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One-dimensional Discrete-time Binary Cellular Neural Networks and Some Examples for Signal Processing

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Abstract: This paper studies on the behavior of one-dimensional discrete-time binary cellular neural networks(abbreviated as a 1-D DBCNN). In general cellular neural networks(CNN) are characterized by the A- and the B-templates and the threshold value θ.Our final aim is to exhaustively examine the behavior of the 1-D DBCNN with arbitrary A-template, B-template and θ from the viewpoint of signal processing. In this paper we will give some partial solutions to this problem. We first show necessary and sufficient conditions for the system to be stable, and then give several numerical examples to illustrate the above results.

Keywords: 1-D CNN, Stability condition, Signal processing

1. Introduction

This paper studies on the behavior of one-dimensional discrete-time binary cellular neural networks (abbreviated as a 1-D DBCNN), which will be defined later. As is well-known, cellular neural networks(CNN) are characterized by the A- and the B-templates and the threshold value θ[1),2)]. Two of most important problems on the CNN are 1) how to find an appropriate template for the prescribed signal processing, and 2) what kinds of signal processing can be performed by CNNs. Roska [5)] published the Software Library collecting many useful templates so far known for image processing. So far most of these templates have been found heuristically or intuitively.

Recently young researcher contest of finding useful A- and B-templates and θ for image processing was proposed, but due to many parameters contained in these 2-D templates, we have not yet understood the entire abilities of CNNs[3)−5)] for signal processing.

Main previous results concerning one-dimensional CNN are as follows: Wolfram[6)] investigated on one-dimensional cellular automata being an extension of the DBCNN from the point of complexity and classified the cellular automata with the periodic boundary. Jen[7)] found the number of limit cycles for a class of one-dimensional cellular automata with the periodic boundary. Shinkai[8)] proved that 1-D DBCNNs with the fixed boundary does not possess a limit cycle with length more than five. Thiran et al[9)] discussed the stability of analog one-dimensional CNNs and determined the number of stable solutions. Sato et al[11)] gave the stability conditions for a kind of DBCNNs. The results of this paper is an extension of our previous paper [11)] together with some new examples which may suggest new signal processing.

Our final aim is to exhaustively examine the behavior of 1-D DBCNNs with arbitrary A-template, B-template and θ, that is, to clarify the relation between the input and the output data, and also to show the whole ability of the DBCNNs. Proir to the above problem, we have to clarify the conditions for 1-D DBCNNs to work well, i.e., stability conditions.

We will give in this paper some partial solution to this problem. First we show that 1-D DBCNNs are useful only for a few kinds of signal processings if we do not use the B-template, in other words, if there is no input. We will next show several necessary and sufficient conditions for the system to be stable and give several numerical examples to illustrate the above results. The examples show that nonautomonous DBCNNs can do more than autonomous ones.

2. Preliminaries

2.1 1-D DBCNN

The behavior of a 1-D DBCNN denoted by S can be described by the equation:

$$x(k+1) = \mathrm{sgn}(Ax(k) + Bu + \theta\mathbf{1}) \tag{1}$$

where $x(k) = [x_1(k), x_2(k), \cdots, x_n(k)]^T$ and $u = [u_1, u_2, \cdots, u_n]^T$ are a binary state vector at discrete

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time k and a binary time-invariant input vector respectively, n is the dimension of S, A and B are $n \times n$ matrices determined by the A- and B-templates, θ is the threshold value, and $\mathbf{1}$ is the n-dimensional column vector consisting of 1 only. In particular $x(0)$ is an initial state vector, which can be regarded as another input data.

We assume that the 1-D DBCNN(or simply D-BCNN) has a neighborhood of radius one, so the A-template can be characterized by three parameters denoted by α, β, and γ. Similarly the B-template by $\hat{\alpha}, \hat{\beta}$, and $\hat{\gamma}$. Then Eq.(1) can be rewritten in a scalar form as:

$$x_i(k+1) = \mathrm{sgn}\,[\beta x_{i-1}(k) + \alpha x_i(k) + \gamma x_{i+1}(k) + \hat{\beta} u_{i-1} + \hat{\alpha} u_i + \hat{\gamma} u_{i+1} + \theta], \quad (i = 1, 2, \cdots, n;\ k = 0, 1, 2, \cdots) \quad (2)$$

Since S can be completely characterized by seven parameters, α, β, γ, $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and θ, we can represent S as $S = S(\alpha, \beta, \gamma, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \theta)$. When we calculate $x_i(k+1)$ by Eq.(2), we have to prescribe the boundary values $x_0(k)$ and $x_{n+1}(k)$ for the state vector x and u_0 and u_{n+1} for the input vector u, respectively.

In this paper, we will exclusively discuss DBCNN with the fixed boundary, i.e., $x_0(k)$ and $x_{n+1}(k)$ are fixed values independent of k, and u_0 and u_{n+1} are also fixed constants.

For an initial binary state vector $x(0) = [x_1(0), x_2(0), \cdots, x_n(0)]$ and a binary input vector u, the state vector $x(k)$ varies along with the time k. Sometimes it converges to a vector x^* depending on both $x(0)$ and u (so strictly speaking x^* should be written as $x^*(x(0), u)$. But we simply write it as x^* hereafter), and in other cases $x(k)$ varies periodically, i.e., we have a limit cycle.

Since $x(0)$ and u are designated as arbitrary binary vector, we can regard $x(0)$, u, or both $x(0)$ and u as input data. But in this paper we regard either $x(0)$ or u as input data.

Definition 1: A 1-D DBCNN S is said to be stable, if no limit cycle occur for any $x(0)$, any u, any fixed boundary values and any value of the dimension n. A DBCNN being not stable is said to be unstable.

2.2 Previous Results

In this section we first consider the case where

$$u = 0, \quad (3)$$

which is equivalent to the case where

$$\hat{\alpha} = \hat{\beta} = \hat{\gamma} = 0. \quad (4)$$

in Eq.(2). Then Eq.(2) can be written as:

$$x_i(k+1) = \mathrm{sgn}\,[\beta x_{i-1}(k) + \alpha x_i(k) + \gamma x_{i+1}(k) + \theta], \quad (i = 1, 2, \cdots, n) \quad (5)$$

In this case the DBCNN is described as $S = S(\alpha, \beta, \gamma, 0, 0, 0, \theta)$ and is said to be autonomous, while if Eqs.(3) and (4) are not satisfied, then S is said to be nonautonomous.

Though the values α, β and γ take any real values, we do not need to consider infinitely many cases so far as we concern only with the ability for signal processing. The following theorems are known from previous literatures (e.g., see [6]):

Theorem 1 Eq.(5) shows only 104 different kinds of behaviors for autonomous DBCNNs.

Theorem 2 Most of 104 kinds of the behaviors of autonomous DBCNNs are rather trivial and does not seem to be useful for signal processing. For example, we reach only a few trivial states such as $x^* = [1, 1, \cdots, 1]^T$ for 2^n different initial states $x(0)$.

Exceptional nontrivial signal processing by autonomous DBCNNs with the prescribed initial state $x(0)$ as follows:

1) noise elimination
2) edge detection
3) isolated pulse detection
4) edge enhancement
5) connected component detection

3. Main Results

3.1 Stability Condition for Nonautonomous 1-D DBCNNs

In this section we assume that $u \neq 0$. Let

$$\Theta \equiv [\theta_1, \theta_2, \cdots, \theta_n]^T = Bu + \theta\mathbf{1} \quad (6)$$

Here Θ represents the information on the input u and the B-template. Since θ_i does not vary along with time, we can regard θ_i as input data instead of u_i. Of course we need to investigate the relation between u and Θ later.

A non-autonomous DBCNN can be described as:

$$x_i(k+1) = \operatorname{sgn}[\beta x_{i-1}(k) + \alpha x_i(k) + \gamma x_{i+1}(k) + \theta_i], \quad (i = 1, 2, \cdots, n;\ k = 0, 1, 2, \cdots) \tag{7}$$

where

$$\theta_i = \hat{\beta} u_{i-1} + \hat{\alpha} u_i + \hat{\gamma} u_{i+1} + \theta \quad (i = 1, 2, \cdots, n). \tag{8}$$

The difference between Eqs.(2) and (7) lies that in the former case θ is constant for all i, but in the latter θ_i varies with i. This difference makes the problem difficult but yields the possibility of variety of signal processing.

Since $\theta_i = \hat{\beta} u_{i-1} + \hat{\alpha} u_i + \hat{\gamma} u_{i+1} + \theta = \pm\hat{\beta} \pm \hat{\alpha} \pm \hat{\gamma} + \theta$, θ_i takes one of eight values, which are denoted by $\theta_{0j} (j = 1, 2, \cdots, 8)$.

Our purpose can be attained by examining sequentially the following three relations: 1) Relation between Θ and x^* from Eq.(5), 2) Relation between Θ and u from Eq.(8), and 3) Relation between u and x^*. In this paper we mainly study on the relation between Θ and x^*.

3.2 Stability Conditions for $\beta = \gamma = 0$

First we consider the simplest case where

$$\beta = \gamma = 0 \tag{9}$$

In this case we have from Eq. (7)

$$x_i(k+1) = \operatorname{sgn}[\alpha x_i(k) + \theta_i], \quad (i = 1, \cdots, n) \tag{10}$$

This means that each $x_i(k)$ behaves independently of other variables $x_j(k) (j \neq i)$.

Theorem 3 Eq.(10) converges to a fixed vector if and only if $\alpha > -|\theta_i|$ and the convergent vector x_i^* is given as $x_i^* = \operatorname{sgn} \theta_i$.

Proof) First assume that

$$\alpha < -|\theta_i| \tag{11}$$

for some i. Then due to (11), the sign of the quantity in the parenthesis in Eq.(11) is opposite to that of $x_i(k)$ irrespective to the sign of $x_i(k)$. That is, $x_i(k+1) = -x_i(k)$. This means that the system does not have any equilibrium point.

Conversely, if Eq. (11) does not hold, then we easily see that it has a fixed point.

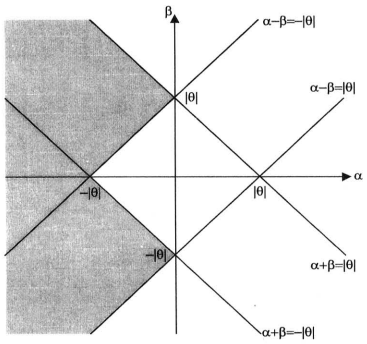

Fig.1 Unstable region for one $\theta = \theta_i$

3.3 Stability Conditions for $\gamma = 0$

Next we consider the case where

$$\gamma = 0 \tag{12}$$

in Eq. (7). Then we have from Eq. (7)

$$x_i(k+1) = \operatorname{sgn}[\beta x_{i-1}(k) + \alpha x_i(k) + \theta_i], \quad (i = 1, 2, \cdots, n) \tag{13}$$

From Eq.(13) we see that $x_i(k)$ depends only on $x_j(k)$ $(j \leq i)$. Since in this paper we regard only one of $x(0)$ and u as input data, we can assume that:

$$x(0) = [x_1(0), x_2(0), \cdots, x_n(0)]^T = \mathbf{0}, \tag{14}$$

$$x_0(k) = x_{n+1}(k) = 0 \quad (k = 0, 1, \cdots) \tag{15}$$

From Eqs.(13)-(15) we easily have

$$x_i(1) = \operatorname{sgn} \theta_i \quad (i = 1, 2, \cdots, n) \tag{16}$$

Theorem 4 In the fixed boundary case $x_i(k)$ converges to a point (depending on u) if and only if the point (α, β) does not belong to the shaded region shown in **Fig. 1**, where θ should be read as θ_i.

Proof of this theorem is omitted because of lack of space.

Since the shaded region in **Fig. 1** corresponds to unstable region for one particular θ_i, by overlapping these figures for θ_{01} through θ_{08}, we obtain the entire unstable region shown in **Fig. 2**.

Theorem 5 In the fixed boundary case $x(k)$ in Eq.(13) converges to a point (depending on u) if and only if (α, β) does not belong to the shaded region shown in **Fig. 2**.

In **Figs. 1** and **2** the straight lines correspond to the equation:

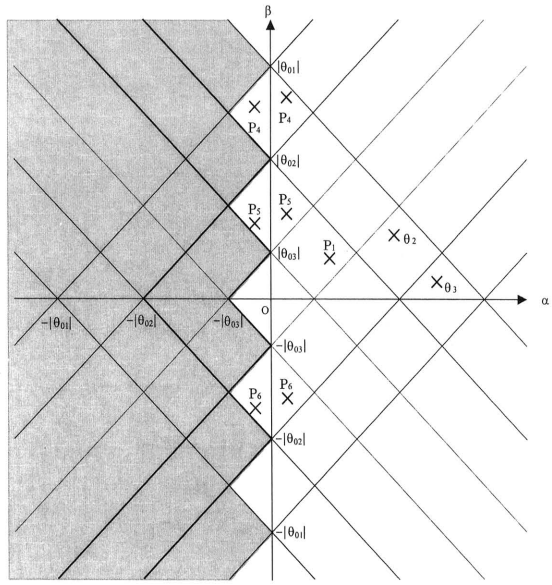

Fig.2 Whole unstable region for several θ's

$$\alpha \pm \beta = \pm\theta_i. \tag{17}$$

Even if we change the values α and β within the rectangular surrounded by four adjacent lines (see **Fig. 2**), the behavior of the system does not change. For example, P_4 and P_4' (P_5 and P_5', P_6 and P_6') have the same behavior, but P_1 and P_2 have different behavior in general.

We easily see from **Fig. 2** that we can change the value of α into the positive value without changing the behavior of the system. That is,

Theorem 6 When we examine the whole behavior of the stable system (13), we can assume without loss of generality that $\alpha > 0$.

Then, due to Theorem 6, the β-θ space can be partitioned into four regions, $R_1 \sim R_4$, as shown in **Fig. 3**.

$$R_1 : \beta - \alpha < \theta_i, \quad 0 < \theta_i, \quad -\beta - \alpha < \theta_i, \tag{18}$$
$$R_2 : \theta_i < -\beta + \alpha, \quad \theta_i < 0, \quad \theta_i < \beta + \alpha, \tag{19}$$
$$R_3 : \beta > 0, \quad |\theta| < \beta - \alpha, \tag{20}$$
$$R_4 : \beta < 0, \quad |\theta| < -\beta - \alpha, \tag{21}$$

If $(\beta, \theta_i) \in R_1$, then we have

$$\begin{aligned} x_i(k+1) &= \text{sgn}\,[\pm\beta + \alpha x_i(k) + \theta_i] \\ &= 1 \text{ or } x_i(k). \end{aligned} \tag{22}$$

Similarly, if $(\beta, \theta_i) \in R_2$, then

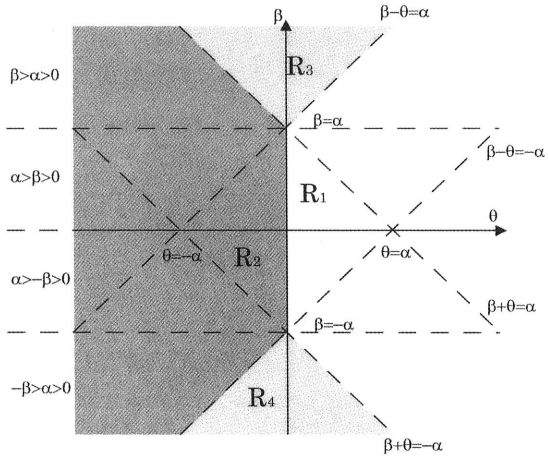

Fig.3 θ-β space for $\alpha > 0$

$$x_i(k+1) = -1 \text{ or } x_i(k). \tag{23}$$

From Eqs. (18),(19),(22), and (23) we see that, if $(\beta, \theta_i) \in R_1 \cup R_2$, then

$$x_i(k+1) = \text{sgn}\,\theta_i. \tag{24}$$

Similarly, if $(\beta, \theta_i) \in R_3$, then

$$x_i(k+1) = x_{i-1}(k), \tag{25}$$

and if $(\beta, \theta_i) \in R_4$, then

$$x_i(k+1) = -x_{i-1}(k). \tag{26}$$

Repeatedly applying the operations (24)-(26), we obtain the following theorems on the convergent vector x^*.

Theorem 7 If $\alpha > |\beta|$ in a 1-D DBCNN $S(\alpha, \beta, 0, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \theta)$ with Eqs.(13) and (14),

$$x_i^* = x_i(2) = x_i(1) = \text{sgn}\,\theta_i. \tag{27}$$

Form this theorem we easily see that the case $0 < \beta < \alpha$ is useless from signal processing viewpoint.

Theorem 8 If $\beta > \alpha > 0$ in a 1-D DBCNN $S(\alpha, \beta, 0, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \theta)$ with Eqs.(13) and (14),

$$x_i^* = \begin{cases} \text{sgn}\,\theta_i & \text{for } i \notin J \\ \text{sgn}\,\theta_j & \text{for } i \in J \end{cases} \tag{28}$$

where J is a set of integer j such that $(\beta, \theta_j) \in R_3$ and θ_j is the nearest leftmost element not belonging

to J.

Theorem 9 If $-\beta > \alpha > 0$ in a 1-D DBCNN $S(\alpha, \beta, 0, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \theta)$ with Eqs.(13) and (14),

$$x_i^* = \begin{cases} \operatorname{sgn} \theta_i & \text{for } i \notin J \\ (-1)^{i-j} \operatorname{sgn} \theta_j & \text{for } i \in J \end{cases} \tag{29}$$

where J is a set of integer j such that $(\beta, \theta_j) \in R_4$ and θ_j is the nearest leftmost element not belonging to J.

3.4 Number of Possible Distinct Signal Processing

Though the system $S(\alpha, \beta, 0, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \theta)$ has 7 real parameters, according to the previous theorems (Theorems 7-9) the number of its behavior (i.e., the relation between x^* and u) is finite. That is, S can be perform only a finite number of signal processing. We will count the number.

As described in Section 3.1, we have to investigate two relations; the relation between $\{\theta_i\}$ and u and those between x^* and $\{\theta_i\}$. The former relation can be classified by the ways of separating eight vertices $(u_{i-1}, u_i, u_{i+1}) = \{(\pm 1, \pm 1, \pm 1)\}$ of the cubic in the u space by the hyperplane:

$$\hat{\beta} u_{i-1} + \hat{\alpha} u_i + \hat{\gamma} u_{i+1} + \theta = 0 \tag{30}$$

Then we can find by carefully enumerating that there is 104 kinds of separations.

We will next consider the relation between x^* and $\{\theta_i\}$.

The case where Theorem 7 can be applied is simple. In this case x^* can be determined only by θ_i independent of the value of α and β and we have at most 104 distinct signal processing.

The cases where Theorems 8 and 9 can be applied are more complicated. In these cases the behavior of S is determined by the relation among the values of α, β, and θ_i. More concretely, in the case of Theorem 8, i.e., in case of $\beta > \alpha$, we have to consider the ways of distribution of θ_i in the following inequality.

$$\begin{aligned} &R_1 : +\beta - \alpha < \theta_i \\ &R_3 : -\beta + \alpha < \theta_i < \beta - \alpha \\ &R_2 : \theta_i < -\beta + \alpha \end{aligned} \tag{31}$$

The number of distinct behavior is equal to the number of ways of separating eight vertices $(u_{i-1}, u_i, u_{i+1}) = \{(\pm 1, \pm 1, \pm 1)\}$ of the cubic in the u space by following two parallel planes.

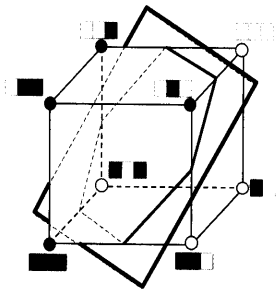

Fig.4 Relation between (u_{i-1}, u_i, u_{i+1}) and θ_i in case of $\alpha > |\beta|$. Three squares in the each top show input (u_{i-1}, u_i, u_{i+1}). The monochrome tone of circle shows whether θ_i belongs to region R_1 or R_2.

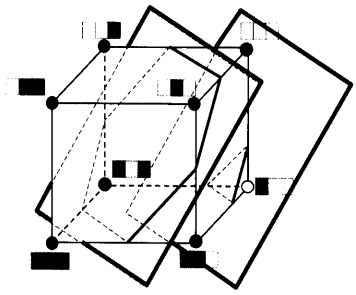

Fig.5 Relation between (u_{i-1}, u_i, u_{i+1}) and θ_i in case of $\beta > \alpha$.

$$\left.\begin{aligned} \hat{\beta} u_{i-1} + \hat{\alpha} u_i + \hat{\gamma} u_{i+1} + \theta = +\beta - \alpha \\ \hat{\beta} u_{i-1} + \hat{\alpha} u_i + \hat{\gamma} u_{i+1} + \theta = -\beta + \alpha \end{aligned}\right\} \tag{32}$$

By carefully enumerating the above, we conclude that there are 993 kinds of separations, i.e., there are distinct 993 kinds of signal processing, though most of them corresponds to rather trivial signal processing.

For the case of Theorem 9 we have the same number (i.e., 993 cases).

4. Examples for Theorems 7-9

We will give several examples corresponding to Theorems 7-9.

Figs. 6(a)-(c) are examples for Theorem 7. In the case of **Fig. 6(a)** we have only one trivial stable solution, all cells having the value -1(black).

In the case of **Fig. 6(b)** the output (x^*) vector consists only of black (-1) cells except for a right edge cell of each black(-1) block of length greater than 1 of the input data.

In the case of **Fig. 6(c)** the output (x^*) con-

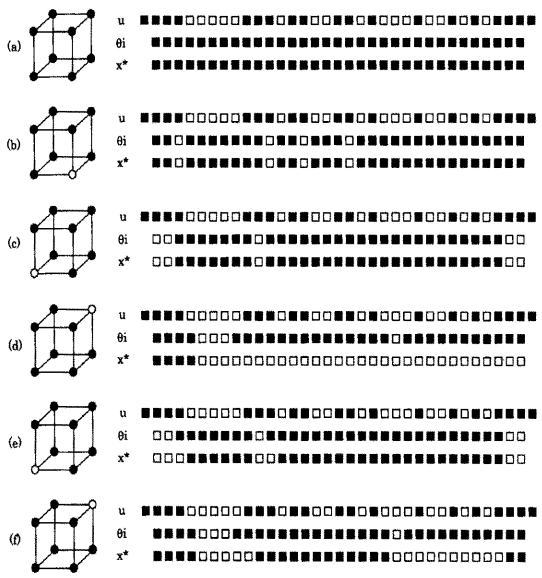

Fig.6 Some examples of stable solutions

sists of only black cells except for mid-cells of black blocks of length greater than 2.

Figs. 6(d)-(e) are examples for Theorem 8. **Fig. 6(d)** where all θ_i belong to R_1 or R_3 but not to R_2, shows that S converges to a vector consisting of only white cells except for some left boundary cells.

Fig. 6(e) shows that only cells except the left end of the block which consists of three or more consecutive black cells -1 has the output value $+1$.

Fig. 6(f) is a example where there are a lot of cells which belongs to R_3. The value (-1) or (+1) is final value respectively in the place where (-1) or (+1) is consecutive three times or more, and the final value of other cells is the final value of the left cell.

For Theorem 9, i.e., for the case of $-\beta > \alpha$, we have similar behavior as the above.

5. Conclusion

We examined 1-D DBCNNs $S(\alpha, \beta, 0, \hat{\alpha}, \hat{\beta}, \hat{\gamma}, \theta)$ with fixed boundary and gave the necessary and sufficient conditions for them to be stable. We also gave several examples to illustrate the theorems. These examples may suggest some kinds of applications to signal processing.

Similar results can be derived for $S(\alpha,\ 0,\ \gamma,\ \hat{\alpha},\ \hat{\beta},\ \hat{\gamma},\ \theta)$.

Periodical boundary cases will be treated in a subsequent paper.

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