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Abstract

In this paper, we consider the popular matching problem with matroid constraints. It is known that if there exists no tie in preference lists of applicants, then this problem can be solved in polynomial time. In this paper, we prove that even if there exist ties in preference lists, this problem can be solved in polynomial time.

1 Introduction

In this paper, we consider some variation of the popular matching problem. The popular matching problem was originally introduced by Abraham, Irving, Kavitha, and Mehlhorn [1]. In this problem, we are give two disjoint sets of applicants and posts, and each applicant has a preference list over posts in which there may exist ties. A matching M between applicants and posts is said to be popular, if there exists no other matching N such that the number of applicants that prefer N to M is larger than the number of applicants that prefer M to N. The concept of popularity was originally proposed by Gärdenfors [10] in the context of matching problems in which agents in both sides have preference lists. The goal of the popular matching problem is to decide whether there exists a popular matching, and find a popular matching if one exists. Abraham, Irving, Kavitha, and Mehlhorn [1] presented polynomial-time algorithms for the popular matching problem with/without ties. Since their seminal paper, several variations of the popular matching problem [12, 19, 21, 24] and related problems [12, 13, 14, 15, 16, 20, 25, 26] have been investigated.

In this paper, we consider a matroid generalization of the popular matching problem. More precisely, in our model, capacity constraints for posts are generalized to matroid constraints. In the study of combinatorial optimization, for example, Fleiner [6] generalized capacity constraints in the stable matching problem to matroid constraints, and Zenklusen [27] considered a matroid generalization of the minimum-cost spanning tree problem with degree constraints. It is known [12] that if there exists no tie in preference lists, then this problem can be solved in polynomial time. In this paper, we prove that even if there exist ties in preference lists, we can solve this problem in polynomial time.

Our algorithm is based on the algorithm of Abraham, Irving, Kavitha, and Mehlhorn [1] for the popular matching problem with ties. However, the algorithm of [1] decomposes

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the vertex set of an input bipartite graph by using Gallai-Edmonds decomposition [5, 8, 9] (see also [18]) of a bipartite graph based on a maximum-size (one-to-one) matching. Thus, we can not straightforwardly generalize the algorithm of [1] to the matroid setting. Although Manlove and Sng [19] proposed an algorithm for a many-to-one variation of the popular matching with ties, their algorithm copies posts and uses the Gallai-Edmonds decomposition in the one-to-one setting. To overcome this difficulty, we decompose the edge set (instead of the vertex set) of an input bipartite graph bases on a maximum-size common independent set. This decomposition can be regarded as a matroid generalization of the Dulmage-Mendelsohn decomposition [3, 4] of a bipartite graph (see, e.g., [22] for the Dulmage-Mendelsohn type decomposition of the matroid intersection problem).

The rest of this paper is organized as follows. In Section 2, we give the formal definition of our model, and basics of matroids and the matroid intersection problem. In Section 3, we give a characterization of a popular matching in our model. In Section 4, we describe our algorithm, and prove its correctness.

2 Preliminaries

For each set X and each element u, we define $X + u := X \cup \{u\}$ and $X - u := X \setminus \{u\}$. For each sets X and Y, we define $X \triangle Y := (X \setminus Y) \cup (Y \setminus X)$. Assume that we are given a set X and a function $\xi\colon X \to \mathbb{Z}$, where $\mathbb{Z}$ is the set of integers. For each subset Y of X, we define

$$\xi(Y) := \sum_{u \in Y} \xi(u).$$

A pair $\mathbf{M} = (U, \mathcal{I})$ is called a *matroid*, if U is a finite set and $\mathcal{I}$ is a family of subsets of U satisfying the following conditions.

(I0) $\emptyset \in \mathcal{I}$.

(I1) If $I \in \mathcal{I}$ and $J \subseteq I$, then $J \in \mathcal{I}$.

(I2) If $I, J \in \mathcal{I}$ and $|I| < |J|$, then $I + u \in \mathcal{I}$ for some element u in $J \setminus I$.

We say that $\mathbf{M}$ is a matroid on the *ground set* U.

In this paper, we are given a finite simple (not necessarily complete) bipartite graph $G = (V, E)$. We assume that V is partitioned into two subsets A and P, and each edge in E connects a vertex in A and a vertex in P. We call a vertex in A an *applicant*, and a vertex in P a *post*. For each applicant a in A and each post p in P, if there exists an edge in E connecting a and p, then we denote by (a, p) this edge. For each vertex v in V and each subset F of E, we define $F(v)$ as the set of edges in F incident to v.

For each applicant a in A, we are given a transitive binary relation $\succsim_a$ on $E(a)$ such that at least one of $e \succsim_a g$ and $g \succsim_a e$ holds for every edges e, g in $E(a)$. Notice that for each applicant a in A and each edges e, g in $E(a)$, there exists a possibility that both $e \succsim_a g$ and $g \succsim_a e$ hold. For each applicant a in A and each edges e, g in $E(a)$, we use the notation $e \succ_a g$, if $e \succsim_a g$ and $g \not\succsim_a e$. For each applicant a in A and each edges e, g in $E(a)$, $e \succ_a g$ means that a preferes e to g. Furthermore, for each applicant a in A and each edges e, g in $E(a)$, if $e \succsim_a g$ and $g \succsim_a e$ hold, then a is indifferent between e and g.

As in [1], we assume that for each applicant a in A, there exists a *last resort post* $\ell(a)$ in P such that $E(\ell(a)) = \{(a, \ell(a))\}$, and $e \succ_a (a, \ell(a))$ for every edge e in $E(a) - (a, \ell(a))$. In addition, we assume that for every applicant a in A, there exists an edge e in $E(a)$ such that $e \neq (a, \ell(a))$. For each post p in P, we are given a matroid $\mathbf{M}_p = (E(p), \mathcal{I}_p)$. We assume that for every edge (a, p) in E, we have $\{(a, p)\} \in \mathcal{I}_p$.

A subset M of E is called a *matching* in G, if

- $|M(a)| = 1$ for every applicant a in A, and
- $M(p) \in \mathcal{I}_p$ for every post p in P.

For each subset F of E and and each applicant a in A such that $|F(a)| = 1$, we denote by $\mu_F(a)$ the unique edge in $F(a)$. For each matchings M, N in G, we denote by $\phi(M; N)$ the number of applicants a in A such that $\mu_M(a) \succ_a \mu_N(a)$. A matching M in G is said to be *popular*, if for every matching N in G, $\phi(M; N) \geq \phi(N; M)$. Then, the goal of the *popular matching problem with ties and matroid constraints* (PMTM for short) is to decide whether there exists a popular matching in G, and find a popular matching if one exists.

2.1 Basics of matroids

Let $\mathbf{M} = (U, \mathcal{I})$ be a matroid. A subset of U belonging to $\mathcal{I}$ is called an *independent set* of $\mathbf{M}$. A subset C of U is called a *circuit* of $\mathbf{M}$, if C is not an independent set of $\mathbf{M}$, but every proper subset of C is an independent set of $\mathbf{M}$. Assume that we are given an independent set I of $\mathbf{M}$ and an element u in $U \setminus I$ such that $I + u \notin \mathcal{I}$. It is known [23, Proposition 1.1.6] that there exists a unique circuit of $\mathbf{M}$ that is a subset of $I + u$, and u belongs to this unique circuit. This unique circuit is called the *fundamental circuit of* u, and denoted by $\mathsf{C}_{\mathbf{M}}(u, I)$. It is known [23, p.20, Exercise 5] that we have

$$\mathsf{C}_{\mathbf{M}}(u, I) = \{w \in I + u \mid I + u - w \in \mathcal{I}\}.$$

Notice that if $\{u\} \in \mathcal{I}$, then $\mathsf{C}_{\mathbf{M}}(u, I) - u \neq \emptyset$.

A maximal independent set of $\mathbf{M}$ is called a *base* of $\mathbf{M}$. Notice that (I2) implies that every base of $\mathbf{M}$ has the same size. For each subset X of U, we define $\mathcal{I}|X$ as the family of subsets I of X such that $I \in \mathcal{I}$, and $\mathbf{M}|X := (X, \mathcal{I}|X)$. It is known [23, p.20] that for every subset X of U, $\mathbf{M}|X$ is a matroid. For each subset X of U, we define $\mathsf{r}_{\mathbf{M}}(X)$ as the size of a base of $\mathbf{M}|X$. For each disjoint subsets X, J of U, we define

$$\mathsf{p}(J; X) := \mathsf{r}_{\mathbf{M}}(J \cup X) - \mathsf{r}_{\mathbf{M}}(X).$$

For each subset X of U, we define

$$\begin{aligned} \mathcal{I}/X &:= \{J \subseteq U \setminus X \mid \mathsf{p}(J; X) = |J|\}, \\ \mathbf{M}/X &:= (U \setminus X, \mathcal{I}/X). \end{aligned}$$

It is known [23, Proposition 3.1.6] that for each subset X of U, $\mathbf{M}/X$ is a matroid.

Lemma 1 (see, e.g., [23, Proposition 3.1.7])**.** *Assume that we are given a matroid* $\mathbf{M} = (U, \mathcal{I})$*, a subset* X *of* U*, and a base* B *of* $\mathbf{M}|X$*. Then, for every element* u *of* $U \setminus X$*,* $\{u\}$ *is an independent set of* $\mathbf{M}/X$ *if and only if* $B + u$ *is an independent set of* $\mathbf{M}$*.*

Lemma 2 (See, e.g., [23, p.15, Exercise 14])**.** *Assume that we are given a matroid $\mathbf{M} = (U, \mathcal{I})$ and circuits C_1, C_2 of $\mathbf{M}$ such that $C_1 \cap C_2 \neq \emptyset$ and $C_1 \setminus C_2 \neq \emptyset$. Then, for every element u in $C_1 \cap C_2$ and every element w in $C_1 \setminus C_2$, there exists a circuit C of $\mathbf{M}$ such that $w \in C$ and C is a subset of $(C_1 \cup C_2) - u$.*

Lemma 3. *Assume that we are given a matroid $\mathbf{M} = (U, \mathcal{I})$, an independent set I, J of $\mathbf{M}$ such that $J \subseteq I$, and an element u in $U \setminus J$ such that $I + u \notin \mathcal{I}$ and $J + u \notin \mathcal{I}$. Then, $\mathsf{C}_{\mathbf{M}}(u, I) = \mathsf{C}_{\mathbf{M}}(u, J)$.*

Proof. Define $C_1 := \mathsf{C}_{\mathbf{M}}(u, I)$ and $C_2 := \mathsf{C}_{\mathbf{M}}(u, J)$. Assume that $C_1 \neq C_2$. Then, Lemma 2 implies that there exists a circuit C of $\mathbf{M}$ such that $C \subseteq (C_1 \cup C_2) - u$. Since $C_1 - u \subseteq I$ and $C_2 - u \subseteq J \subseteq I$, we have $C \subseteq I$. This contradicts the fact that $I \in \mathcal{I}$. □

Lemma 4. *Assume that we are given a matroid $\mathbf{M} = (U, \mathcal{I})$, an independent set I of $\mathbf{M}$, and a subset J of $U \setminus I$ such that $I \cup J \in \mathcal{I}$. Furthermore, we assume that we are given subsets X, Y of $U \setminus J$ such that $(I \cup X) \setminus Y \in \mathcal{I}$ and $I + x \notin \mathcal{I}$ for every element x in X. Then, $((I \cup X) \setminus Y) \cup J$ is an independent set of $\mathbf{M}$.*

Proof. We prove this lemma by contradiction. Assume that $((I \cup X) \setminus Y) \cup J \notin \mathcal{I}$. This implies that there exists a circuit C of $\mathbf{M}$ such that $C \subseteq ((I \cup X) \setminus Y) \cup J$. If $C \cap J = \emptyset$, then this contradicts the fact that $(I \cup X) \setminus Y$ is an independent set of $\mathbf{M}$. Thus, there exists an element u in $J \cap C$. If $C \cap X = \emptyset$, then this contradicts the fact that $I \cup J$ is an independent set of $\mathbf{M}$. Thus, $X \cap C$ is not empty. Assume that $X \cap C = \{x_1, x_2, \ldots, x_k\}$. Since $u \in J$, $J \cap X = \emptyset$, and $J \cap I = \emptyset$, we have $u \notin \mathsf{C}_{\mathbf{M}}(x_i, I)$ for every $i = 1, 2, \ldots, k$.

Here we consider the following procedure.

Step 1. Set $i := 1$ and $C_0 := C$.

Step 2. If $i \leq k$, then do the following steps.

- **(2-a)** If $x_i \notin C_{i-1}$, then set $C_i := C_{i-1}$ and go to **(2-c)**.
- **(2-b)** Find a circuit C_i of $\mathbf{M}$ such that $u \in C_i$ and $C_i \subseteq (C_{i-1} \cup \mathsf{C}_{\mathbf{M}}(x_i, I)) - x_i$.
- **(2-c)** Update $i := i + 1$ and go back to the beginning of **Step 2**.

Step 3. Output C_k.

In **Step (2-b)**, since $x_i \in C_{i-1} \cap \mathsf{C}_{\mathbf{M}}(x_i, I)$ and $u \in C_{i-1} \setminus \mathsf{C}_{\mathbf{M}}(x_i, I)$, Lemma 2 implies that there exists a circuit C_i of $\mathbf{M}$ such that $u \in C_i$ and $C_i \subseteq (C_{i-1} \cup \mathsf{C}_{\mathbf{M}}(x_i, I)) - x_i$. It is not difficult to see that C_k is a subset of $I \cup J$, which contradicts the fact that $I \cup J \in \mathcal{I}$. This completes the proof. □

Assume that we are given k matroids $\mathbf{M}_1 = (U_1, \mathcal{I}_1), \mathbf{M}_2 = (U_2, \mathcal{I}_2), \ldots, \mathbf{M}_k = (U_k, \mathcal{I}_k)$ such that $U_i \cap U_j = \emptyset$ for every distinct $i, j = 1, 2, \ldots, k$. Define

$$\bigoplus_{i=1}^{k} \mathcal{I}_i := \Big\{ X \subseteq \bigcup_{i=1}^{k} U_i \;\Big|\; X \cap U_i \text{ is an independent set of } \mathbf{M}_i \text{ for every } i = 1, 2, \ldots, k \Big\}.$$

Furthermore, we define

$$\bigoplus_{i=1}^{k} \mathbf{M}_i := \Big(\bigcup_{i=1}^{k} U_i,\ \bigoplus_{i=1}^{k} \mathcal{I}_i\Big).$$

It is not difficult to see that $\bigoplus_{i=1}^{k} \mathbf{M}_i$ is a matroid.

Let $\mathbf{M} = (U, \mathcal{I})$ and $\mathbf{N} = (U, \mathcal{J})$ be matroids. A subset I of U is called a *common independent set* of $\mathbf{M}$ and $\mathbf{N}$, if $I \in \mathcal{I} \cap \mathcal{J}$. We denote by $\mathcal{B}_{\mathbf{MN}}$ and $\gamma_{\mathbf{MN}}$ the family and the size of maximum-size common independent sets of $\mathbf{M}$ and $\mathbf{N}$, respectively. We can find a maximum-size common independent set of $\mathbf{M}$ and $\mathbf{N}$ in $O(|U|^{2.5}\mathsf{EO})$ by using the algorithm of Cunningham [2], where EO is the time required to decide whether $X + u$ is an independent set of $\mathbf{L}$ for every matroid $\mathbf{L}$ in $\{\mathbf{M}, \mathbf{N}\}$, every common independent set X of $\mathbf{M}$ and $\mathbf{N}$, and every element u in $U \setminus X$. Furthermore, for every function $\xi \colon U \to \mathbb{Z}$, we can find a maximum-size common independent set I of $\mathbf{M}$ and $\mathbf{N}$ such that

$$\xi(I) = \max\{\xi(J) \mid J \in \mathcal{B}_{\mathbf{MN}}\}$$

in $O(|U|^3\mathsf{EO})$ time by using the algorithm of Frank [7].

2.2 Auxiliary graphs and decomposition

Assume that we are given matroids $\mathbf{M} = (U, \mathcal{I})$ and $\mathbf{N} = (U, \mathcal{J})$, and a common independent set I of $\mathbf{M}$ and $\mathbf{N}$. Then, we define a directed graph $\mathsf{D}_{\mathbf{MN}}(I)$ as follows. The vertex set of $\mathsf{D}_{\mathbf{MN}}(I)$ is U. For each elements u of $U \setminus I$ and v of I,

- there exists an arc from v to u in $\mathsf{D}_{\mathbf{MN}}(I)$ if and only if $I + u \notin \mathcal{I}$ and $v \in \mathsf{C}_{\mathbf{M}}(u, I)$,
- there exists an arc from u to v in $\mathsf{D}_{\mathbf{MN}}(I)$ if and only if $I + u \notin \mathcal{J}$ and $v \in \mathsf{C}_{\mathbf{N}}(u, I)$.

These are all arcs of $\mathsf{D}_{\mathbf{MN}}(I)$. Define

$$\mathsf{T}_{\mathbf{M}}(I) := \{u \in U \setminus I \mid I + u \in \mathcal{I}\},$$
$$\mathsf{T}_{\mathbf{N}}(I) := \{u \in U \setminus I \mid I + u \in \mathcal{J}\}.$$

Lemma 5 (see, e.g., [17, Lemma 13.30])**.** *Assume that we are given matroids* $\mathbf{M}$ *and* $\mathbf{N}$ *on the same ground set* U *and a common independent set* I *of* $\mathbf{M}$ *and* $\mathbf{N}$*. Then,* I *is a maximum-size common independent set of* $\mathbf{M}$ *and* $\mathbf{N}$ *if and only if there exists no directed path in* $\mathsf{D}_{\mathbf{MN}}(I)$ *from a vertex in* $\mathsf{T}_{\mathbf{M}}(I)$ *to a vertex in* $\mathsf{T}_{\mathbf{N}}(I)$*.*

Assume that I is a maximum-size common independent set of $\mathbf{M}$ and $\mathbf{N}$. We denote by $\Omega^{+}_{\mathbf{MN}}(I)$ the set of elements u in U such that there exists a directed path in $\mathsf{D}_{\mathbf{MN}}(I)$ from a vertex in $\mathsf{T}_{\mathbf{M}}(I)$ to u. In addition, we define $\Omega^{-}_{\mathbf{MN}}(I)$ as the set of elements u in U such that there exists a directed path in $\mathsf{D}_{\mathbf{MN}}(I)$ from u to a vertex in $\mathsf{T}_{\mathbf{N}}(I)$. Notice that since I is a maximum-size common independent set of $\mathbf{M}$ and $\mathbf{N}$, Lemma 5 implies that $\Omega^{+}_{\mathbf{MN}}(I)$ and $\Omega^{-}_{\mathbf{MN}}(I)$ are disjoint. Define

$$\Omega_{\mathbf{MN}}(I) := U \setminus \Big(\Omega^{+}_{\mathbf{MN}}(I) \cup \Omega^{-}_{\mathbf{MN}}(I)\Big).$$

Lemma 6. *Assume that we are given matroids* $\mathbf{M}$ *and* $\mathbf{N}$ *on the same ground set* U *and a common independent set* I *of* $\mathbf{M}$ *and* $\mathbf{N}$*. Furthermore, we assume that we are given a vertex* u *in* $\mathsf{T}_{\mathbf{M}}(I) \cup I$ *and a vertex* v *in* $\mathsf{T}_{\mathbf{N}}(I) \cup I$*, and a shortest directed path* L *in* $\mathsf{D}_{\mathbf{MN}}(I)$ *from* u *to* v*. Then,* $I \triangle K$ *is a common independent set of* $\mathbf{M}$ *and* $\mathbf{N}$*, where* K *is the set of elements in* U *that* L *goes through* (*notice that since* L *is a shorted directed path,* L *goes through each vertex at most once*)*.*

Proof. This lemma immediately follows from [17, Lemma 13.27] and Lemma 4. □

Lemma 7 (see, e.g., [11, Lemma 4.2])**.** *Assume that we are given matroids* $\mathbf{M}$ *and* $\mathbf{N}$ *on the same ground set. Then, for every maximum-size common independent sets* I, J *of* $\mathbf{M}$ *and* $\mathbf{N}$*,*

$$J \cap \Omega^-_{\mathbf{MN}}(I) \text{ is a base of } \mathbf{M}|\Omega^-_{\mathbf{MN}}(I), \tag{1}$$
$$J \cap (\Omega_{\mathbf{MN}}(I) \cup \Omega^-_{\mathbf{MN}}(I)) \text{ is a base of } \mathbf{M}|(\Omega_{\mathbf{MN}}(I) \cup \Omega^-_{\mathbf{MN}}(I)), \tag{2}$$
$$J \cap \Omega^+_{\mathbf{MN}}(I) \text{ is a base of } \mathbf{N}|\Omega^+_{\mathbf{MN}}(I), \tag{3}$$
$$J \cap (\Omega^+_{\mathbf{MN}}(I) \cup \Omega_{\mathbf{MN}}(I)) \text{ is a base of } \mathbf{N}|(\Omega^+_{\mathbf{MN}}(I) \cup \Omega_{\mathbf{MN}}(I)). \tag{4}$$

Although the following lemma is well-known, we give its proof for completeness.

Lemma 8. *Assume that we are given matroids* $\mathbf{M} = (U, \mathcal{I})$ *and* $\mathbf{N} = (U, \mathcal{J})$*. Then, for every maximum-size common independent sets* I, J *of* $\mathbf{M}$ *and* $\mathbf{N}$*, we have*

$$\Omega^+_{\mathbf{MN}}(I) = \Omega^+_{\mathbf{MN}}(J),$$
$$\Omega_{\mathbf{MN}}(I) = \Omega_{\mathbf{MN}}(J),$$
$$\Omega^-_{\mathbf{MN}}(I) = \Omega^-_{\mathbf{MN}}(J).$$

Proof. For simplicity, we define $\mathsf{D}_{\mathbf{MN}}(\cdot) := \mathsf{D}(\cdot)$, $\Omega^+_{\mathbf{MN}}(\cdot) := \Omega^+(\cdot)$, and $\Omega^-_{\mathbf{MN}}(\cdot) := \Omega^-(\cdot)$. Let I, J be maximum-size common independent sets of $\mathbf{M}$ and $\mathbf{N}$.

We first prove that $\Omega^+(I) = \Omega^+(J)$. For proving this, we first prove that $\Omega^+(J) \subseteq \Omega^+(I)$. Assume that $\mathsf{T}_{\mathbf{M}}(J) \not\subseteq \Omega^+(I)$, and let u be an element in $\mathsf{T}_{\mathbf{M}}(J) \setminus \Omega^+(I)$. Then, $J + u \in \mathcal{I}$ and (I1) imply that

$$(J + u) \cap (\Omega(I) \cup \Omega^-(I)) = (J \cap (\Omega(I) \cup \Omega^-(I))) + u \in \mathcal{I},$$

which contradicts (2). Assume that there exists an arc in $\mathsf{D}(J)$ from a vertex u in $\Omega^+(I)$ to a vertex v in $\Omega(I) \cup \Omega^-(I)$. If u is in I, then $J + v - u \in \mathcal{I}$. This and (I1) imply that

$$(J + v - u) \cap (\Omega(I) \cup \Omega^-(I)) = (J \cap (\Omega(I) \cup \Omega^-(I))) + v \in \mathcal{I}.$$

This contradicts (2). If u is not in I, then $J + u - v \in \mathcal{J}$. This and (I1) imply that

$$(J + u - v) \cap \Omega^+(I) = (J \cap \Omega^+(I)) + u \in \mathcal{J}.$$

This contradicts (3). By exchanging the roles of I and J, we can prove that $\Omega^+(I) \subseteq \Omega^+(J)$, which implies that $\Omega^+(J) = \Omega^+(I)$.

Next we prove that $\Omega^-(J) = \Omega^-(I)$. For proving this, we first prove that $\Omega^-(J) \subseteq \Omega^-(I)$. Assume that $\mathsf{T}_{\mathbf{N}}(J) \not\subseteq \Omega^-(I)$, and let u be an element in $\mathsf{T}_{\mathbf{N}}(J) \setminus \Omega^-(I)$. Then, $J + u \in \mathcal{J}$ and (I1) imply that

$$(J+u) \cap (\Omega^+(I) \cup \Omega(I)) = (J \cap (\Omega^+(I) \cup \Omega(I))) + u \in \mathcal{J}.$$

This contradicts (4). Assume that there exists an arc in $\mathsf{D}(J)$ from u in $\Omega^+(I) \cup \Omega(I)$ to v in $\Omega^-(I)$. If u is in I, then $J + v - u \in \mathcal{I}$. This and (I1) imply that

$$(J+v-u) \cap \Omega^-(I) = (J \cap \Omega^-(I)) + v \in \mathcal{I}.$$

This contradicts (1). If u is not in I, then $J + u - v \in \mathcal{J}$. This and (I1) imply that

$$(J+u-v) \cap (\Omega^+(I) \cup \Omega(I)) = (J \cap (\Omega^+(I) \cup \Omega(I))) + u \in \mathcal{J}.$$

This contradicts (4). By exchanging the roles of I and J, we can prove that $\Omega^-(I) \subseteq \Omega^-(J)$, which implies that $\Omega^-(J) = \Omega^-(I)$. This completes the proof. $\square$

Lemma 8 implies that for every matroids $\mathbf{M}$ and $\mathbf{N}$ on the same ground set, $\Omega^+_{\mathbf{MN}}(I)$, $\Omega_{\mathbf{MN}}(I)$, and $\Omega^-_{\mathbf{MN}}(I)$ do not depend a choice of a maximum-size common independent set I of $\mathbf{M}$ and $\mathbf{N}$. Thus, for each matroids $\mathbf{M}$ and $\mathbf{N}$ on the same ground set and some maximum-size common independent set I, we define

$$\begin{aligned}\Omega^+_{\mathbf{MN}} &:= \Omega^+_{\mathbf{MN}}(I),\\ \Omega_{\mathbf{MN}} &:= \Omega_{\mathbf{MN}}(I),\\ \Omega^-_{\mathbf{MN}} &:= \Omega^-_{\mathbf{MN}}(I),\end{aligned}$$

respectively.

3 Characterization

In this section, we give a characterization of a popular matching. Define

$$\mathcal{U} := \left\{ F \subseteq E \;\middle|\; |F(a)| \le 1 \text{ for every applicant } a \text{ in } A \right\},$$

and $\mathcal{A} := (E,\mathcal{U})$. It is not difficult to see that $\mathcal{A}$ is a matroid. Define

$$\mathbf{P} := \bigoplus_{p \in P} \mathbf{M}_p.$$

Notice that for each subset M of E, M is a matching in G if and only if M is a common independent set of $\mathbf{A}$ and $\mathbf{P}$ such that $|M| = |A|$. For each applicant a in A, we define $f(a)$ by

$$f(a) := \left\{ e \in E(a) \;\middle|\; e \succsim_a g \text{ for every edge } g \text{ in } E(a) \right\}.$$

Notice that for each applicant a in A, there exists a possibility that $|f(a)| > 1$. Define

$$\begin{aligned}
&\Gamma := \bigcup_{a \in A} f(a),\\
&\mathbf{B} := \mathbf{A}|\Gamma,\\
&\mathbf{Q} := \mathbf{P}|\Gamma \quad \Big(= \bigoplus_{p \in P} (\mathbf{M}_p|\Gamma(p)) \Big).
\end{aligned}$$

In addition, we define

$$T := \Big\{ e \in E \setminus \Gamma \ \Big| \ \{e\} \text{ is an independent set of } \mathbf{P}/(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}}) \Big\}.$$

Notice that for each applicant a in A, since there exists the last resort post $\ell(a)$, $T(a) \neq \emptyset$. For each applicant a in A, we define $s(a)$ by

$$s(a) := \Big\{ e \in T(a) \ \Big| \ e \succsim_a g \text{ for every edge } g \text{ in } T(a) \Big\}.$$

In addition, we define

$$\begin{aligned}
S &:= \bigcup_{a \in A} s(a),\\
\Pi &:= \Gamma \cup S.
\end{aligned}$$

The following characterization can be regarded as a generalization of [1, Lemma 3.5].

Theorem 9. *For each matching M in G, M is a popular matching in G if and only if*

(P1) *$M \cap \Gamma$ is a maximum-size common independent set of $\mathbf{B}$ and $\mathbf{Q}$, and*

(P2) *M is a subset of Π.*

We first give lemmas that are necessary for proving the *only if*-part of Theorem 9.

Lemma 10. *Assume that we are given a popular matching M in G. Then, $M \cap \Gamma$ is a maximum-size common independent set of $\mathbf{B}$ and $\mathbf{Q}$.*

Since the proof of Lemma 10 is long, we leave it to Section 3.1.

Lemma 11. *Assume that we are given a popular matching M in G. Then, there exists no applicant a in A such that $e \succ_a \mu_M(a) \succ_a g$ for edges e in $f(a)$ and g in $s(a)$.*

Proof. For proving this lemma by contradiction, we assume that there exists an applicant a in A such that $e \succ_a \mu_M(a) \succ_a g$ for edges e in $f(a)$ and g in $s(a)$. Since M is a popular matching in G, Lemma 10 implies that $M \cap \Gamma$ is a maximum common independent set of $\mathbf{B}$ and $\mathbf{Q}$. This and Lemma 7 imply that $M \cap (\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})$ is a base of $\mathbf{Q}|(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})$ $(= \mathbf{P}|(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}}))$. Define

$$N := (M \cap (\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})) + \mu_M(a). \tag{5}$$

Since $N \subseteq M$ and M is an independent set of $\mathbf{P}$, N is an independent set of $\mathbf{P}$. Thus, (5) and Lemma 1 imply that $\mu_M(a)$ is an independent set of $\mathbf{P}/(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})$. However, this contradicts that fact that $\mu_M(a) \succ_a g$ for an edge g in $s(a)$. This completes the proof. $\square$

Lemma 12. *Assume that we are given a popular matching M in G. Then, there exists no applicant a in A such that $\mu_M(a) \notin \Pi$.*

We leave the proof of Lemma 12 to Section 3.2.

Next we give a lemma that is necessary for proving the *if*-part of Theorem 9. For each matching M in G, we denote by $\mathsf{b}(M)$ the set of edges $e = (a, p)$ in E such that $e \succ_a g$ for an edge g in $s(a)$.

Lemma 13. *For every matching M in G, we have $\gamma_{\mathbf{BQ}} \geq |\mathsf{b}(M)|$.*

Proof. For proving this lemma by contradiction, we assume that there exists a matching M in G such that $\gamma_{\mathbf{BQ}} < |\mathsf{b}(M)|$. Let N be a maximum-size common independent set in $\mathbf{B}$ and $\mathbf{Q}$ such that

$$\forall N' \in \mathcal{B}_{\mathbf{BQ}}\colon\ |M \cap N| \geq |M \cap N'|. \tag{6}$$

Since $\gamma_{\mathbf{BQ}} < |\mathsf{b}(M)|$, we have $|N| < |\mathsf{b}(M)|$. In addition, since M is an independent set of $\mathbf{P}$, (I1) implies that $\mathsf{b}(M)$ is an independent set of $\mathbf{P}$. Furthermore, N is an independent set of $\mathbf{P}$. Thus, (I2) implies that there exists an edge $e = (a, p)$ in $\mathsf{b}(M) \setminus N$ such that $N + e$ is an independent set of $\mathbf{P}$. Notice that since e is in $\mathsf{b}(M)$, $e \succ_a g$ for an edge g in $s(a)$.

We first consider the case where $e \notin \Gamma$. Since $N + e$ is an independent set of $\mathbf{P}$, (I1) implies that

$$(N \cap (\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})) + e$$

is an independent set of $\mathbf{P}$. Since it follows from Lemma 7 that $N \cap (\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})$ is a base of $\mathbf{Q}|(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})$ $(= \mathbf{P}|(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}}))$, $\{e\}$ is an independent set of $\mathbf{P}/(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})$. This contradicts the fact that $e \succ_a g$ for an edge g in $s(a)$.

Next we consider the case where $e \in \Gamma$, i.e., $N + e \subseteq \Gamma$. Since N is a maximum-size common independent set in $\mathbf{B}$ and $\mathbf{Q}$, $N + e$ is not an independent set of $\mathbf{A}$. That is, $|N(a)| = 1$. In addition, since $e \in M$ and M is a matching in G, $\mu_N(a) \notin M$. Thus,

$$|(N + e - \mu_N(a)) \cap M| = |M \cap N| + 1. \tag{7}$$

Since N is an independent set of $\mathbf{B}$, $N + e - \mu_N(a)$ is an independent set of $\mathbf{B}$. Since $N + e$ is an independent set of $\mathbf{Q}$, (I1) implies that $N + e - \mu_N(a)$ is a maximum-size common independent set in $\mathbf{B}$ and $\mathbf{Q}$. These and (7) contradict (6), which completes the proof. $\square$

We are now ready to prove Theorem 9.

Proof of Theorem 9. Since the *only if*-part follows from Lemmas 10 and 12, we prove the *if*-part. Let M be a matching in G satisfying (P1) and (P2), and assume that we are given a matching N in G. We denote by A_M and A_N be the sets of applicants a in A such that $\mu_M(a) \succ_a \mu_N(a)$ and $\mu_N(a) \succ_a \mu_M(a)$, respectively. If we can prove that $|A_M| \geq |A_N|$, then the proof is done. For proving this, it is sufficient to construct an injective function $\tau\colon A_N \to A_M$.

Lemma 13 and (P1) imply that there exists an injective function

$$\varphi\colon \mathsf{b}(N) \setminus (M \cap \Gamma) \to (M \cap \Gamma) \setminus \mathsf{b}(N).$$

Let a be an applicant in A_N. Since $\mu_N(a) \succ_a \mu_M(a)$, (P2) implies that $\mu_M(a)$ is in $s(a)$. Thus, $\mu_N(a)$ is in $\mathsf{b}(N) \setminus (M \cap \Gamma)$. Here we consider the following procedure.

Step 1. Set $i := 1$, and define $e_1 = (a_1, p_1)$ as $\varphi(\mu_N(a))$.

Step 2. If we have $e_i \succ_{a_i} \mu_N(a_i)$, then we define $\tau(a) := a_i$ and halt. Otherwise, define $e_{i+1} := (a_{i+1}, p_{i+1})$ as $\varphi(\mu_N(a_i))$ and update $i := i + 1$. Then, repeat **Step 2**.

For proving that this procedure is well-defined, it is sufficient to prove in **Step 2**, $\mu_N(a_i)$ is in $\mathsf{b}(N) \setminus (M \cap \Gamma)$. Since $e_i \in (M \cap \Gamma) \setminus \mathsf{b}(N)$, if $e_i \in N$, then $e_i \in f(a_i)$ implies that $e_i \in \mathsf{b}(N)$. This contradicts the fact that $e_i \in (M \cap \Gamma) \setminus \mathsf{b}(N)$. Thus, $e_i \notin N$. In addition, since $e_i \in f(a_i)$, if $e_i \not\succ_{a_i} \mu_N(a_i)$, then $\mu_N(a_i) \in f(a_i)$. Thus, $\mu_N(a_i) \in \mathsf{b}(N) \setminus (M \cap \Gamma)$.

Since G is finite and φ is injective, this procedure halts. Thus, since φ is injective, τ is clearly injective. This completes the proof. □

3.1 Proof of Lemma 10

Here we give a proof of Lemma 10. In the sequel, for each common independent set I of $\mathbf{B}$ and $\mathbf{Q}$, we do not distinguish between each vertex of $\mathsf{D}_{\mathbf{BQ}}(I)$ and the edge in Γ that it corresponds to. That is, we may call a vertex of $\mathsf{D}_{\mathbf{BQ}}(I)$ an edge. Since $\mathsf{D}_{\mathbf{BQ}}(I)$ is a directed graph (i.e., it contains only arcs), it does not make any confusion.

Since M is a matching in G, M is a common independent set of $\mathbf{A}$ and $\mathbf{P}$. This and (I1) imply that $M \cap \Gamma$ is a common independent set of $\mathbf{B}$ and $\mathbf{Q}$. For proving this lemma by contradiction, we assume that $M \cap \Gamma$ is not a maximum-size common independent set of $\mathbf{B}$ and $\mathbf{Q}$. It follows from this and Lemma 5 that there exist an edge $g = (b, q)$ in $\mathsf{T}_{\mathbf{B}}(M \cap \Gamma)$ and an edge $h = (c, r)$ in $\mathsf{T}_{\mathbf{Q}}(M \cap \Gamma)$ such that there exists a directed path in $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ from g to h. Let L be a shortest directed path in $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ from g to h, and we denote by $K = \{e_1, e_2, \ldots, e_k\}$ the set of edges in Γ (i.e., vertices of $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$) that L goes through. Furthermore, we assume that $e_i := (a_i, p_i)$ for each $i = 1, 2, \ldots, k$, and L goes through $e_1, e_2, \ldots, e_k$ in this order. Notice that $e_1 = g$, $e_k = h$, k is odd, and $e_i \in M$ for every $i = 2, 4, \ldots, k-1$. Define $N := M \triangle K$ and $N_0 := N - \mu_M(b)$. Lemma 6 implies that $N \cap \Gamma$ $(= N_0 \cap \Gamma)$ is an independent set of $\mathbf{Q}$.

For completing a proof of Lemma 10, we first prove necessary lemmas (Section 3.1.1), and then complete a proof of Lemma 3.1 (Section 3.1.2).

3.1.1 Necessary lemmas

Here we prove necessary lemmas for proving Lemma 10.

Lemma 14. *For every post p in $P - r$, we have $N(p) \in \mathcal{I}_p$ and $N_0(p) \in \mathcal{I}_p$.*

Proof. Let p be a post in $P - r$. By using Lemma 4, we prove that $N(p) \in \mathcal{I}_p$. If we can prove this, then (I1) implies that $N_0(p) \in \mathcal{I}_p$.

Define $I := M(p) \cap \Gamma$, $J := M(p) \setminus \Gamma$, $X := K(p) \setminus M$, and $Y := M(p) \cap K$. Then,

$$(I \cup X) \setminus Y = N(p) \cap \Gamma,$$
$$((I \cup X) \setminus Y) \cup J = N(p).$$

Since M is an independent set of $\mathbf{P}$, the definition of $\mathbf{P}$ implies that $I \cup J \in \mathcal{I}_r$. In addition, the definition of $\mathbf{Q}$ implies that $(I \cup X) \setminus Y$ is an independent set of $\mathbf{M}_p|\Gamma(p)$, i.e., $\mathbf{M}_p$. Furthermore, for every edge e in X, it follows from the definition of $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ that $(M \cap \Gamma) + e$ is not an independent set of $\mathbf{Q}$. Thus, since $M \cap \Gamma$ is an independent set of $\mathbf{Q}$, the definition of $\mathbf{Q}$ implies that $I + e$ is not an independent set of $\mathbf{M}_p|\Gamma(p)$, i.e., $\mathbf{M}_p$. Thus, Lemma 4 implies that $N(p) \in \mathcal{I}_p$. $\square$

Lemma 15. *For every applicant a in $A - b$,* (i) $|N(a)| = 1$, *and* (ii) $\mu_N(a) \succsim_a \mu_M(a)$.

Proof. For every $i = 2, 4, \ldots, k-1$, since there exists an arc in $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ from e_i to e_{i+1}, $M + e_{i+1} - e_i$ is an independent set of $\mathbf{B}$. Thus, the definitions of $\mathbf{A}$ implies that we have $a_i = a_{i+1}$ for every $i = 2, 4, \ldots, k-1$. This implies that for every applicant a in $A - b$, since $|M(a)| = 1$, we have $|N(a)| = 1$. Furthermore, since $K \subseteq \Gamma$, $\mu_N(a) \in f(a)$ for every applicant a in $A - b$, which implies (ii). This completes the proof. $\square$

Lemma 16. (i) $g \succ_b \mu_M(b)$, *and* (ii) $|N_0(b)| = 1$.

Proof. Since $g \in \mathsf{T}_{\mathbf{B}}(M \cap \Gamma)$, $M(b) \cap \Gamma = \emptyset$. Thus, $\mu_M(b) \notin \Gamma$. This and $g \in \Gamma$ imply that $g \succ_b \mu_M(b)$. As proved in the proof of Lemma 15, $a_i = a_{i+1}$ for every $i = 2, 4, \ldots, k-1$. Thus, since $\mu_M(b) \notin \Gamma$, we have $K(b) = \{g\}$. This and $M(b) = \{\mu_M(b)\}$ imply (ii). $\square$

We denote by L' the subpath of L from g to e_{k-1}. Since L is a shortest directed path in $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ from g to h, L' is a shortest directed path in $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ from g to e_{k-1}. Define $K' := K - h$. Then, $N - h = M \triangle K'$. Lemma 6 implies that $(N - h) \cap \Gamma$ is an independent set of $\mathbf{Q}$.

Lemma 17. $N(r) - h \in \mathcal{I}_r$ *and* $N_0(r) - h \in \mathcal{I}_r$.

Proof. By using Lemma 4, we prove that $N(r) - h \in \mathcal{I}_r$. If we can prove this, then (I1) implies that $N_0(r) - h \in \mathcal{I}_r$.

Define $I := M(r) \cap \Gamma$, $J := M(r) \setminus \Gamma$, $X := K'(r) \setminus M$, and $Y := M(r) \cap K'$. Then,

$$(I \cup X) \setminus Y = (N(r) - h) \cap \Gamma,$$
$$((I \cup X) \setminus Y) \cup J = N(r) - h.$$

Since M is an independent set of $\mathbf{P}$, the definition of $\mathbf{P}$ implies that $I \cup J$ is in $\mathcal{I}_r$. In addition, the definition of $\mathbf{Q}$ implies that implies that $(I \cup X) \setminus Y$ is in $\mathcal{I}_r$. For every edge e in X, the definition of $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ implies that $(M \cap \Gamma) + e$ is not an independent set of $\mathbf{Q}$. Thus, since $M \cap \Gamma$ is an independent set of $\mathbf{Q}$, this and the definition of $\mathbf{Q}$ imply that $I + e$ is not an independent set of $\mathbf{M}_r$. Thus, Lemma 4 implies that $N(r) - h \in \mathcal{I}_r$. $\square$

3.1.2 Completing a proof

We are now ready to complete a proof of Lemma 10. If $N_0(r) \in \mathcal{I}_r$, then it follows from Lemmas 14, 15(i), and 16(ii) that N_0 is a matching in G. In addition, Lemma 15(ii) and 16(i) imply that $\phi(N_0; M) \geq 1$, which contradicts the fact that M is a popular matching in G. Thus, in the sequel, we can assume that $N_0(r) \notin \mathcal{I}_r$. In this case, since $N_0(r) \subseteq N(r)$, $N(r)$ is not in $\mathcal{I}_r$. Thus, Lemma 17 implies that $\mathsf{C}_{\mathbf{M}_r}(h, N(r) - h)$ and $\mathsf{C}_{\mathbf{M}_r}(h, N_0(r) - h)$ are well-defined. Define

$$C := \mathsf{C}_{\mathbf{M}_r}(h, N(r) - h),$$
$$C_0 := \mathsf{C}_{\mathbf{M}_r}(h, N_0(r) - h).$$

Notice that Lemma 3 implies that $C = C_0$.

Since $N_0 \cap \Gamma$ is an independent set of $\mathbf{Q}$, it follows from the definition of $\mathbf{Q}$ that $N_0(r) \cap \Gamma$ is in $\mathcal{I}_r$. Thus, $C_0 \subseteq N_0(r)$ implies that $C_0 \not\subseteq \Gamma$. Let $g_1 = (b_1, r)$ be an edge in $C_0 \setminus \Gamma$. Since $g \in \Gamma$ and $g_1 \notin \Gamma$, we have $g \neq g_1$. Thus, since $g, g_1 \in N_0$, we have $b \neq b_1$. Define $N_1 := N_0 - g_1$. Then, $g_1 \in C_0$ and Lemmas 14, 15(i), and 16(ii) imply that

- $|N_1(a)| = 1$ for every applicant a in $A - b_1$, and $N_1(b_1) = \emptyset$,
- $N_1(p) \in \mathcal{I}_p$ for every post p in P.

Let $h_1 = (b_1, q_1)$ be an edge in $f(b_1)$. We first consider the case where $N_1(q_1) + h_1 \in \mathcal{I}_{q_1}$. Define $N_2 := N_1 + h_1$. In this case, N_2 is a matching in G. Furthermore, since it follows from $g_1 \notin \Gamma$, Lemmas 15(ii) and 16(i) that

$$\begin{aligned}
&\mu_{N_2}(b) = g \succ_b \mu_M(b),\\
&\mu_{N_2}(b_1) = h_1 \succ_{b_1} g_1 = \mu_N(b_1) \succsim_{b_1} \mu_M(b_1),\\
&\forall a \in A \setminus \{b, b_1\} \colon \mu_{N_2}(a) = \mu_N(a) \succsim_a \mu_M(a),
\end{aligned}$$

we have $\phi(N_2; M) \geq 2$. These contradict the fact that M is a popular matching in G.

Next we consider the case where $N_1(q_1) + h_1 \notin \mathcal{I}_{q_1}$. Define

$$C_1 := \mathsf{C}_{\mathbf{M}_{q_1}}(h_1, N_1(q_1)).$$

We first assume that at least one of $q_1 \neq q$ and $C_1 \setminus \{g, h_1\} \neq \emptyset$ holds. Let $g_2 = (b_2, q_1)$ be an edge in $C_1 \setminus \{g, h_1\}$. Since $g_2 \neq g, h_1$ and $g, g_2 \in N_1$, we have $b_2 \neq b, b_1$. Define

$$N_3 := N_2 - g_2 + \ell(b_2).$$

Then, N_3 is a matching in G. Since Lemmas 15(ii) and 16(i) imply that

$$\begin{aligned}
&\mu_{N_3}(b) = g \succ_b \mu_M(b),\\
&\mu_{N_3}(b_1) = h_1 \succ_{b_1} g_1 = \mu_N(b_1) \succsim_{b_1} \mu_M(b_1),\\
&\mu_M(b_2) \succ_{b_2} \ell(b_2) = \mu_{N_3}(b_2),\\
&\forall a \in A \setminus \{b, b_1, b_2\} \colon \mu_{N_3}(a) = \mu_N(a) \succsim_a \mu_M(a),
\end{aligned}$$

we have $\phi(N_3; M) \geq 1$. This contradicts the fact that M is a popular matching in G.

Next we consider the case where $q_1 = q$ and $C_1 = \{g, h_1\}$. Define

$$N_4 := N_2 - g + \mu_M(b).$$

Assume that $\mu_M(b) = (b, q')$. Notice that $q' \neq q$. If $q' = r$, then

$$N_4(r) = N_2(r) + \mu_M(b) = N_1(r) + \mu_M(b) = N_0(r) + \mu_M(b) - g_1 = N(r) - g_1.$$

Since $g_1 \in C_0 = C$, $N(r) - g_1 \in \mathcal{I}_r$ and $N_4(r)$ is an independent set of $\mathbf{M}_r$. If $q' \neq r$, then

$$N_4(q') = N_2(q') + \mu_M(b) = N_1(q') + \mu_M(b) = N_0(q') + \mu_M(b) = N(q').$$

Lemma 14 implies that $N_4(q')$ is an independent set of $\mathbf{M}_r$. In both cases, N_4 is a matching in G. Furthermore, since Lemma 15(ii) implies that

$$\begin{aligned}
&\mu_{N_4}(b) = \mu_M(b),\\
&\mu_{N_4}(b_1) = h_1 \succ_{b_1} g_1 = \mu_N(b_1) \succsim_{b_1} \mu_M(b_1),\\
&\forall a \in A \setminus \{b, b_1\} \colon \mu_{N_4}(a) = \mu_N(a) \succsim_a \mu_M(a),
\end{aligned}$$

we have $\phi(N_4; M) \geq 1$. This contradicts the fact that M is a popular matching in G, and completes the proof.

3.2 Proof of Lemma 12

In this subsection, we give a proof of Lemma 12.

For proving this lemma by contradiction, we assume that there exists an applicant b in A such that $\mu_M(b) \notin \Pi$. Then, Lemma 11 implies that $e \succ_b \mu_M(b)$ for an edge e in $s(b)$. Let $g = (b, q)$ be an edge in $s(b)$. Define $N := M + g - \mu_M(b)$. Since $N(q) = M(q) + g$, if $M(q) + g \in \mathcal{I}_q$, then (I1) implies that N is a matching in G. Furthermore, $\phi(N; M) = 1$. This contradicts the fact that M is a popular matching in G. Thus, in the sequel, we can assume that $M(q) + g \notin \mathcal{I}_q$.

We first consider the case where

$$\mathsf{C}_{\mathbf{M}_q}(g, M(q)) - g \not\subseteq \Gamma.$$

Let $e_1 = (a_1, q)$ be an edge in

$$(\mathsf{C}_{\mathbf{M}_q}(g, M(q)) - g) \setminus \Gamma.$$

Since $g \neq e_1$ holds, we have $a_1 \neq b$. Let $e_2 = (a_1, q_1)$ be an edge in $f(a_1)$. Since $e_1 \notin \Gamma$ and $e_2 \in \Gamma$ (i.e., $e_1 \neq e_2$), we have $q_1 \neq q$. Define $N' := N + e_2 - e_1$. If $N(q_1) + e_2 \in \mathcal{I}_{q_1}$, then N' is a matching in G. Furthermore, we have $\phi(N'; M) = 2$, which contradicts the fact that M is a popular matching in G. Assume that $N(q_1) + e_2 \notin \mathcal{I}_{q_1}$. Let $e_3 = (a_2, q_1)$ be an edge in $\mathsf{C}_{\mathbf{M}_{q_1}}(e_2, N(q_1)) - e_2$. Since $q_1 \neq q$, we have $e_3 \neq g$. Define $N'' := N' - e_3 + \ell(a_2)$. Then, N'' is a matching in G and we have $\phi(N''; M) = 1$. This contradicts the fact that M is a popular matching in G.

Next we consider the case where

$$\mathsf{C}_{\mathbf{M}_q}(g, M(q)) - g \subseteq \Gamma. \tag{8}$$

Since g is in $s(a)$, Lemmas 1, 7, and 10 imply

$$(M \cap (\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})) + g$$

is an independent set of $\mathbf{P}$. This and the definition of $\mathbf{P}$ imply that

$$(M(q) \cap (\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})) + g \in \mathcal{I}_q.$$

Thus,

$$\mathsf{C}_{\mathbf{M}_q}(g, M(q)) - g \not\subseteq (M(q) \cap (\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})).$$

This implies that there exists an edge in $\mathsf{C}_{\mathbf{M}_q}(g, M(q)) \cap \Omega^-_{\mathbf{BQ}}$.

Since (8) implies that

$$\mathsf{C}_{\mathbf{M}_q}(g, M(q)) \subseteq (M(q) \cap \Gamma) + g,$$

we have $(M(q) \cap \Gamma) + g \notin \mathcal{I}_q$. Since (I1) and $M(q) \in \mathcal{I}_q$ imply that $M(q) \cap \Gamma \in \mathcal{I}_q$, this and Lemma 3 imply that

$$\mathsf{C}_{\mathbf{M}_q}(g, M(q) \cap \Gamma) = \mathsf{C}_{\mathbf{M}_q}(g, M(q)).$$

Thus,

$$\mathsf{C}_{\mathbf{M}_q}(g, M(q) \cap \Gamma) \cap \Omega^-_{\mathbf{BQ}} \neq \emptyset. \tag{9}$$

Define $\mathbf{C} := \mathbf{A}|(\Gamma + g)$ and $\mathbf{R} := \mathbf{P}|(\Gamma + g)$. It is not difficult to see that $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ is obtained from $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$ as follows.

Step 1. Add g to the vertex set of $\mathsf{D}_{\mathbf{BQ}}(M \cap \Gamma)$.

Step 2. Add an arc from g to every edge in $\mathsf{C}_{\mathbf{M}_q}(g, M(q) \cap \Gamma)$.

Furthermore, since $\mu_M(b) \notin f(b)$, we have

$$\mathsf{T}_{\mathbf{C}}(M \cap \Gamma) = \mathsf{T}_{\mathbf{B}}(M \cap \Gamma) + g,$$
$$\mathsf{T}_{\mathbf{R}}(M \cap \Gamma) = \mathsf{T}_{\mathbf{Q}}(M \cap \Gamma).$$

It follows from (9) that there exists an edge h in $\mathsf{T}_{\mathbf{R}}(M \cap \Gamma)$ such that there a directed path in $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ from g to h. Let L be a shortest directed path in $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ from g to h. We denote by $K = \{e_1, e_2, \ldots, e_k\}$ the set of edges in $\Gamma + g$ that L goes through. Assume that $e_i := (a_i, p_i)$ for each $i = 1, 2, \ldots, k$, and L goes through $e_1, e_2, \ldots, e_k$ in this order. Notice that $e_1 = g$, $e_k = h$, k is odd, and $e_i \in M$ for every $i = 2, 4, \ldots, k-1$. Define $N := M \triangle K$ and $N_0 := N - \mu_M(b)$. Lemma 6 implies that $N \cap (\Gamma + g)$ $(= N_0 \cap (\Gamma + g))$ is an independent set of $\mathbf{R}$.

For completing a proof of Lemma 12, we first necessary lemmas (Section 3.2.1), and complete a proof of Lemma 12 (Section 3.2.2).

3.2.1 Necessary lemmas

Here we give necessary lemmas for completing the proof of Lemma 12.

Lemma 18. *For every post p in $P - r$, we have $N(p) \in \mathcal{I}_p$ and $N_0(p) \in \mathcal{I}_p$.*

Proof. Let p be a post in $P - r$. By using Lemma 4, we prove that $N(p) \in \mathcal{I}_p$. If we can prove this, then (I1) implies that $N_0(p) \in \mathcal{I}_p$.

Define $I := M(p) \cap \Gamma$, $J := M(p) \setminus \Gamma$, $X := K(p) \setminus M$, and $Y := M(p) \cap K$. Then,

$$(I \cup X) \setminus Y = N(p) \cap (\Gamma + g),$$
$$((I \cup X) \setminus Y) \cup J = N(p).$$

Since M is an independent set of $\mathbf{P}$, the definition of $\mathbf{P}$ implies that $I \cup J$ is in $\mathcal{I}_p$. In addition, the definition of $\mathbf{R}$ implies that implies that $(I \cup X) \setminus Y$ is in $\mathcal{I}_p$. For every edge e in X, the definition of $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ implies that $(M \cap \Gamma) + e$ is not an independent set of $\mathbf{R}$. Thus, since $M \cap \Gamma$ is an independent set of $\mathbf{R}$, this and the definition of $\mathbf{R}$ imply that $I + e$ is not an independent set of $\mathbf{M}_p$. Thus, Lemma 4 implies that $N(p) \in \mathcal{I}_p$. $\square$

Lemma 19. *For every applicant a in $A - b$,* (i) $|N(a)| = 1$, *and* (ii) $\mu_N(a) \succsim_a \mu_M(a)$.

Proof. For every $i = 2, 4, \ldots, k-1$, since there exists an arc in $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ from e_i to e_{i+1}, $M + e_{i+1} - e_i$ is an independent set of $\mathbf{C}$. Thus, the definitions of $\mathbf{A}$ implies that $a_i = a_{i+1}$ for every $i = 2, 4, \ldots, k-1$. This implies that for every applicant a in $A - b$, since $|M(a)| = 1$, we have $|N(a)| = 1$. Furthermore, since $K - g \subseteq \Gamma$, $\mu_N(a) \in f(a)$ for every applicant a in $A - b$, which implies (ii). This completes the proof. □

Lemma 20. (i) $g \succ_b \mu_M(b)$, *and* (ii) $|N_0(b)| = 1$.

Proof. Since $g \in s(b)$ and $e \succ_b \mu_M(b)$ for an edge e in $s(b)$, we have $g \succ_b \mu_M(b)$. Furthermore, as proved in Lemma 19, we have $a_i = a_{i+1}$ for every $i = 1, 3, \ldots, k-1$. Thus, since $\mu_M(b) \notin \Gamma$, we have $K(b) = \{g\}$. This and $M(b) = \{\mu_M(b)\}$ imply (ii). □

We denote by L' the subpath of L from g to e_{k-1}. Since L is a shortest directed path in $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ from g to h, L' is a shortest directed path in $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ from g to e_{k-1}. Define $K' := K - h$. Then, $N - h = M \triangle K'$. Lemma 6 implies that $(N - h) \cap (\Gamma + g)$ is an independent set of $\mathbf{R}$.

Lemma 21. $N(r) - h \in \mathcal{I}_r$ *and* $N_0(r) - h \in \mathcal{I}_r$.

Proof. By using Lemma 4, we prove that $N(r) - h \in \mathcal{I}_r$. If we can prove this, then (I1) implies that $N_0(r) - h \in \mathcal{I}_r$.

Define $I := M(r) \cap \Gamma$, $J := M(r) \setminus \Gamma$, $X := K'(r) \setminus M$, and $Y := M(r) \cap K'$. Then,

$$(I \cup X) \setminus Y = (N(r) - h) \cap (\Gamma + g),$$
$$((I \cup X) \setminus Y) \cup J = N(r) - h.$$

Since M is an independent set of $\mathbf{P}$, the definition of $\mathbf{P}$ implies that $I \cup J$ is in $\mathcal{I}_r$. In addition, the definition of $\mathbf{R}$ implies that $(I \cup X) \setminus Y$ is in $\mathcal{I}_r$. For every edge e in X, the definition of $\mathsf{D}_{\mathbf{CR}}(M \cap \Gamma)$ implies that $(M \cap \Gamma) + e$ is not an independent set of $\mathbf{R}$. Thus, since $M \cap \Gamma$ is an independent set of $\mathbf{R}$, this and the definition of $\mathbf{R}$ imply that $I + e$ is not an independent set of $\mathbf{M}_r$. Thus, Lemma 4 implies that $N(r) - h \in \mathcal{I}_r$. □

3.2.2 Completing a proof

We are now ready to complete a proof of Lemma 12. If $N_0(r) \in \mathcal{I}_r$, then it follows from Lemmas 18, 19(i), and 20(ii) that N_0 is a matching in G. Furthermore, Lemma 19(ii) and 20(i) imply that $\phi(N_0; M) \geq 1$, which contradicts the fact that M is a popular matching in G. Thus, in the sequel, we can assume that $N_0(r) \notin \mathcal{I}_r$. In this case, since $N_0(r) \subseteq N(r)$, $N(r)$ is not in $\mathcal{I}_r$. Lemma 21 implies that $\mathsf{C}_{\mathbf{M}_r}(h, N(r) - h)$ and $\mathsf{C}_{\mathbf{M}_r}(h, N_0(r) - h)$ are well-defined. Define

$$C := \mathsf{C}_{\mathbf{M}_r}(h, N(r) - h),$$
$$C_0 := \mathsf{C}_{\mathbf{M}_r}(h, N_0(r) - h).$$

Lemma 3 implies that $C = C_0$.

Since $N_0 \cap (\Gamma + g)$ is an independent set of $\mathbf{R}$, the definition of $\mathbf{R}$ implies that $N_0(r) \cap (\Gamma + g)$ is in $\mathcal{I}_r$. Thus, if $C_0 \subseteq \Gamma + g$, then $C_0 \subseteq N_0(r)$ implies that $C_0 \subseteq N_0(r) \cap (\Gamma + g)$.

This contradicts the fact that $N_0(r) \cap (\Gamma + g) \in \mathcal{I}_r$. Thus, $C_0 \not\subseteq \Gamma + g$. Let $g_1 = (b_1, r)$ be an edge in $C_0 \setminus (\Gamma + g)$. Notice that since $g \neq g_1$ and $g, g_1 \in N_0$, we have $b \neq b_1$. Define $N_1 := N_0 - g_1$. Then, $g_1 \in C_0$ and Lemmas 18, 19(i), and 20(ii) imply that

- $|N_1(a)| = 1$ for every applicant a in $A - b_1$, and $N_1(b_1) = \emptyset$,
- $N_1(p) \in \mathcal{I}_p$ for every post p in P.

Let $h_1 = (b_1, q_1)$ be an edge in $f(b_1)$. We first consider the case where $N_1(q_1) + h_1 \in \mathcal{I}_{q_1}$. Define $N_2 := N_1 + h_1$. In this case, N_2 is a matching in G. Furthermore, since it follows from $g_1 \notin \Gamma$, Lemmas 19(ii) and 20(i) that

$$
\begin{aligned}
&\mu_{N_2}(b) = g \succ_b \mu_M(b),\\
&\mu_{N_2}(b_1) = h_1 \succ_{b_1} g_1 = \mu_N(b_1) \succsim_{b_1} \mu_M(b_1),\\
&\forall a \in A \setminus \{b, b_1\}\colon \mu_{N_2}(a) = \mu_N(a) \succsim_a \mu_M(a),
\end{aligned}
$$

we have $\phi(N_2; M) \geq 2$. These contradict the fact that M is a popular matching in G.

Next we consider the case where $N_1(q_1) + h_1 \notin \mathcal{I}_{q_1}$. Define

$$C_1 := \mathsf{C}_{\mathbf{M}_{q_1}}(h_1, N_1(q_1)).$$

We first assume that at least one of $q_1 \neq q$ and $C_1 \setminus \{g, h_1\} \neq \emptyset$ holds. Let $g_2 = (b_2, q_1)$ be an edge in $C_1 \setminus \{g, h_1\}$. Since $g_2 \neq g, h_1$ and $g, g_2 \in N_1$, we have $b_2 \neq b, b_1$. Define

$$N_3 := N_2 - g_2 + \ell(b_2).$$

Then, N_3 is a matching in G. Since Lemmas 19(ii) and 20(i) imply that

$$
\begin{aligned}
&\mu_{N_3}(b) = g \succ_b \mu_M(b),\\
&\mu_{N_3}(b_1) = h_1 \succ_{b_1} g_1 = \mu_N(b_1) \succsim_{b_1} \mu_M(b_1),\\
&\mu_M(b_2) \succ_{b_2} \ell(b_2) = \mu_{N_3}(b_2),\\
&\forall a \in A \setminus \{b, b_1, b_2\}\colon \mu_{N_3}(a) = \mu_N(a) \succsim_a \mu_M(a),
\end{aligned}
$$

we have $\phi(N_3; M) \geq 1$. This contradicts the fact that M is a popular matching in G.

Next we consider the case where $q_1 = q$ and $C_1 = \{g, h_1\}$. Define

$$N_4 := N_2 - g + \mu_M(b).$$

Assume that $\mu_M(b) = (b, q')$. Notice that $q' \neq q$. If $q' = r$, then

$$N_4(r) = N_2(r) + \mu_M(b) = N_1(r) + \mu_M(b) = N_0(r) + \mu_M(b) - g_1 = N(r) - g_1.$$

Since $g_1 \in C_0 = C$, $N(r) - g_1 \in \mathcal{I}_r$ and $N_4(r)$ is an independent set of $\mathbf{M}_r$. If $q' \neq r$, then

$$N_4(q') = N_2(q') + \mu_M(b) = N_1(q') + \mu_M(b) = N_0(q') + \mu_M(b) = N(q').$$

Lemma 18 implies that $N_4(q')$ is an independent set of $\mathbf{M}_r$. In both cases, N_4 is a matching in G. Furthermore, since Lemma 19(ii) implies that

$$
\begin{aligned}
&\mu_{N_4}(b) = \mu_M(b),\\
&\mu_{N_4}(b_1) = h_1 \succ_{b_1} g_1 = \mu_N(b_1) \succsim_{b_1} \mu_M(b_1),\\
&\forall a \in A \setminus \{b, b_1\}\colon \mu_{N_4}(a) = \mu_N(a) \succsim_a \mu_M(a),
\end{aligned}
$$

we have $\phi(N_4; M) \geq 1$, which contradicts the fact that M is a popular matching in G. This completes the proof.

4 Algorithm

In this section, we propose our algorithm for PMTM. Define a function $\xi\colon \Pi \to \{0,1\}$ by

$$\xi(e) := \begin{cases} 1 & \text{if } e \in \Gamma \\ 0 & \text{if } e \in \Pi \setminus \Gamma. \end{cases}$$

Our algorithm `PMTM` is described as follows.

Algorithm `PMTM`

Step 1. Compute Π, and define $\mathbf{A}' := \mathbf{A}|\Pi$ and $\mathbf{P}' := \mathbf{P}|\Pi$.

Step 2. Find a maximum-size common independent set M of $\mathbf{A}'$ and $\mathbf{P}'$ such that

$$\xi(M) = \max\{\xi(N) \mid N \in \mathcal{B}_{\mathbf{A}'\mathbf{P}'}\}.$$

Step 3. If $\xi(M) = \gamma_{\mathbf{BQ}}$ and $|M| = |A|$ hold, then output M and halt (in this case, M is a popular matching in G). Otherwise, output **null** and halt (in this case, there exists no popular matching in G).

End of Algorithm

For proving the correctness of the algorithm `PMTM`, we need the following lemma.

Lemma 22. *Assume that we are given a popular matching M in G. Then, M is a common independent set of $\mathbf{A}'$ and $\mathbf{P}'$ such that $\xi(M) = \gamma_{\mathbf{BQ}}$ and $|M| = |A|$.*

Proof. Since M is a matching in G, M is a common independent set of $\mathbf{A}$ and $\mathbf{P}$ such that $|M| = |A|$. In addition, (P2) of Theorem 9 implies that M is a subset of Π. Thus, M is a common independent set of $\mathbf{A}'$ and $\mathbf{P}'$. Since (P1) of Theorem 9 implies that $M \cap \Gamma$ is a maximum-size common independent set of $\mathbf{B}$ and $\mathbf{Q}$, we have $|M \cap \Gamma| = \gamma_{\mathbf{BQ}}$, i.e., $\xi(M) = \gamma_{\mathbf{BQ}}$. This completes the proof. □

We are now ready to prove the correctness of the algorithm `PMTM`.

Theorem 23. *The algorithm `PMTM` can correctly solve PMTM.*

Proof. Let M be a common independent set of $\mathbf{A}'$ and $\mathbf{P}'$ that is found in **Step 2** of the algorithm `PMTM`. If the algorithm `PMTM` outputs M, then M is a matching and Theorem 9 implies that M is a popular matching in G.

Assume that the algorithm `PMTM` outputs **null**. Since M is a common independent set of $\mathbf{A}$ and $\mathbf{P}$, (I1) implies that $M \cap \Gamma$ is a common independent set of $\mathbf{B}$ and $\mathbf{Q}$. Thus, $|M \cap \Gamma| \le \gamma_{\mathbf{BQ}}$, which implies that $\xi(M) \le \gamma_{\mathbf{BQ}}$. Furthermore, since $|M(a)| \le 1$ for every applicant a in A, we have $|M| \le |A|$. Thus, $\xi(M) < \gamma_{\mathbf{BQ}}$ and/or $|M| < |A|$. We prove that in this case, there exists no popular matching in G by contradiction. Assume that there exists a popular matching N in G. Then, Lemma 22 implies that N is a common independent set of $\mathbf{A}'$ and $\mathbf{P}'$ such that $\xi(N) = \gamma_{\mathbf{BQ}}$ and $|N| = |A|$. If $|M| < |A|$, then the existence of N contradicts the fact that M is a maximum-size common independent set of $\mathbf{A}'$ and $\mathbf{P}'$. If $\xi(M) < \gamma_{\mathbf{BQ}}$ and $|M| = |A|$, then the existence of N contradicts the fact that

$$\xi(M) = \max\{\xi(N') \mid N' \in \mathcal{B}_{\mathbf{A}'\mathbf{P}'}\}.$$

Thus, there exists no popular matching in G. This completes the proof. $\square$

Here we consider the time complexity of the algorithm PMTM. We denote by EO the time required to decide whether $I + e \in \mathcal{I}_p$ for every post p in P, every independent set I of $\mathbf{M}_p$, and every edge e in $E(p) \setminus I$. Define $m := |E|$. For simplicity, we assume that $E(p) \neq \emptyset$ for every post p in P and $\mathsf{EO} = \Omega(m)$. Furthermore, we assume that for every applicant a in A and every edges e, g in $E(a)$, we can decide in $O(1)$ time whether $e \succsim_a g$ holds.

We first consider the time complexity of **Step 1**. It is not difficult to see that we can compute $f(a)$ for all applicants a in $O(m)$ time. For computing $s(a)$ for all applicants a in A, we first compute $\Omega^+_{\mathbf{BQ}}$, $\Omega_{\mathbf{BQ}}$, and $\Omega^-_{\mathbf{BQ}}$ in $O(m^{2.5}\mathsf{EO})$ time by finding a maximum-size common independent set of $\mathbf{B}$ and $\mathbf{Q}$ via the algorithm of Cunningham [2]. Then, we find a base B of $\mathbf{P}|(\Omega^+_{\mathbf{BQ}} \cup \Omega_{\mathbf{BQ}})$ in $O(m\mathsf{EO})$ time. Lemma 1 implies that by using the base B, we can compute $s(a)$ for all applicants a in A in $O(m\mathsf{EO})$ time. Thus, we can compute Π in $O(m^{2.5}\mathsf{EO})$ time.

Next we consider the time complexity of **Step 2**. It is not difficult to see that we can decide in $O(m)$ time whether $M + e$ is an independent set of $\mathbf{A}'$ for every independent set M of $\mathbf{A}'$ and every edge e in $\Pi \setminus M$. Furthermore, it is not difficult to see that we can decide in $O(\mathsf{EO})$ time whether $M + e$ is an independent set of $\mathbf{P}'$ for every independent set M of $\mathbf{P}'$ and every edge e in $\Pi \setminus M$. Thus, in **Step 2**, we can find a desired maximum-size common independent set of $\mathbf{A}'$ and $\mathbf{P}'$ in $O(m^3\mathsf{EO})$ time by using the algorithm of Frank [7]. Thus, the time complexity of the algorithm PMTM is $O(m^3\mathsf{EO})$.

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