ (18) seem to suggest mathematical hypotheses (i) ~ (x) under the change in v from $-c$ to c. (i) If $v=-c$, then this gives a singular point, where the size of the toy space model is 0 and the density of the toy energy model is ∞. This singularity is under the control of the invariant (4). (ii) If it is impossible or unstable for the singular point (i) to be present, then v begins to change from $-c$ (namely $|v|$ begins to decrease from $|c|$). This leads to the birth of a tiny toy space model from 0. (iii) Immediately after the birth, the newborn tiny toy space model shows an early rapid expansion at the speed greater than c [(12) ~ (15), for example], under the decelerated expansion that continues until the inflection point occurring at $v = -c/2$ [(8)] with a redshift of 0.732 [(16)]. (iv) The inflection point is the point at which the toy space model turns into the accelerated expansion from the decelerated expansion. If the rate at which v changes with time is constant, then the inflection point ($v = -c/2$) occurs at the time of the middle between the birth ($v = -c$) and the present ($v = 0$) of the toy space model. (v) This accelerated expansion of the toy space model continues until v becomes c, where the speed of expansion exceeds c [(17) ~ (18), for example]. (vi) If $v = c$, then this gives a singular point, where the size of the toy space model is ∞ and the density of the toy energy model is 0. This singularity is under the control of the invariant (4). (vii) If it is impossible or unstable for the singular point (vi) to be present, then the toy space model shows an explosion or an everlasting expansion. Is it possible that there will be other cases (the explosion → the next birth, or the contraction → the next birth)? The fate of the toy space model depends on the rate at which v will change with time. (viii) If based on the equivalence between Bondi K–factor and exponential function [(1)], then exponential function is related to the relativistic expansion of the toy space model. (ix) Fig. 1 shows the relative size of the toy space model [$B(v)$] under the change in v from –1.0 to 0.5 (c = 1.0), namely $-c \le v \le 0.5c$. (x) This relativistic expansion of the toy space model reflects the result of the battle between the attractive force and the repulsive force.

Expression (4) with the change in v from $-c$ to c is mathematically equivalent to expression (19) with the change in v from c to $-c$, where there is an exchangeability between $B(v)$ and $E(v)$,

$$\sqrt{\frac{c+v}{c-v}} \cdot \sqrt{\frac{c-v}{c+v}} = 1, \quad (-c \le v \le c), \quad (4)$$

$$\sqrt{\frac{c-v}{c+v}} \cdot \sqrt{\frac{c+v}{c-v}} = 1, \quad (c \ge v \ge -c). \quad (19)$$

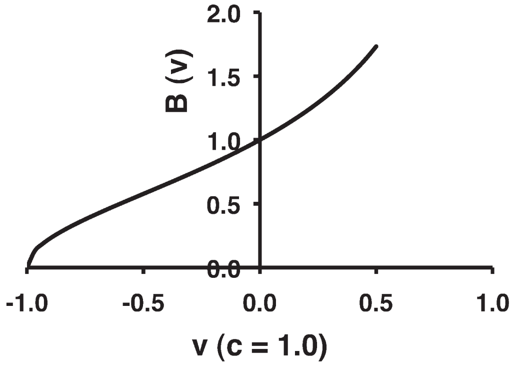

Fig. 1. Relative size of the toy space model [$B(v)$] under the change in v (velocity) from –1.0 to 0.5 (c = 1.0), namely $-c \le v \le 0.5c$.

Singular points of the toy space model and the toy energy density model

Singular points of the toy space model and the toy energy density model are given by applying $v = -c$ and $v = c$ to expresssion (4), respectively. Thus,

$$0 \cdot \infty = 1, \ (20) \qquad \infty \cdot 0 = 1. \ (21)$$

Do these seem to suggest a hypothesis that the beginning of the toy space model (20) is mathematically the same as the ending of the toy space model (21)?

The natural logarithm of singular points (20) and (21) gives expressions (22) ~ (26),

$$\ln(0) + \ln(\infty) = \ln(\infty) + \ln(0) = \ln(1), \ (22)$$

$$-\infty + \infty = \infty + (-\infty), \ (23)$$

$$= 2n\pi\boldsymbol{i}, \ (24)$$

$$= 2n\pi \cdot \frac{\psi(x, t) - \psi^*(x, t)}{2A\sin(x, t)}, \ (25)$$

$$= 2n\pi \cdot \left(\frac{xp - px}{h/2\pi} \right), \ (26)$$

where n = integers, π = circular constant, $\boldsymbol{i}$ = imaginary unit, x = position, p = momentum, $\psi(x, t)$ = wave function, $\psi^*(x, t)$ = conjugate complex of $\psi(x, t)$, $\sin(x, t)$= sine function, A = amplitude, t = time.

Expressions (20) ~ (26) seem to suggest mathematical hypotheses (a) ~ (e). (a) Something like a renormalization of infinities occurs in order to resolve the problem of singular points. (b) If $n = 0$, then the tiny toy space model with the huge energy density model appears from 0 [(24)]. (c) If $n = 1$, then the tiny toy space model with the huge energy density model appears from the quantum phenomenon [(25) and (26)]. (d)Which of (b) and (c) occurs? (e) The hypothetical explosion of the toy space model [(vii) in the preceding section, the right–hand side of (23) in this section] leads to 0 when $n = 0$, or to the quantum phenomenon when $n = 1$. Which of the two cases occurs?

Childish playing with mathematical expressions

If expressions (4) and (23) ~ (26) are forcedly applied to Euler's identity (27), then do these seem to suggest hypothetical expressions (28) ~ (30)?

$$0 = \exp(\boldsymbol{i}\,\pi) + 1, \ (27)$$

$$= \exp\left(\frac{-\infty + \infty}{2\pi} \cdot \pi \right) + \sqrt{\frac{c+v}{c-v}} \cdot \sqrt{\frac{c-v}{c+v}}, \ (28)$$

$$= \exp\left(\frac{\psi(x, t) - \psi^*(x, t)}{2A\sin(x, t)} \cdot \pi \right) + \sqrt{\frac{c+v}{c-v}} \cdot, \sqrt{\frac{c-v}{c+v}}, \quad (29)$$

$$= \exp\left(\frac{xp - px}{h/2\pi} \cdot \pi\right) + \sqrt{\frac{c+v}{c-v}} \cdot \sqrt{\frac{c-v}{c+v}} \ . \ (30)$$

If exponential function in Planck's law (31) is forcedly replaced with the reciprocal of Bondi K–factor (3), then do expressions (32) and (33) seem to suggest a hypothetical incorporation of redshift (z) into Planck's law?

$$I\,(v,\ T) = \frac{2hv^3}{c^2} \cdot \frac{1}{\exp(hv/k_B T) - 1} \ , \ (31)$$

$$\rightarrow \frac{2hv^3}{c^2} \cdot \frac{1}{\sqrt{(c-v)/(c+v)} - 1} \ , \ (32)$$

$$= \frac{2hv^3}{c^2} \cdot \frac{1}{z} \ . \ (33)$$

Do the following seem to suggest hypothetical relationships between something like a renormalization of infinities (34) and relativistic phenomena (35) ~ (37)?

$$\frac{-\infty + \infty}{2\pi} = \boldsymbol{i} \ , \ (34)$$

$$= \sqrt{\frac{m^2c^4}{p^2c^2 - E^2}} \ , \ (35)$$

$$= \sqrt{\frac{(ct)^2}{s^2 - (x^2 + y^2 + z^2)}} \ , \ (36)$$

$$= \sqrt{\frac{\Lambda g_{\mu v}}{G_{\mu v} - kT_{\mu v}}} \ , \ (37)$$

Conclusions

This study suggests a mathematical hypothesis that Bondi K–factor and its reciprocal give a toy space model showing a relativistic expansion under the change in v from $-c$ to c.

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