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A Topological Condition for an RC Active Circuit Containing Op-amps to be Unstable

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Abstract: This paper deals with the instability of RC active circuits containing non-ideal op-amps. It is very often that, when we design a circuit with a prescribed transfer function, installed RC active circuits are unstable, even though the designed transfer function itself is stable. One of the reasons for it is the existence of inevitable deviations of circuit element characteristics from ideal ones. In this paper we assume that the voltage gain of each op-amp is represented by the one-pole model and give a topological sufficient condition for the denominator polynomial of RC active circuits with these op-amps to be able to possess negative coefficient terms.

Keywords: RC active circuit, Op-amp, Topological condition, Instability

1. Introduction

When we design an active circuit with a prescribed transfer function, there often arises an instability problem, though the transfer function itself is stable, i.e., all poles of this transfer function lie only in the left-half plane of the complex variable s. Similar instability problem occurs for an equilibrium point of a nonlinear dc circuit composed of dc sources, nonlinear resistors, and active elements. There are some explanations for these instabilities in the literatures [1)−6)] as follows: 1)Deviation of real circuit-element characteristics from ideal ones (frequency characteristics, element losses, etc.), 2) Existence of parasitic elements(capacitances, inductances), 3) Existence of a positive feedback with the gain greater than unity. These explanations are plausible in some aspects, but are not necessarily reasonable. Further the relations among them have not been clarified yet.

In this paper we assume that gain characteristics g_i of each op-amp are represented by one pole model, i.e., $g_i = g_{0i}/(1 + s/\omega_{BTi})$, and discuss the instability of a circuit consisting of these op-amps and resistors and capacitors.

Let the prescribed transfer function be $T(s) = Q(s)/P(s)$, where $P(s)$ is a Hurwitz polynomial of order n. If op-amps are ideal, i.e., g_{0i} are infinite, then the designed circuit surely realizes the above $T(s)$ and therefore is stable. However due to the fact that g_{0i} are finite there arise many unwanted parasitic terms in the denominator of the real transfer function in addition to designated terms.

Since among these parasitic terms those having order lower than or equal to n will be embedded into the designated terms with large values, they will hardly affect the stability of the circuit. It is however very probable that those having order higher than n and having negative coefficients will cause the instability with high probability.

In some cases these negative coefficient terms may be canceled with positive ones but we cannot optimistically expect it, because the above parasitic values of these higher order terms as well as the values g_{0i} and ω_{BTi} are not predictable a priori, when we design the circuit. Hence it is desirable to know a priori the possibility of negative coefficient term of order higher than n.

In this paper we investigate the condition that the denominator polynomial of an actual transfer function can possess a negative coefficient term of order higher than n independent of values of circuit parameters, such as resistances, capacitances, and op-amp's gains. The obtained condition is graph-theoretic, and the result seems to be a generalization of that of total feedback gain, which is very familiar to circuit designers.

2. Preliminaries

2.1 Graph representation

Consider an RC active circuit with op-amps as shown in **Fig.1**, which is composed of m resistors, R_L, and $R_i(= 1/G_i)$ $(i = 1, \cdots, m-1)$, n capacitors, $C_i(i = 1, \cdots, n)$, k op-amps, and an independent current source, J. Here R_L is the load resistor.

From a practical point of view, we assume as follows:

Assumption 1: One of output terminals of each op-amp is grounded. □

In the subsequence an op-amp $\#i$ is replaced by a cascade of a voltage-controlled current source (VCCS) with the gain g_i and an ideal gyrator with gyration ratio g_{Gi} as shown in **Fig.2(b)**, where $g_{Gi} = 1$. The gain g_i has the frequency characteristic described below and should be written as

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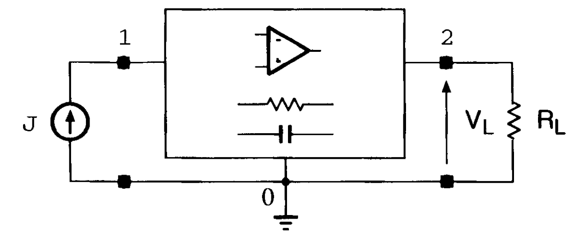
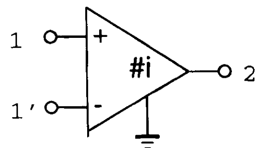
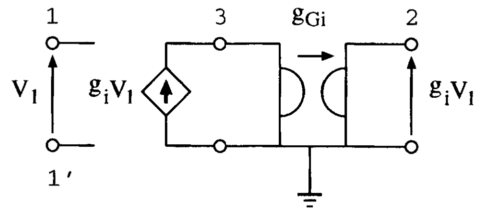


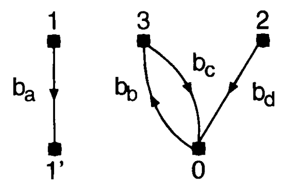
Fig.1 An RC active circuit with op-amps



(a) Op-amp



(b) Equivalent circuit of an Op-amp



(c) Directed graph

Fig.2 Op-amp $\#i$, its equivalent circuit using a VCCS and a gyrator and its corresponding graph.

$g_i(s)$.

We derive a graph denoted by G corresponding to the above-mentioned modified circuit with VCCSs and gyrators in the following way:

1) Replace a resistor R_i and a capacitor C_j by directed branches b_{Ri} and b_{Cj}, respectively. They are called resistor- and capacitor-branches or simply R- and C-branches, respectively.
2) Replace an op-amp by four directed branches corresponding to the VCCS and the gyrator as shown in **Fig.2(c)**, where b_a and b_b respectively correspond to controlling and controlled branches of the VCCS, and branches b_c and b_d correspond to the gyrator. The direction of each branch is also shown in **Fig.2(c)**. We call branches b_a, b_b, b_c, and b_d a-, b-, c-, and d-branches, respectively.
3) Replace an independent current source J by a directed branch b_J.

2.2 Circuit analysis

Let $A = [a_{ij}]$ be a reduced incidence matrix of the obtained graph G, where the ground is regarded as the reference node of A. Let V_{br} and I_{br} denote a branch voltage vector and a branch current vector. We assume that the first element of V_{br} (I_{br}) corresponds to the current source, the second element to the load resistor, the 3rd to $4k+2$th elements to the k op-amps and the remainings to other resistors and capacitors. Let the total number of nodes including the ground be $\rho + 1$ and let ρ-dimensional vector U denote the node potential vector with respect to the reference node 0. Note that nodes 1 and 2 are designated as in **Fig.1**.

Then the equations of this circuit can be described as :

$$
\begin{aligned}
\text{KCL} \quad &: AI_{br} = 0 \\
\text{KVL} \quad &: V_{br} = A^T U \\
\text{Ohm's Law} \quad &: I_{br} = Y_{br} V_{br}
\end{aligned}
\tag{1}
$$

where Y_{br} is the constitution matrix representing the characteristics of all elements. Concretely, it has the form:

$$
\begin{aligned}
Y_{br} = \; & (0) \oplus \left(\frac{1}{\mathrm{R_L}}\right) \oplus \begin{pmatrix} 0 & 0 \\ g_1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & g_{G1} \\ -g_{G1} & 0 \end{pmatrix} \\
& \oplus \begin{pmatrix} 0 & 0 \\ g_2 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & g_{G2} \\ -g_{G2} & 0 \end{pmatrix} \oplus \cdots \\
& \oplus \begin{pmatrix} 0 & 0 \\ g_k & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & g_{Gk} \\ -g_{Gk} & 0 \end{pmatrix} \\
& \oplus \operatorname{diag}\left(\frac{1}{\mathrm{R_1}}, \cdots, \frac{1}{\mathrm{R_{m-1}}}, \mathrm{sC_1}, \cdots, \mathrm{sC_n}\right)
\end{aligned}
\tag{2}
$$

where $\oplus$ denotes the direct sum of matrices. The first term of the right-hand side of Eq.(2) is the internal conductance of the independent current source, the second one the load conductance, the third and fourth ones the characteristics of the gyrator corresponding to the first op-amp, the fifth and sixth ones the characteristics of the VCCS corresponding to the second op-amp, and so on.

Combining equations in (1), we have the nodal equation

$$
AY_{br}A^T U = A\boldsymbol{J} \tag{3}
$$

where $\boldsymbol{J} = (J, 0, \cdots, 0)^T$.

Let the coefficient matrix of the nodal equation be

$$
Y = AY_{br}A^T. \tag{4}
$$

The incidence matrix A is an $\rho \times b$ matrix where $b = m + n + 4k + 1$. The matrix A has the form in (5).

$$A = \left(b_J \middle| b_{R_L} \middle| b_{a1} \middle| b_{b1} \middle| b_{c1} \middle| b_{d1} \middle| b_{a2} \middle| b_{b2} \middle| b_{c2} \middle| b_{d2} \middle| \cdots \cdots \middle| b_{ak} \middle| b_{bk} \middle| b_{ck} \middle| b_{dk} \middle| b_{R1} \middle| \cdots \middle| b_{C1} \middle| \cdots \right) \tag{5}$$

Here the label of each column of A represents the corresponding branch. Then the matrix AY_{br} can be formally represented as in (6).

$$AY_{br} = \left(\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c} b_J & b_{R_L} & b_{b1} & b_{a1} & b_{d1} & b_{c1} & b_{b2} & b_{a2} & b_{d2} & b_{c2} & \cdots & b_{bk} & b_{ak} & b_{dk} & b_{ck} & b_{R1} & \cdots & b_{C1} & \cdots \\ \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\ 0 & \frac{1}{R_L} & g_1 & 0 & -g_{G1} & g_{G1} & g_2 & 0 & -g_{G2} & g_{G2} & \cdots & g_k & 0 & -g_{Gk} & g_{Gk} & \frac{1}{R_1} & \cdots & sC_1 & \cdots \end{array} \right) \tag{6}$$

The matrix AY_{br} in (6) shows how columns of AY_{br} are related to those of A in (5). That is, the first column of AY_{br} is equal to the first column of A multiplied by zero(= the conductance of the independent current source), the second column of AY_{br} is equal to the second column of A multiplied by G_L, the third column of AY_{br} is equal to the fourth column of A multiplied by g_1, and so on.

2.3 Transfer function of an RC active circuit

The transfer function $T = V_2/J$ of the RC active circuit as shown in **Fig.1** is given

$$T = \frac{V_2}{J} = \frac{\Delta_{21}}{\Delta} \tag{7}$$

where Δ is a determinant of the coefficient matrix Y in (4) and Δ_{21} denotes a cofactor of the $(2,1)$ element of Y.

The determinant of Y in (4) can be calculated by means of the well-known Binet-Cauchy's theorem. That is, Δ can be written as:

$$\Delta = \sum_{1 \le i_1 < \cdots < i_\rho \le b} |A_{[i_1,\cdots,i_\rho]}| \cdot |AY_{br[i_1,\cdots,i_\rho]}| \tag{8}$$

where $A_{[i_1,\cdots,i_\rho]}$ denotes the $\rho \times \rho$ submatrix composed of the i_1th, $\cdots$, i_ρth columns of A. The other symbol $AY_{br[i_1,\cdots,i_\rho]}$ has also the similar meaning.

As is easily seen from Y in (4) the ith column of AY_{br} is the jth column of A multiplied by y_{ij} for some j. Then the second term of the right-hand side of Eq.(8) is given as

$$|AY_{br[i_1,\cdots,i_\rho]}| = y_{i_1j_1} \cdots y_{i_\rho j_\rho} |A_{[j_1,\cdots,j_\rho]}|. \tag{9}$$

Therefore we have from Eqs.(8) and (9),

$$\Delta = \sum_{1 \le i_1 < \cdots < i_\rho \le b} y_{i_1j_1} \cdots y_{i_\rho j_\rho} |A_{[i_1,\cdots,i_\rho]}| \cdot |A_{[j_1,\cdots,j_\rho]}|. \tag{10}$$

In (10), $|A_{[i_1,\cdots,i_\rho]}|$ and $|A_{[j_1,\cdots,j_\rho]}|$ are either 0 or ± 1 because of the totally unimodularity of an incidence matrix. Since Δ is a multilinear function of the op-amp gain g_i $(i = 1, 2, \cdots, k)$, Eq.(10) can be written as follows:

$$\Delta = g_1 g_2 \cdots g_k \left(P + \frac{P_1}{g_1} + \frac{P_2}{g_2} + \cdots + \frac{P_k}{g_k} + \frac{P_{1,2}}{g_1 g_2} + \frac{P_{1,3}}{g_1 g_3} + \cdots + \frac{P_{k-1,k}}{g_{k-1} g_k} + \frac{P_{1,2,3}}{g_1 g_2 g_3} + \frac{P_{1,2,4}}{g_1 g_2 g_4} + \cdots + \frac{P_{1,2,\cdots,k}}{g_1 g_2 \cdots g_k} \right) \tag{11}$$

where P, P_1, and $P_{1,2,\cdots,k}$ denote multilinear forms of some $1/R_i$'s and sC_j's, hence polynomials in s.

The cofactor Δ_{21} can also be written as

$$Delta_{21} = g_1 g_2 \cdots g_k \left(Q + \frac{Q_1}{g_1} + \frac{Q_2}{g_2} + \cdots + \frac{Q_k}{g_k} + \frac{Q_{1,2}}{g_1 g_2} + \frac{Q_{1,3}}{g_1 g_3} + \cdots + \frac{Q_{k-1,k}}{g_{k-1} g_k} + \frac{Q_{1,2,3}}{g_1 g_2 g_3} + \frac{Q_{1,2,4}}{g_1 g_2 g_4} + \cdots + \frac{Q_{1,2,\cdots,k}}{g_1 g_2 \cdots g_k} \right) \tag{12}$$

where Q, Q_1, and $Q_{1,2,\cdots,k}$ are also multilinear forms of some $1/R_i$'s and sC_j's only and polynomials in s. Other symbols also have the similar meanings.

Concerning actual op-amps, we assume:

Assumption 2: The gain characteristic of each op-amp can be represented by one-pole model:

$$g_i = \frac{g_{0i}}{1 + s/\omega_{BTi}}. \tag{13}$$

where g_{0i} has a large positive value. □

Combining (7), (11), (12), and (13), we obtain

$$T = \frac{Q + \frac{Q_1}{g_1} + \frac{Q_2}{g_2} + \cdots + \frac{Q_{1,2,\cdots,k}}{g_1 g_2 \cdots g_k}}{P + \frac{P_1}{g_1} + \frac{P_2}{g_2} + \cdots + \frac{P_{1,2,\cdots,k}}{g_1 g_2 \cdots g_k}} \triangleq \frac{\widetilde{Q}(s)}{\widetilde{P}(s)}. \tag{14}$$

Remark 1: If all op-amps are ideal, i.e, $g_{0i} = \infty$ in (13), then the actual transfer function T equals the prescribed transfer function Q/P. □

Concerning the prescribed transfer function Q/P, we assume the followings.

Assumption 3: $P \not\equiv 0$ and $\deg P = n$. □

Assumption 4: Two polynomials P and Q do not have a common factor. □

Assumption 5: $P(0) > 0$. □

Remark 2: Assumption 3 is reasonable because, if $P \equiv 0$, then some of op-amps are not required for the realization of the circuit. The latter part of Assumption 3 means that the circuit is a minimum capacitor circuit, i.e., the number of capacitors equals the degree of the transfer function. □

Remark 3: Though Assumption 4 does not necessarily hold in conventional realization methods, it is reasonable for low sensitive realization. □

We also make the following assumption:

Assumption 6: The op-amp gains g_{0i} $(i = 1, \cdots, k)$ are sufficiently large, but may be relatively quite different from each other. □

Remark 4: The latter part of Assumption 6 is reasonable because we cannot exactly predict them in general and because this paper deals rather with the worst case in some sense. □

Concerning the polynomial $\widetilde{P}(s)$ in (14), the following lemma holds.

Lemma 1: If the degree of a polynomial, say $P_{i_1,i_2,\cdots,i_\nu}$ in (11) equals n, then it is possible to choose op-amp parameters such that the coefficient of degree $n+\nu$ in the denominator polynomial $\widetilde{P}(s)$ almost equals the coefficient of the highest order in $P_{i_1,i_2,\cdots,i_\nu}$ for some g_{0i} and ω_{BTi}. □

Proof: If $\deg P_{i_1,i_2,\cdots,i_\nu} = n$, then it follows from (13) that $\det(P_{i_1,i_2,\cdots,i_\nu}/g_{i_1}g_{i_2}\cdots g_{i_\nu}) = n+\nu$. Thus Lemma 1 holds when we choose g_{0i} and ω_{BTi} $(i = 1, 2, \cdots, k)$ as follow:

$$g_{0i} = \begin{cases} \gamma & i = i_1, i_2, \cdots, i_\nu \\ \Gamma & \text{otherwise} \end{cases}$$

and

$$\omega_{BTi} = \begin{cases} \omega_m & i = i_1, i_2, \cdots, i_\nu \\ \omega_M & \text{otherwise} \end{cases}$$

where $\Gamma \gg \gamma$ and $\omega_M \gg \omega_m$. □

3. Main result

3.1 Instability of RC active circuits containing op-amps

In this paper, instability of RC active circuits containing op-amps is defined as follows:

Definition 1: If at least one zero of the denominator polynomial $\widetilde{P}(s)$ in (14) exists in the left half s-plane for *any parameter values* such as g_{0i} and ω_{BTi} $(i = 1, 2, \cdots, k)$ in (13), resistance and capacitance , then the RC active circuit is said to be *potentially unstable.* □

According to the above definition for instability of RC active circuits and Lemma 1, the following lemma holds.

Lemma 2: If there exists a negative term of order n in the denominator polynomials except P in (14), then the circuit is potentially unstable. □

3.2 Main theorem

In order to describe our main result we will introduce some graph operations and a special class of graphs.

Let a connected graph be composed only of sub-

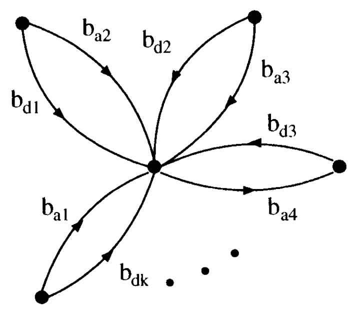


Fig.3 Example of a cactus graph

sets of a-branches and d-branches, denoted by B_a and B_d, respectively. If each of B_a and B_d forms a tree of the graph, then the graph is said to have *a complementary tree structure with respect to B_a and B_d*, or simply have *a complementary tree structure.*

Let G be the graph defined in Sec.2.1. We apply the following graph operations:

(I) Open-circuit the branch b_J and all b-branches.

(II) Short-circuit all capacitor branches and all c-branches.

(III) Open-circuit or short-circuit each resistor branch arbitrarily.

(IV) Open-circuit some (nonzero) a-branches and short-circuit the associated d-branches.

(V) Move one node of a-branches which are not grounded along other a-branches to the ground.

Remark 5: By applying the graph operations (III) and (IV) arbitrarily, many subgraphs can be derived. Some subgraphs have a complementary tree structure with respect to the remaining a-branches and d-branches. □

Remark 6: Unless a subgraph has a complementary tree structure, either $|A_{[k_1,\cdots,k_n]}|$ or $|A_{[k'_1,\cdots,k'_n]}|$ in (10) is singular, that is, the degree of the corresponding polynomial is less than m. Thus this subgraph does not satisfy Lemma 1. □

For the subsequent discussion let the subgraph obtained by applying the graph operations have a complementary tree structure with respect to the remaining a-branches and d-branches.

Let one of the resulting graphs with a complementary tree structure be composed of κ a-branches, for example, b_{a1}, b_{a2}, $\cdots$, and $b_{a\kappa}$ and the corresponding κ d-branches, b_{d1}, b_{d2}, $\cdots$, and $b_{d\kappa}$. If b_{di} and b_{ai+1} $(i = 1, 2, \cdots, \kappa - 1)$ form a loop and $b_{d\kappa}$ and b_{a1} form a loop, then we say that the resulting graph is said to be *a cactus graph* (See the reference[8]).

Remark 7: The order of the subscripts of branches is not essential in the definition of cactus graphs. □

Fig.3 shows an example of a cactus graph. In **Fig.3** we say that the branches b_{d1} and b_{a2} form *a similarly directed cutset* and form *an opposite directed loop.* Similarly we say that the branches b_{d3} and b_{a4} form *a similarly directed loop* and form *an* *opposite directed cutset.*

We can now state our main theorem:

Theorem 1: The circuit is potentially unstable, if we can obtain a cactus graph with an even number of similarly directed loops by applying the graph operations (I)-(V) to the original graph G. □

Remark 8: Theorem 1 gives a topological condition for the denominator polynomial $\tilde{P}(s)$ to have a negative term of higher order than n. □

The proof of Theorem 1 is very involved but is similar to the proof of theorems in the literature[8]. We only describe an outline of the proof in Appendix.

3.3 Example

Consider an RC active circuit shown in **Fig.4** consisting of 3 nullors[9)10)]. The transfer function of this nullor circuit is given as follows:

$$T = \frac{\Delta_{21}}{\Delta}$$

$$\Delta = s^2C_1C_2(R_2^{-1}R_4^{-1} + R_2^{-1}R_5^{-1}) + sC_2R_4^{-1}R_6^{-1}(R_1^{-1} + R_2^{-1} + R_3^{-1}) + R_3^{-1}R_4^{-1}R_6^{-1}R_7^{-1} + R_3^{-1}R_5^{-1}R_6^{-1}R_7^{-1},$$

and

$$\Delta_{21} = -(s^2C_1C_2R_2^{-1} + R_3^{-1}R_6^{-1}R_7^{-1}).$$

It is well known that a nullor is equivalent to an ideal op-amp. Thus, we can derive 48 different op-amp circuits[11)] which realize the above transfer function. One of these op-amp circuits is illustrated in **Fig.5**. **Fig.6(a)** shows a corresponding directed graph where all b- and c-branches are omitted. Based on the mentioned graph operations (I)-(V), open-circuit branches b_J, b_{R2}, b_{R4}, b_{R5}, b_{R6}, b_{R7} and b_{a1}, and short-circuit branches b_{C1}, b_{C2}, b_{R1}, b_{R3}, and b_{d1}. Then we have a subgraph as shown in **Fig.6(b)**.

There exists two similarly directed loops in the obtained subgraph as shown in **Fig.6(b)**. It follows from Theorem 1 that the RC op-amp circuit in **Fig.5** turns out to be unstable. This assertion accords with the result of the experiment in the reference[11)].

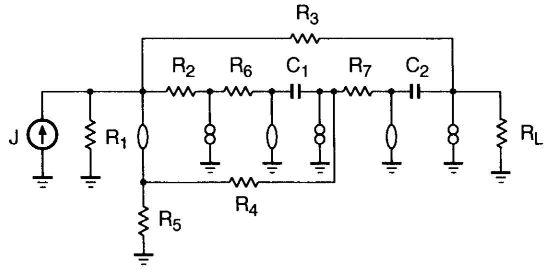


Fig.4 Example of a nullor circuit

4. Conclusion

We showed a topological condition for a negative coefficient due to the frequency characteristic of op-amps to possibly arise. The result is given in terms of a cactus graph with an even number of similarly directed loops.

This condition is identical to the existence of a positive feedback in a wide sense. Since a positive feedback can apparently produce a negative coefficient, we showed that the existence of a positive feedback is also necessary for a negative coefficient.

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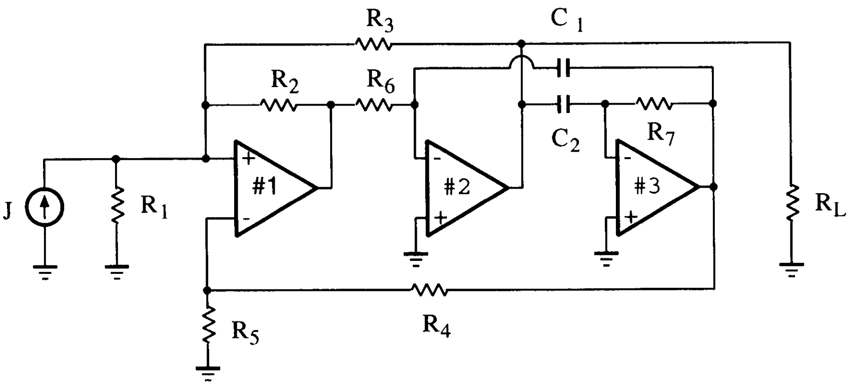


Fig.5 Example of an op-amp circuit equivalent to the nullor circuit in Figure 4.

Appendix

A Proof of Theorem 1

We will describe an outline of the proof of Theorem 1 mainly for the polynomial P_1 in (14).

According to Eq.(8), a term of order n in P_1 is calculated by $|A_1|\cdot|A_2|$ where A_1 and A_2 are $\rho\times\rho$ submatrices of A and AY_{br}, respectively.

Remark 9: All columns associated with c-branches are always included in a submatrix of A. □

Submatrices A_1 and A_2 are represented as follows:

$$A_1 = \left(\begin{array}{c|c|c|c|c|c|c} b_{c1} & b_{d1} & b_{a2} & b_{c2} & \cdots & b_{ak} & b_{ck} \end{array}\right. \\ \left.\begin{array}{|c|c|c|c|c|c}b_{R1} & \cdots & b_{R\mu} & b_{C1} & \cdots & b_{Cn}\end{array}\right) \tag{A.1}$$

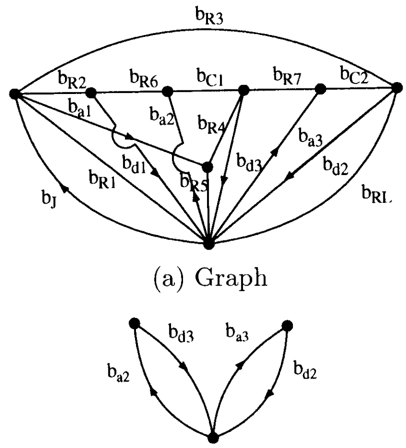
and

$$A_2 = \left(\begin{array}{c|c|c|c|c|c|c} b_{c1} & b_{d1} & b_{b2} & b_{d2} & \cdots & b_{bk} & b_{dk} \\ \times & \times & \times & \times & \times & \times & \times \\ -g_{G1} & g_{G1} & g_2 & -g_{G2} & \cdots & g_k & -g_{Gk} \end{array}\right. \\ \left.\begin{array}{|c|c|c|c|c|c} b_{R1} & \cdots & b_{R\mu} & b_{C1} & \cdots & b_{Cn} \\ \times & \times & \times & \times & \times & \times \\ \frac{1}{R_1} & \cdots & \frac{1}{R_\mu} & sC_1 & \cdots & sC_n \end{array}\right). \tag{A.2}$$

Remark 10: From topological point of view, deleting columns of an incidence matrix means an open-circuiting associated branches from the original directed graph. □

Submatrices A_1 and A_2 are obtained by deleting some columns of A and AY_{br}, respectively. In this case, we open-circuit branches corresponding to columns not appearing in both A_1 and A_2. Thus, branches b_J, b_{R_L}, b_{a1}, b_{b1}, $b_{R\mu+1}$, $\cdots$, b_{Rm-1} are open-circuited from the graph G. Let G' denote a directed graph obtained by this operation.

Observing the direction of b_{bi} and b_{ci} (See **Fig.2(c)**) and interchanging columns of A_1 and A_2 appropriately, we have



(a) Graph

(b) Obtained subgraph

Fig.6 Example of the application of Theorem 1

$$|A_1|\cdot|A_2| = s^n(-1)^{k-1}g_2g_3\cdots g_k g_{G1}^2 g_{G2}\cdots g_{Gk} \\ \frac{1}{R_1}\cdots\frac{1}{R_\mu}C_1\cdots C_n|A_1'|\cdot|A_2'| \tag{A.3}$$

where matrices A_1' and A_2' take the form

$$A_1' = \left(\begin{array}{c|c|c|c|c|c|c|c} b_{a2} & b_{a3} & \cdots & b_{ak} & b_{c1} & \cdots & b_{ck} & b_{d1} \end{array}\right. \\ \left.\begin{array}{|c|c|c|c|c|c} b_{R1} & \cdots & b_{R\mu} & b_{C1} & \cdots & b_{Cn}\end{array}\right) \tag{A.4}$$

and

$$A_2' = \left(\begin{array}{c|c|c|c|c|c|c|c} b_{d2} & b_{d3} & \cdots & b_{dk} & b_{c1} & \cdots & b_{ck} & b_{d1} \end{array}\right. \\ \left.\begin{array}{|c|c|c|c|c|c} b_{R1} & \cdots & b_{R\mu} & b_{C1} & \cdots & b_{Cn}\end{array}\right). \tag{A.5}$$

Since the gyrator ratio g_{Gi} $(i = 1, 2, \cdots, k)$ equals 1, the following lemma holds.

Lemma 3: $|A_1|\cdot|A_2|$ is negative if and only if $|A_1'|\cdot|A_2'| = (-1)^k$. □

In a similar way as in the case of the polynomial P_1, we have the following lemma for a general polynomial, say $P_{i_1,i_2,\cdots,i_\nu}$.

Lemma 4: If $|A_1'|\cdot|A_2'| = (-1)^{\nu+1}$, there exists a

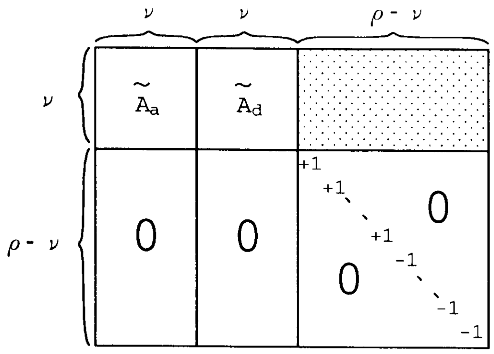


Fig.7 Matrix $\widetilde{A}$

negative term of order n in $P_{i_1,i_2,\cdots i_\nu}$. □

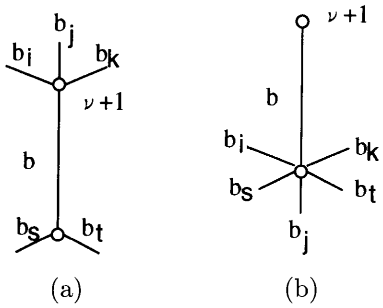
Let us partition the submatrix A'_1 (resp. A'_2) into two submatrices $\begin{pmatrix} A_a & A_c \end{pmatrix}$ (resp.$\begin{pmatrix} A_d & A_c \end{pmatrix}$). Here we call branches corresponding to columns in A_c (i.e., common columns in both A'_1 and A'_2) as *common branches.* For example, in the case where submatrices A'_1 and A'_2 are given as (A.4) and (A.5), respectively, branches $b_{c1}, \cdots, b_{ck}$ $b_{d1}, b_{R1}, \cdots, b_{R\mu}$, $b_{C1}, \cdots$, and b_{Cn} are common branches.

Submatrix A' is a $\rho \times (\rho+\nu)$ incidence matrix of the directed graph G' whose branches correspond to the columns in A'_1 and A'_2.

Suppose without loss of generality that the $(\nu+1, 2\nu+1)$ element of A' is nonzero. Let us multiply the $2\nu+1$th column by 0 or ± 1 appropriately, and add this column to other columns so that elements of the $\nu+1$th row except the $(\nu+1, 2\nu+1)$ element become 0. This operation is said to be *an sweeping out by the* $(\nu+1, 2\nu+1)$ *element as a pivot.*

Let us consider that the $2\nu+1$th column of A' is associated with an branch b in the graph G'. Assume that one of endpoints of the branch b is node $\nu+1$. Then the graph-theoretic interpretation of the sweeping out by the $(\nu+1, 2\nu+1)$ can be stated as follows.

Remark 11: All branches connected to node $\nu+1$ except the branch b are moved along b so that they are attached to another endpoint of the branch b as shown in **Fig.8**. □



(a) (b)

Fig.8 Graph-theoretic interpretation of a sweeping out operation in the incidence matrix

Applying the sweeping out operations by some pivots, we obtain a matrix $\widetilde{A}$ as shown in **Fig.7(b)** from A'.

Remark 12: We can identify that $\nu \times 2\nu$ submatrix $\begin{pmatrix} \widetilde{A}_a & \widetilde{A}_d \end{pmatrix}$ is an incidence matrix with respect to the remaining a-branches and d-branches of a graph $\widetilde{G}$ obtained from G' by short-circuiting common branches. □

Because of the topology of d-branches, the submatrix $\widetilde{A}_d$ always equals an identity matrix of order ν. Thus we have $|A'_1| \cdot |A'_2| = |\widetilde{A}_a| \cdot |\widetilde{A}_d| = |\widetilde{A}_a|$.

Let $\widetilde{A}_a$ be a reducible matrix. By interchanging rows and columns appropriately, then $\begin{pmatrix} \widetilde{A}_a & \widetilde{A}_d \end{pmatrix}$ has the following structure:

$$\begin{array}{cc} & \begin{array}{cccc} B_{a1} & B_{a2} & B_{d1} & B_{d2} \end{array} \\ \begin{pmatrix} \widetilde{A}_a & \widetilde{A}_d \end{pmatrix} = & \begin{pmatrix} \widetilde{A}_{11} & \mathbf{0} & \mathbf{1}_{\nu 1} & \mathbf{0} \\ \widetilde{A}_{21} & \widetilde{A}_{22} & \mathbf{0} & \mathbf{1}_{\nu 2} \end{pmatrix} \end{array} \tag{A.6}$$

where $\mathbf{1}_{\nu i} (i = 1, 2)$ is an identity matrix of order ν_i. Submatrix $\widetilde{A}_{ii} (i = 1, 2)$ denotes an irreducible matrix of order ν_i, and $\nu_1 + \nu_2 = \nu$. It follows from (A.6) that

$$|\widetilde{A}_a| \cdot |\widetilde{A}_d| = |\widetilde{A}_{11}| \cdot |\widetilde{A}_{22}|. \tag{A.7}$$

If Lemma 4 holds, it follows from (A.6) and (A.7) that either $|\widetilde{A}_{11}| = (-1)^{\nu_1+1}$ or $|\widetilde{A}_{22}| = (-1)^{\nu_2+1}$ always holds. Thus it is sufficient to consider the case where the following lemma holds.

Lemma 5: Matrix $\widetilde{A}_a$ is an irreducible matrix. □

According to Lemma 5, a-branches in the directed graph G' form a strongly connected component. Applying sweeping out operations to the matrix $\widetilde{A}_a$ and interchanging rows and columns appropriately so that a-branches still form a strongly connected component, then $\begin{pmatrix} \widetilde{A}_a & \widetilde{A}_d \end{pmatrix}$ can take the form

$$\begin{pmatrix} \widetilde{A}_a & \widetilde{A}_d \end{pmatrix} = \left(\begin{array}{cccccc|c} 0 & \varepsilon_2 & 0 & \cdots & & 0 & \\ 0 & 0 & \varepsilon_3 & 0 & \cdots & 0 & \\ \vdots & \ddots & & \ddots & \ddots & \vdots & \mathbf{1} \\ 0 & \cdots & 0 & \varepsilon_{\nu-1} & & 0 & \\ 0 & \cdots & & 0 & & \varepsilon_\nu & \\ \varepsilon_1 & 0 & & \cdots & & 0 & \end{array} \right) \tag{A.8}$$

and we have

$$|\widetilde{A}_a| \cdot |\widetilde{A}_d| = (-1)^{\nu-1} \varepsilon_1 \varepsilon_2 \cdots \varepsilon_\nu. \tag{A.9}$$

When $\varepsilon_1 \varepsilon_2 \cdots \varepsilon_\nu = +1$, Lemma 4 holds, from which Theorem 1 follows.