Dynamics of Japan's Stock Market Volatility : A Comparison of GARCH and Stochastic Volatility Models

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Dynamics of Japan's Stock Market Volatility: A Comparison of GARCH and Stochastic Volatility Models

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Abstract

This paper compares generalised autoregressive conditional heteroscedastic (GARCH) and stochastic variance (SV) modelling approaches in analysing the dynamics of Japan's stock market volatility using monthly time series from January, 1984 through April, 2013. GARCH models essentially model the conditional variance of returns given past returns and observations. While SV models, have different data-generating process, with variance that is specified to follow some unobserved stochastic process. We examine estimates of GARCH models with and without breaks accounting for market crash and financial crises to assess their impact on stock market returns volatility and SV-type models. Using a variety of return transformations, we find persistence and variability in the relevant parameters of both GARCH and SV models. We observe that the volatility persistence parameter in the SV model which indicates volatility clustering is comparable with the persistence measure of GARCH models and a similarity in the trend of the estimated SV model with that of the IGARCH model. Finally, we investigate whether GARCH and SV-type models differ significantly in their ability to predict the volatility of Japan's stock index returns over horizons ranging from 1, 3, 6 to 12 months.

Keywords: Stock Market Volatility, GARCH, Breaks, Persistence, Stochastic Volatility, Forecasting
JEL Classification Codes: C22; C53; C58; G01; G17

1. Introduction

It is established in finance literature that the variance of financial time series (e.g. exchange rates, bonds and stock returns) tends to change over time and often exhibit volatility clustering. Attempts to model this feature have motivated the development of *autoregressive conditional heteroscedastic* (ARCH) models by Engle (1982), Bollerslev (1986) and Nelson (1991), among others and stochastic volatility (SV) models by Taylor (1986), Harvey *et al.* (1994), Kim *et al.* (1998),

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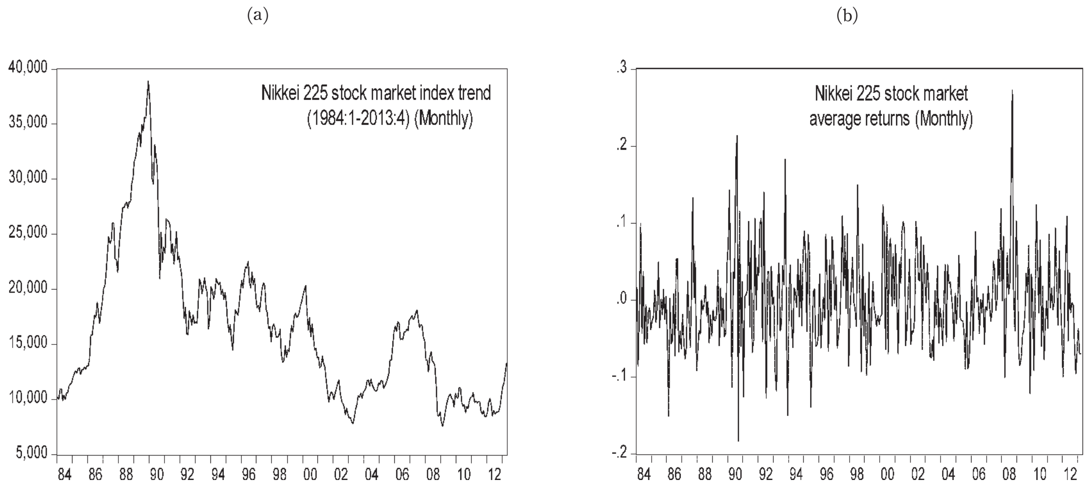
Omori *et al.* (2007), etc. The conditional variances for ARCH-type models are specified as a function of past squared innovations and lagged conditional variances while SV models' variances are modelled to follow some unobserved stochastic process. Although the likelihood functions of ARCH models could be readily derived, the estimation methods proposed for SV models are often more computationally involved ranging from quasi-maximum likelihood approach (QML), exact likelihood, to a variety of simulation techniques. For an overview of SV techniques and their connection to continuous-time option pricing models, see Ghysels *et al.* (1996), Campbell *et al.* (1997), Tsay (2005), Shephard and Andersen (2009), Teräsvirta *et al.* (2010), among several others.

The main distinction between GARCH and SV models is that SV models have separate disturbance terms in the mean and variance equations, precluding direct observation of the variance process (see, Koopman and Uspensky, 2002). The recent surge in popularity of models of conditional heteroscedasticity is partly explained by the fact that they can be used for forecasting volatility of financial assets [Koopman *et al.* (2005), Teräsvirta *et al.* (2010)]. We will examine and compare the forecast performance of these alternative models within the context of Japan's financial time series.

Japan's equity market based on total value is the second largest in the world and its financial markets have witnessed significant fluctuations recently [Hoshi and Kashyap (2004), Kang *et al.* (2009)]. The Nikkei-225 index experienced persistent fluctuations over the last two decades with implications on investors and risk level in the Japanese financial markets. Japan's equity markets were driven by high economic growth and expectations before declining drastically in the late 1980s, with the main stock indexes never regaining their previous levels. Observe the growth of equity prices before 1989 and sharp drops probably attributed to the Asian currency crisis in the period 1997-1998, the dot-com bubble during the 1998-99 periods and the global financial crisis of 2007-09 (see Figure 1a). For example, the Nikkei-225 index had in late 1989 recorded its highest value of 38,916 points before declining to its lowest level in the 2000s; subsequently leading to one of the worst bear markets to be experienced in a major industrialised country [Kang, *et al.* (2009)]. Recently, policies aimed at tackling deflation and jumpstarting growth through quantitative easing (QE) by the Bank of Japan (BOJ), and aggressive fiscal spending known as *"Abenomics"* have contributed in influencing Japan's financial markets towards an upward trajectory. Further, these have encouraged shifts of assets from Japanese government bonds (JGB) to stocks thereby directly helping to move equity prices up [see also Shiller, Kon-Ya and Tsutsui (1996), Ito (2005), Tsutsui and Hirayama (2013)].

Since the 1980s when ARCH models were introduced, several generalisations and extensions aimed at increasing the flexibility of the original models have been proposed [see also Baillie *et al.* (1996), Engle, and Rangel (2008)]. These variants include *generalised* ARCH, *threshold* GARCH [Glosten, Jaganathan and Runkle (GJR), 1993], *exponential* and *integrated* GARCH

Figure 1: Time Series Plot of Monthly Nikkei 225 Stock Market Average (1984:1–2013:4):
(a) Nikkei-225 Stock Market Index (b) Nikkei-225 Stock Market Index Log Returns.



(Nelson, 1991), *fractionally integrated* GARCH, *quadratic* GARCH, Markov-switching GARCH, etc. Engle (2003) stressed that these extensions recognise the presence of nonlinearities, asymmetry and long memory properties associated with volatility and the non-normally distributed nature of returns. The SV approach is employed to model series particularly those with nonstationarity in their variances. Often unobserved volatility in asset returns is subject to a dynamic process (Jungbacker and Koopman, 2009). As a result, SV models are now one of the main ways time-varying volatility is modelled in financial markets. One of the difficulties in applications of SV models is that compared with their ARCH counterparts, they are hard to estimate efficiently due to the unobservable nature of the volatility state variable. Furthermore, the likelihood functions of ARCH-type models are more readily available [see, Harvey *et al.* (1994), Kitagawa (1996) and Shephard and Andersen (2009)].

The main approaches to modelling volatility are: conditional volatility, stochastic volatility (SV) and realised volatility (RV). An alternative method, known as implied volatility is based on the Black-Scholes option pricing model. The estimation methods and analysis have been within both the classical and Bayesian frameworks. Harvey (2013) notes that GARCH and SV models have provided the principal means of analysing, modelling and monitoring volatility changes in the last three decades. Shephard (2005) highlights that "the development of this subject has been highly multidisciplinary, with results drawn from financial economics, probability theory and econometrics, blending to produce methods and models which have aided our understanding of the realistic pricing of options, efficient asset allocation and accurate risk assessment". Asset pricing theory in particular is underscored by the idea that higher rewards may be expected when we face higher risks, but these risks change through time in complex ways.

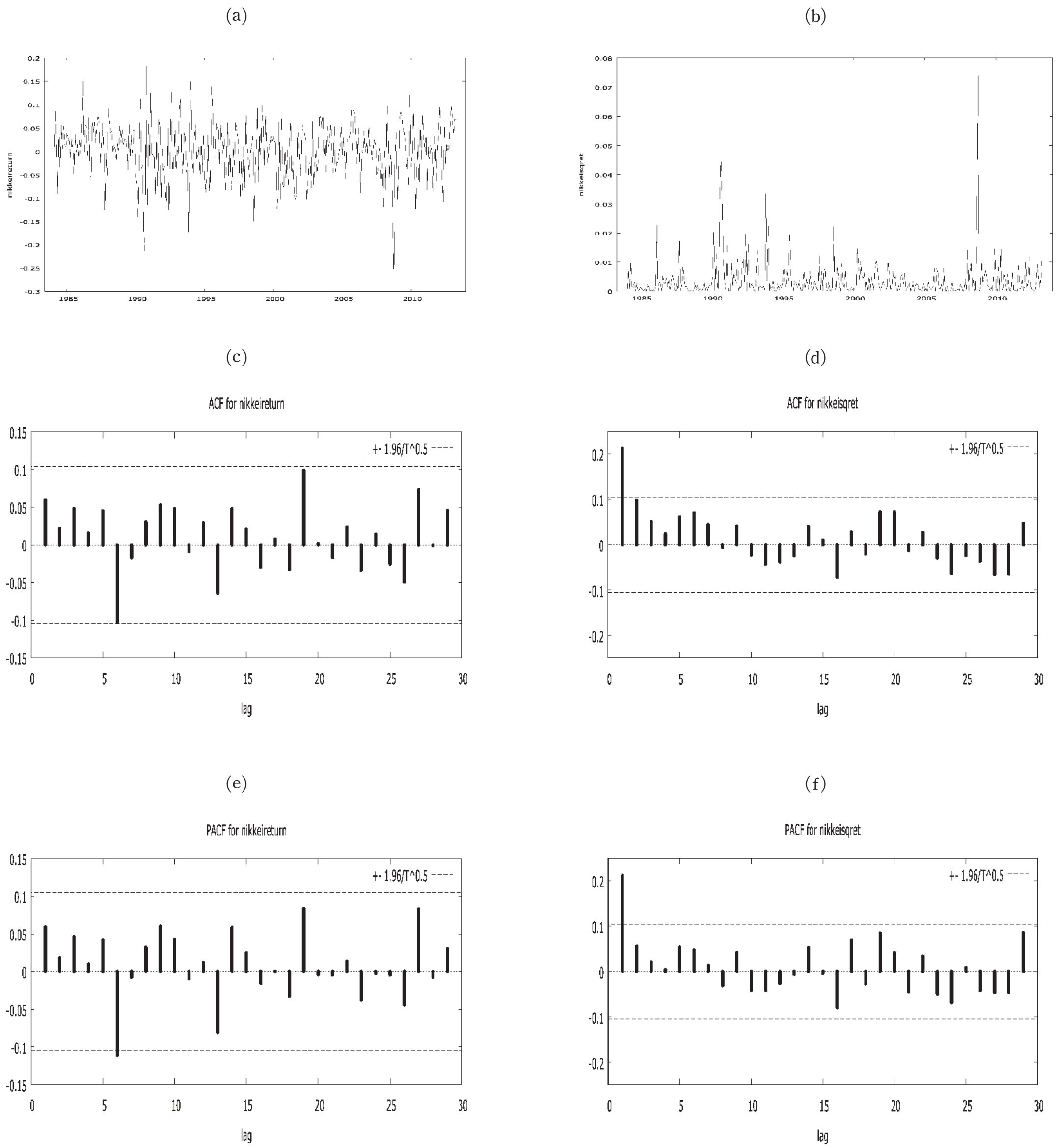
We employ GARCH and SV models using monthly Nikkei-225 index return from January, 1984 to April, 2013 to examine Japan's stock market volatility dynamics. The fit of these models are compared within the context of model selection criteria, persistence coefficients and volatility forecast performance results. The rest of the paper is organised as follows: Section 2 reviews related previous studies and asset returns stylised facts and characteristics, while Section 3 discusses simple and asymmetric GARCH models and basic SV processes and their distributional assumptions. Section 4 presents data, analysed normality of returns and tests for GARCH errors. The estimation results and forecast evaluation of the models are discussed in Section 5 while Section 6 concludes.

2. Previous Studies, Asset Returns Characteristics and Stylised Facts

There has been substantial growth in the literature on the behaviour of stock prices and financial market volatility and since the invention of GARCH and SV models, interest in the analysis of financial series increased due to fluctuations experienced by major financial markets arising from financial crises, market crashes and the need to forecast volatility of financial assets. Most financial models that attempt to describe the evolution of financial assets over time involve the use of returns, instead of prices of assets due to certain statistical advantages [Campbell *et al.*, 1997; Tsay, 2005]. There are several definitions of returns ranging from simple return, log return, absolute return to squared return and the actual time interval is vital in the analysis and in the comparison of returns. However, the basic patterns of simple and log returns tend to be similar. Given monthly stock index data, let P_t denote stock index at the end of month t and P_{t-1} be the stock index at the end of month $t-1$ with r_t denoting the index return. The monthly log-return which is the sum of daily returns over the month can be defined as: $r_t=(lnP_t-lnP_{t-1})$.

Figure 2 plots monthly log-returns and squared log returns, with their autocorrelation (defined as: $\hat{\rho}(r_t,\ r_{t-s})$ with s as the number of lags) and partial autocorrelation (calculated using the Durbin-Levinson algorithm) functions. The persistence shown in the first panel represents volatility clustering (i.e. low values of volatility followed by low values and high values of volatility followed by high values. The correlations in the squares of the Nikkei-225 index returns shown in Figure 2, indicates the presence of conditional heteroscedasticity associated with the return. An important feature of return series is that they are characterised by some attractive statistical properties such as stationarity and ergodicity (Campbell *et al.*, 1997). Problems associated with nonstationarity of many financial time series are minimised with the use of return series thereby enabling us to take advantage of the theory for stationary models. Hence, r_t may be assumed to consist of independent and identically distributed (i.i.d.) random variables (see Terasvirta *et al.*, 2010). The most important volatility stylised facts from the literature include: volatility clustering, fat tails, volatility mean reversion and asymmetry. During periods of financial turmoil asset price return volatilities often exhibit jumps and breaks leading to extreme values and distributions with fatter tails than that of a normal distribution. Given that the fourth order moment exists, Bollerslev (1986) showed that the Kurtosis implied by a GARCH (1, 1) model with normal errors is greater than 3. Zivot (2009) note that most often, a GARCH model with non-normal error distribution is required to fully capture the observed

Figure 2: Monthly Returns, Squared Returns, Autocorrelation and Partial Autocorrelation



fat-tails in return series.

Recent studies have been focusing on assessing the impact of breaks on asset market volatility [e.g. Granger and Hyung (2004), Rapach and Strauss (2008), Babikir *et al.*, (2012)]. Hammoudeh and Li (2008) find that most Gulf-area stock markets are more sensitive to major global events than to local and regional factors. For example, the 1997 Asian financial crisis, oil-price collapse in 1998 and the 9/11 attacks were found to have significant impact and accounting for these large shifts in volatility in GARCH models reduces volatility persistence in stock markets. Liu and Hung (2010) examine volatility forecasting for S&P-100 stock index from 1997 to 2003 and identified the essential source of performance improvements between distributional assumption and volatility specification using distribution and asymmetry-type models. Their results indicate that asymmetry-type models achieve more accurate volatility forecasts. Koopman *et al.* (2005) compares the predictive abilities of realized volatility (RV) models with those of SV and GARCH models for daily returns series [see also Heynan and Kat (1994), Kobayashi and Shi (2005), Franses *et al.* (2008)].

Kim *et al.* (1998) used *Markov chain Monte Carlo* (MCMC) methods to provide a likelihood-based framework for SV models and find that the simple SV model typically fits the data as well as more parameterised GARCH models. Similarly, Koopman and Uspensky (2002) regard SV and SV-*in-mean* (SVM) models as practical alternatives to their ARCH-type counterparts, and present evidence of a negative but weak relationship between returns and volatility. Omori *et al.* (2007) using daily Tokyo stock price index (TOPIX) returns conducted a Bayesian analysis of SV with leverage by extending Kim *et al's* (1998) method that was developed for SV models without leverage. Omori *et al's* approach relies on approximating the joint distribution of the outcome and volatility innovations through a constructed ten-component mixture of bivariate normal distributions. The procedure is found to be fast and highly efficient. Cho and Yoo (2011) investigate volatility of Korean stock market during the Asian currency crisis of 1997–98 and financial crisis of 2007–09. They find that volatility of transitory component of stock return rose during the currency crisis, but did not increase much during the financial crisis.

Bauwens *et al* (2010) using Bayesian estimation with Gibbs sampling algorithm develop an Markov-Switching-GARCH (MS-GARCH) model where both the mean and variance switch in time from one GARCH process to another with the switching governed by a hidden Markov chain. Billio and Pelizzon (2003) use multivariate MS models to analyse whether deregulation, globalisation, financial crises, convergence of EU economies and introduction of the Euro have produced some effects on the return distribution of the world market index and on volatility spillover. Moore and Wang (2007) investigate the stock market volatility for five new EU members using MS model. Their model detects two and three volatility states for the new EU emerging markets. Their key finding is that there is a tendency for emerging stock markets to move from high volatility regime in the earlier transition period into the low volatility regime as they move into the EU. Thus, entry to the EU appears to be associated with a reduction of volatility in unstable emerging markets.

Wang and Theobald (2008) investigate regime-switching behaviour in the returns of six East-Asian emerging stock markets from 1970 to 2004 and examine the features of each regime using an MS model. Their result shows evidence of more than one regime in each of these markets (namely Malaysia, The Philippines and Taiwan where characterised by two regime, while three regimes characterise Indonesia, Korea, and Thailand markets) and the conditional probabilities in each regime provide mixed evidence of the impact of financial liberalisation on return volatility.

3. Methodology and Theoretical Framework

3.1. The Generalized Autoregressive Heteroscedastic (GARCH) Volatility Process

The variance of innovation term is often assumed to be constant in previous studies, but Figure 1 (b) shows that the Nikkei-225 index returns experience periods of large volatility, followed by relative tranquillity (e.g. the 1990 and 2008 periods of high volatility and the 1998-2007 periods of low volatility). Let ε_t denote a stochastic process and Ω_{t-1} the information set through time $t-1$. The ARCH and GARCH (p, q) processes proposed by Engle (1982) and Bollerslev (1986) are given by

$$y_t = X\beta + \varepsilon_t, \quad \varepsilon_t|\Omega_{t-1} \sim N(0, \quad \sigma^2_{t|t-1}), \tag{1}$$

$$\sigma^2_{t|t-1} = Var(\varepsilon_t|\Omega_{t-1}) = E[\varepsilon^2_t|\Omega_{t-1}] = \alpha_0 + \textstyle\sum_{i=1}^{q} \alpha_i\varepsilon^2_{t-i}, \tag{2}$$

$$\sigma^2_{t|t-1} = \alpha_0 + \textstyle\sum_{i=1}^{q} \alpha_i\varepsilon^2_{t-i} + \sum_{i=1}^{p} \beta_i\sigma^2_{t|t-1}, \tag{3}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$ ensuring that the conditional variance $\sigma^2_{t|t-1}$ is always positive and ε_t form a sequence of i.i.d. random variable. The mean equation is specified in (1) while the ARCH and GARCH specifications are given by eqns. (2) and (3) respectively. The GARCH equation is related to (1) wherein $\sigma^2_{t|t-1}$ is conditional variance of the ε_t sequence. The GARCH model is specified as a function of three terms: α_0, ε^2_{t-i} and $\sigma^2_{t|t-i}$. The persistence of $\sigma^2_{t|t-1}$ is captured by $(\alpha+\beta)$ and covariance stationarity requires that $\alpha+\beta<1$, while the unconditional variance is equal to $\alpha_0/(1-\alpha_1-\beta)$. The EGARCH model allows for asymmetric effects between positive and negative asset returns and is given by

$$log(\sigma^2_t) = \alpha_0 + \textstyle\sum_{i=1}^{p}\beta_i log(\sigma^2_t) + \sum_{i=1}^{q}\alpha_i \dfrac{|\varepsilon_{t-i}| + \gamma_i\varepsilon_{t-i}}{\sigma_t}. \tag{4}$$

Note that when ε_{t-i} is positive "good news", the total effect of ε_{t-i} is $(1+\gamma_i)|\varepsilon_{t-i}|$; while when ε_{t-i} is negative "bad news", the total effect of ε_{t-i} is $(1-\gamma_i)|\varepsilon_{t-i}|$. The EGARCH is covariance stationary provided that $\sum_{i=1}^{p}\beta_i < 1$ (Zivot, 2009). If parameters of GARCH models are restricted to sum to one, and the constant term is dropped, it gives the IGARCH model defined as

$$\sigma_t^2 = \sum_{i=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{j=1}^{q} \alpha_i \varepsilon_{t-i}^2. \tag{5}$$

Engle *et al.* (1987) extended the ARCH model to allow the mean of a sequence to depend on its own conditional variance. By introducing standard deviation into the mean equation in (1), we get the ARCH-*in-Mean* (ARCH-M) model expressed as

$$y_t = X\beta + \sigma_{t|t-1} + \varepsilon_t \tag{6}$$

The ARCH-M model can be extended to a GARCH-M specification in applications where the expected return on an asset is related to the expected risk. The estimated coefficient on the expected risk is a measure of the risk-return trade-off. This relation has received much attention in finance, considered consistent with the capital asset pricing model (CAPM). Recent empirical studies have argued that GARCH models tend to overestimate volatility persistence when regime shifts are not taken into account. We therefore modify the above models as follows

$$\sigma_{t|t-1}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t|t-1}^2 + d_i gbfc_t + \lambda_i mcrash_i, \tag{7}$$

where $gbfc_i$ and $mcrash_i$ stands for global financial crisis and stock market crash dummies. The coefficient d_i and λ_i in (7) measures the impact of financial crisis and market crash to conditional variance of returns. We identify shifts in the Nikkei-225 index return volatility and estimate GARCH models augmented with dummy variables related to the sudden change points. Recently, time series research seems to be focusing on SV models, considered as practical alternatives to GARCH models that relied on simultaneous modelling of the first and second moments. SV models avoid problems of simultaneous estimation of the mean and variance respectively (Koopman and Uspensky, 2002).

3.2. The Stochastic Volatility (SV) Process

SV models are used in estimating unobserved volatility and they have different data generating process (i.e. are parameter-driven) compared with observation-driven GARCH models. There are several approaches in the estimation of SV models such as the QML, frequency domain and MCMC methods. Following Harvey *et al.* (1994), we employ the QML approach with parameter estimation implemented via the Kalman filter. The stock market log-return data y_t is assumed to have a constant (zero) mean and a time-varying variance. The basic SV model for r_t is given by:

$$r_t = \mu + exp\left(\frac{1}{2}h_t\right)\varepsilon_t, \quad \varepsilon_t \sim N(0,\ 1), \qquad t = 1, \ldots,\ n \tag{8a}$$

$$h_{t+1} = \gamma + \phi(h_t - \mu) + \eta_t, \quad \eta_t \sim N(0,\ \sigma_\eta^2), \qquad |\phi| \leq 1 \tag{8b}$$

where μ, y_t, h_t are the mean, return and the log-volatility at time t, assumed to follow a stationary process, while ϕ measures persistence in volatility. The disturbances ε_t and η_t are mutually and serially uncorrelated, where $E(log\varepsilon_t^2) = -1.2704$, $var(log\varepsilon_t^2) = \pi^2/2 \cong 4.93$, and the observation error

($log\varepsilon_t^2$) is not standard normal but a $log\chi^2$ distributed error. Equation (8) is a nonlinear model as both h_t and ε_t, in $exp\left(\frac{1}{2}h_t\right)\varepsilon_t$, are stochastic. The unconditional mean of the log-volatility process from (8) is $\gamma_h=(1-\phi)^{-1}\gamma$, interpreted as the long term log variance of stock index return series r_t while the unconditional variance is $\sigma_h^2=(1-\phi^2)^{-1}\sigma_\eta^2$. The stochastic varying variance of the log returns r_t conditional on h_t is given by $\sigma_t^2=E(r_t-\mu)^2=exp\ h_t$.

From (8), the measurement equation describes the relationship between the observations and the latent factors and the state equation describes the dynamic properties of the latent factors while the relative variance is given by $\phi=log(\sigma_\eta^2/\sigma_\varepsilon^2)$. Harvey *et al.* (1994) observe that the SV model is quite close to an EGARCH model in some respects and that although the Kalman filter can be applied to it, it will only yield minimum mean square linear estimators (MMSEs) of the state and future observations. But since the model is not conditionally Gaussian, the exact likelihood cannot be obtained from the prediction errors. Nevertheless, estimates can be obtained by treating ε_t in (8) as though it were Gaussian and maximising the resulting likelihood function (Harvey *et al.*, 1994; Jungbacker and Koopman, 2008).

3.3. Distributional Assumptions

Several studies have shown that financial time series mostly exhibit excess kurtosis and fat tails to the extent that the assumption of normality becomes unrealistic. Since this is important in empirical finance, using a more appropriate distribution would help to account for excess kurtosis [see Bollerslev, Chou and Kroner (1992) and Engle, 2003]. We consider both the student's t and normal distributions for the standardised residuals of returns innovations. From standard distribution theory, the likelihood of any realisation of ε_t is $L_t=(1/\sqrt{2\pi\sigma^2})exp(-\varepsilon_t/2\sigma^2)$; where L_t is the likelihood of ε_t. For normal distribution, the log-likelihood function is defined as

$$l_t=-\frac{1}{2}\log(2\pi)-\frac{1}{2}\log\sigma_t^2-\frac{1}{2}(y_t-X\beta)^2/\sigma_t^2 \tag{9}$$

For student's t-distribution, log-likelihood function is given by

$$l_t=-\frac{1}{2}\log\left(\frac{\pi[\nu-2]\Gamma[\upsilon/2]^2}{\Gamma[(\upsilon+1)/2]^2}\right)-\frac{1}{2}\log\sigma_t^2-\frac{(\nu+1)}{2}\log\left(1+\frac{(y_t-X\beta)^2}{\sigma_t^2(\nu-2)}\right) \tag{10}$$

The degree of freedom $\nu>2$ controls the tail behaviour and $\Gamma(.)$ is the gamma function. The t distribution approaches the normal as $\nu\to\infty$. It places a greater likelihood on large realisations than does the normal distribution. In the SV approach the unknown coefficients to be estimated are $\theta_{SV}=(\phi,\ \sigma_\eta^2,\ \sigma_\varepsilon^2$ and $\gamma)$ respectively and are collected in the parameter vector θ_{SV} by maximising the likelihood function expressed as

$$\log p(y_t|h_t) = -\frac{1}{2}\log(2\pi\sigma^2) - \frac{1}{2}h_t - \frac{y_t^2}{2\sigma^2}\exp(-h_t) \tag{11}$$

where, $\log p(y_t|h_t)$, y_t and h_t are the conditional log density, time series of stock index log-returns, and log-volatility at time t.

4. The Data and Estimation Results

4.1. The Data

Data used comprises monthly Nikkei-225 stock index closing series and returns over the period January, 1984 through April, 2013. The data was retrieved from the Yahoo! Finance website. The stock market average is priced in yen and not adjusted for dividends, see French *et al.* (1987) and Poon and Taylor (1992), who noted that inclusion of dividends affected results only marginally. In examining the series, the continuously compounded return is utilised as it has attractive statistical properties such as stationarity and ergodicity (Campbell *et al.,* 1997 and Tsay, 2005). The monthly simple and mean corrected returns are computed as: $r_t = ln(P_t/P_{t-1})$ and $r_t = [(ln\ P_t - ln\ P_{t-1}) - 1/n\sum_{i=1}^{n}(ln\ P_i - ln\ P_{i-1})]$. Table 1 reports the summary statistics for Nikkei-225 index series, log returns and squared log returns for the sample period and the Jarque-Bera (JB) test for normality. The JB test is computed by $JB = T/6\{\widehat{\text{skew}}^2 + (\widehat{\text{kurt}} - 3)^2/4\}$, where $\widehat{\text{skew}}$ is the skewness, and $\widehat{\text{kurt}}$ denotes the kurtosis of the sample. Under the null hypothesis that the data are i.i.d. normal, JB is asymptotically distributed as chi-square with 2 degrees of freedom (Zivot, 2009).

The return series exhibit negative mean and evidence of fat tails with the kurtosis above 3 while the squared returns obviously has a positive mean. The log returns display evidence of positive skewness and kurtosis that is characterised with tails that are significantly thicker than

Table 1: Summary Statistics of the Monthly Nikkei-225 (N-225) Index and Returns

	N-225 Series	Log Returns	Squared Returns
Mean	16490.08	−0.0008	0.0038
Median	15931.00	−0.0061	0.0016
Maximum	38916.00	0.2722	0.0741
Minimum	7568.42	−0.1828	$3.84e^{-07}$
Std. Dev.	6540.78	0.0617	0.0067
Skewness (Skew)	1.03618	0.5559	5.0722
Kurtosis (Kurt)	3.8296	4.1589	42.0470
Jarque-Bera (JB)	73.0829	37.7202	23803.37
Excess Kurtosis	0.8296	1.1589	39.047
Observation	353	352	351

Notes: Std. Dev. represents standard deviation and N-225 stands for Nikkei-225 stock index. The full sample period is 1984:1–2013:4 covering 353 observations.

that of a normal distribution. This means that if we base our analysis on the normal distribution, we would underestimate the probability that in any one period a large increase or decrease in stock index return can occur. Other important features of the series can be observed in Table 1 and Figures 1, 2, and 3 respectively. One of these features in Figure 1(b) is that as stock prices are falling, volatility becomes higher. Engle (2003) argued that financial price volatility is caused by arrival of new information and that news intensity is high during economic distress or major economic news announcements. This has been corroborated by many studies including Zivot (2009), Koopman, Jungbacker and Hol (2005), Granger and Hyung (2004), etc.

4.2. Checks for Normality

The graphs in Figure 3 showing stock returns distribution and its quantile-quantile (q-q) plots are used to examine and conduct checks for normality of the series among other tests. The q-q graph is designed to be a straight line if returns are normally distributed and will have elongated s-shape if there are more extremes. The normal qq-plots show departure from normality for the returns (for a normally distributed random series, skewness is 0 and kurtosis is 3). When kurtosis is higher than normal, it implies that there is too much concentration of observations around the mean to be consistent with a normal distribution. De Grauwe (2012) reports that models with this feature tend to underestimate the probability of extremely large asset price changes (i.e. they underestimate the probability of large bubbles and crashes). Figure 3(c) compares the actual returns with standard normal and t densities (with 3 degrees of freedom). Accordingly, the Ljung-Box (Q) test (LB-Q) defined by $Q_{LB}=T(T+2)\sum_{j=1}^{k}/(T-j)$ where τ_j^2, and T are the j-th autocorrelation and number of observations is usually implemented on the standardised residuals (devolatised returns) to test for serial correlation and on their squared returns, to test for heteroscedasticity [see Engle (2003), Enders (2012)].

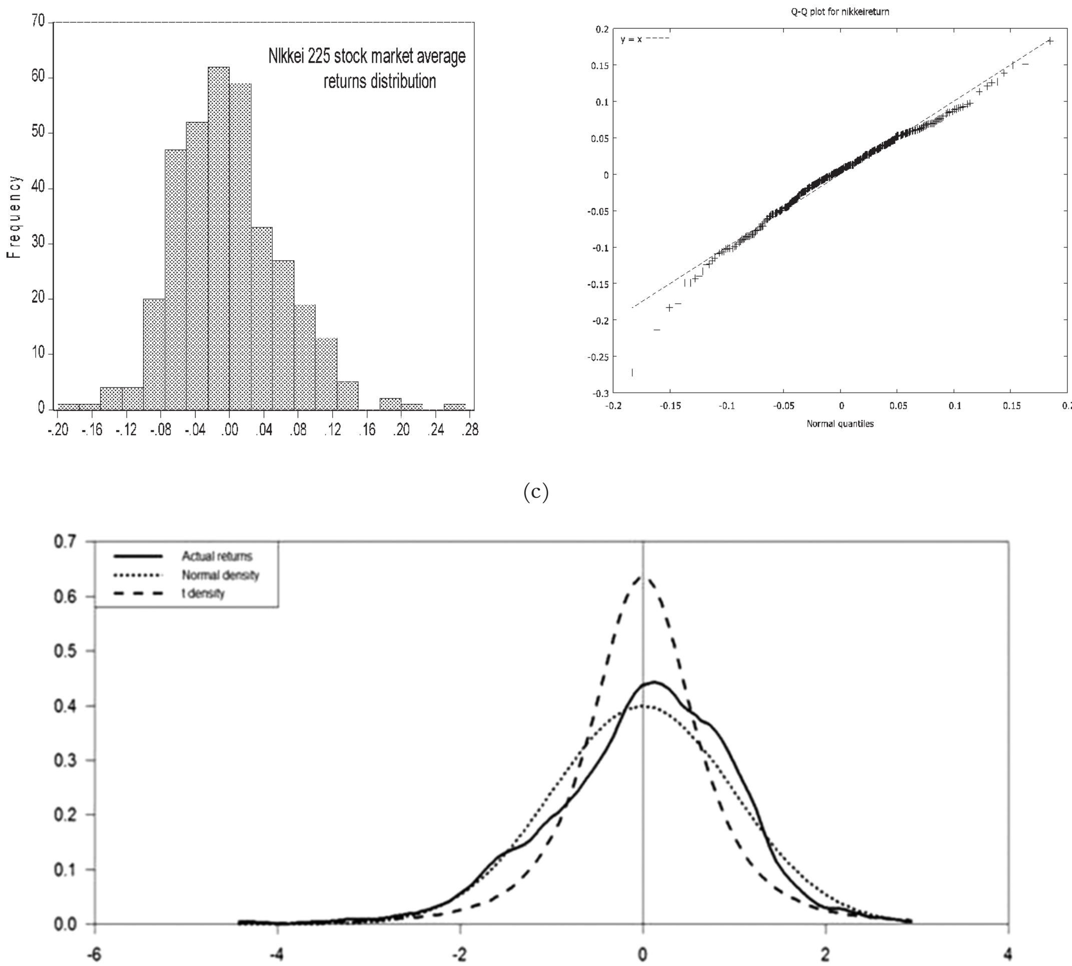
4.3. Tests for ARCH errors

Enders (2010) states that the key feature of ARCH models is for the conditional variance of the series disturbance to constitute an autoregressive moving average (ARMA) process and for the squared residuals to display this characteristic pattern. The correlogram of the squared residuals should be suggestive of such process (see Engle, 1982; Bollerslev, 1986 and Zivot, 2009).

The Lagrange multiplier (LM) test statistic result with 20 lags is presented in Table 2. In testing for conditional heteroscedasticity, we also used the McLeod and Li (1983) test. The LM test computed by $\varepsilon_t^2=\beta_0+(\sum_{s=1}^{q}\beta_s\varepsilon_{t-s}^2)+\nu_t$ regress the squared residuals on lagged squared residuals and a constant up to lag order q[1]. This is less formal than the McLeod and Li test. Using a sample of T residuals, under the null hypothesis of "no ARCH" errors, the test statistic (TR^2) normally converges to a chi-squared distribution with q degrees of freedom (Enders, 2010).

Figure 3: Nikkei 225 Index Returns Distribution and Quantile-Quantile (Q-Q) Plots

(a) (b) (c)



Estimated with 20 lags, the McLeod-Li test statistic is 16.44. Since test statistic (TR^2) is sufficiently large, we however reject the null hypothesis of no ARCH errors.

5. Estimation Results and Discussions

This section present results and examines implications of the estimated volatility models. Tables 5, 6, 7 and 8 present results of ARCH, GARCH, EGARCH, GARCH_D, IGARCH and SV

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1) Even though the LM Test is constructed from an ARCH model, it has been shown that it has power against more general GARCH alternatives, thus it can be used as a general specification test for GARCH effects (Engle, 2002; Zivot, 2009).

Table 2: Autocorrelation and ARCH Characteristics of Nikkei 225 Index Squared Returns

	McLeod-Li (1983) Test		ARCH (LM) test (λ_{LM})		
Series	Test stat (TR^2)	Sig level	Test stat (TR^2)	Probabilities	
Nikkei 225 returns	16.443 (20)	0.6888	17.6785 (20)	Prob F(20,309)	0.6201
				Prob chi sq (20)	0.6086
				F-stat	0.8745

Notes: prob., test stat, and sig level represent probability value, test statistic, and significance level. T represents the number of observations.

models of Japan's Nikkei-225 index returns for the period January, 1984 to April, 2013. The coefficients of the key parameters are positive, satisfying the necessary and sufficient conditions for ARCH-type processes. The significance of the coefficient for ARCH effects in (Table 5, column (2)) suggests that the returns are significant at conventional levels. Furthermore, returns reveal significant GARCH effect, implying that current volatility can be explained to some degree by their lagged volatility in the GARCH (1, 1) model in Table 5. The measure of persistence of the impact of shocks to returns $(\alpha_1+\beta)$ shows high level of persistence ranging from 0.847, 0.849 to 0.850 and the value of 1.000 with respect to the IGARCH model. The models satisfy the covariance stationary condition that $\alpha+\beta<1$. The unconditional variance of returns $\alpha_0/(1-\alpha_1-\beta)$ ranges from 0.0003 to 0.0038. The unconditional standard deviation of returns, $\bar{\sigma}=\sqrt{\alpha_0/(1-\alpha_1-\beta)}$, implied by the GARCH (1, 1) model is 0.060 and is very close to the sample standard deviation of returns earlier reported in Table 1. This is almost the same with the sample standard deviation of returns.

Nelson (1991) and Enders (2012) stressed that constraining $(\alpha+\beta)$ to 1.000 can yield a parsimonious representation of the distribution of an asset's return and this in some respect forces the conditional variance to act like a process with a unit root. The IGARCH (1, 1) results in (Table 5) designed to have $\alpha+\beta=1$ reveals that both α_1 and β are statistically significant at conventional levels. Estimates of GARCH-M model in (Table 5) where σ_1 or σ_t^2 can be added as a regressor to the mean equation, shows an insignificant but negative GARCH-M term (ρ). This implies that rational risk-averse investors require higher expected returns during more volatile periods, consistent with CAPM [see French *et al.* (1987) and Poon and Taylor (1992)]. The σ_1 (0.038) reveals a positive ARCH effect. The coefficient measuring the effect of the financial crisis is 0.004 and is statistically significant at conventional levels. Shocks to ε_{t-1} act to increase conditional variance, leading to periods of tranquillity and volatility. Koopman and Uspensky (2002) argued that if expected volatility and expected returns are positively related and future cash flows are unaffected, the current stock price index should fall.

Since GARCH models cannot capture asymmetric effects of negative or positive returns on volatility, we estimate EGARCH model with results shown in column (4) of Table 5. EGARCH

Table 5: GARCH Estimation Results for Nikkei 225 Stock Index Returns

Dependent variable: Nikkei 225 stock market index return (January, 1984-April, 2013).

(1) Parameter	(2) ARCH	(3) GARCH(1,1)	(4) EGARCH	(5) GARCH_D	(6) IGARCH(1,1)	(7) GARCH-M
Constant	0.0032	0.0049	0.0031	-0.0013	0.0047	0.0051
	(0.0032)	(0.0031)	(0.0032)	(0.0033)	(0.0038)	(0.0119)
ρ	-	-	-	-	-	-0.0579
						(3.1485)
AR(1)	0.0480	0.0606	0.0854	0.0846	0.0687	0.0605
	(0.0547)	(0.0569)	(0.0554)	(0.0438)	(0.0561)	(0.0559)
α_0	0.0032	0.0006	-1.3191	0.0001	-	0.0006
	(0.0004)*	(0.0005)	(0.5574)**	(0.0005)**		(0.0005)
α_1	0.1379	0.1358	0.0685	0.0676	0.0617	0.1357
	(0.0703)**	(0.0688)**	(0.0944)	(0.0576)	(0.0625)	(0.0744)
β	-	0.7141	0.7786	0.5818	0.9383	0.7145
		(0.1769)*	(0.0971)*	(0.1850)**	(0.0625)*	(0.1859)*
γ	-	-	-0.2220	-	-	-
			(0.0706)*			
$gbfc_t$	-	-	-	0.0049	-	-
				(0.0023)**		
$mcrash_t$	-	-	-	0.0071	-	-
				(0.0081)		
$(\alpha_1+\beta)$	-	0.8499	0.8471	0.6494	1.0000	0.8502
$\alpha_0/(1-\alpha_1-\beta)$	-	0.0038	-	0.0003	-	0.0031
t-dist. d.f	13.9393	10.8802	19.9360	-	7.8759	10.8802
	(10.7513)	(6.1997)	(24.0956)		(2.5774)**	(6.1997)
loglik	485.824	489.549	495.692	491.5846	484.197	489.550
AIC	-2.753	-2.769	-2.798	-2.6918	-2.738	-2.763
SBC	-2.709	-2.714	-2.732	-2.6918	-2.683	-2.697
Ljung-Box(Q)	28.92	28.92	28.92	28.92	28.92	28.92

Notes: numbers in parenthesis indicate the standard errors. *,**, indicates significance at 1%, and 5% levels. Log Lik, SBC, t-dist. d.f, AR (1) and Q stands for Log likelihood, Akaike and Schwarz information criteria, t-distributed degree of freedom and Ljung-box Q statistics. Estimation methods: Maximum Likelihood (ML)-BFGS and BHHH for GARCH_D.

models assume that volatility could respond asymmetrically to past forecast errors. The α_1 is insignificant but β and the leverage effect terms, γ (which measures asymmetry of shocks) are all significant. The tendency for volatility to decline when returns rise and to rise when returns fall is referred to as the leverage effect. Given the value of σ_{t-1}, a one-unit increase in ε_{t-1} will induce a change in the logarithm of the conditional variance by 0.21 [0.24−0.03=0.21]. However, if ε_{t-1} falls by one unit, the logarithm of conditional volatility declines by −0.267 [−0.236 −0.030]. The implication is that "good news" has smaller effect on the conditional volatility than "bad news". Black (1976), French *et al.* (1987) and Schwert (1989) argued that leverage alone cannot account for the magnitude of the negative relationship. Over the years, yen appreciation has led to declines in the Nikkei-225 stock average and recently with yen depreciation due to monetary easing by the BOJ; the stock market has begun to show signs of recovery. This is evident as Japan engages in substantial foreign trade and much of these exports and imports are sensitive to exchange-rate changes. Since major exporting companies operate in the market, exchange-rate fluctuation usually affects their equity and stock prices.

From Table 5, column (5), given that α_1 is not statistically different from zero (0.0679), we can conclude that there is no much volatility clustering evidence from the estimated GARCH model with dummies (GARCH_D). However, there exist a sharp volatility break as the intercept of the variance equation was 0.0001 before December, 1989 and declined to −0.0069 (i.e. 0.0001−0.0071) during stock market crash of the late-1980s and a drop to −0.0048 (i.e. 0.0001−0.0049) around September, 2008 associated with the impact of the global financial crisis (GFC). Estimated degrees-of-freedom (d.o.f) for the shape of t distributed errors are shown in Table 5. Asymptotically, the t distribution approaches the normal as the d.o.f increase and a large value for the d.o.f estimates indicate that the series are approaching normally distributed errors (see the EGARCH results). Meanwhile, Figure 4 shows Japan's stock returns volatility from 1984 to 2013

Figure 4: Nikkei-225 Stock Market Index Returns Volatility Dynamics (GARCH Models)

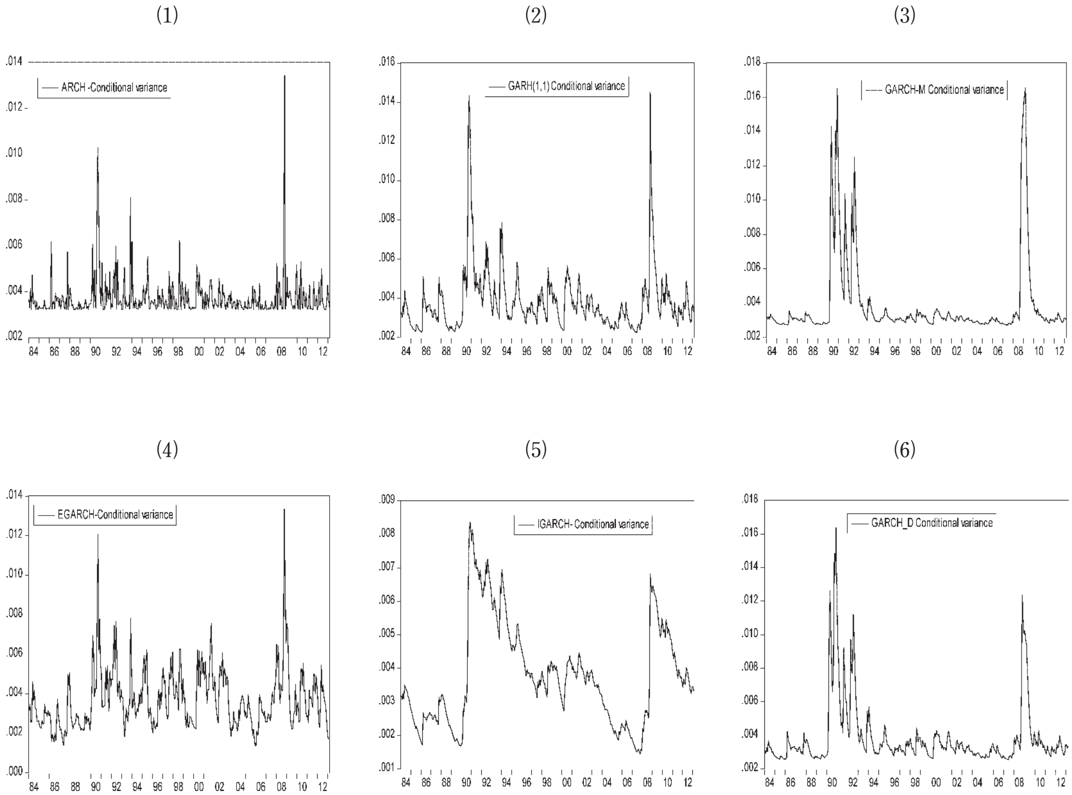


Table 6: Model Selection Criteria for Estimated GARCH (p, q) for Nikkei-225 Index Returns

(q, p)	AIC	SBC	Likelihood
(1,0)	-2.759	-2.726	487.2879
(2,0)	-2.762	-2.718	488.8091
(3,0)	-2.766	-2.711	490.4007
(4,0)	-2.760	-2.694	490.4173
(5,0)	-2.759	-2.682	491.2332
(1,1)	-2.774	-2.730	490.7963
(1,2)	-2.768	-2.713	490.8431
(2,1)	-2.768	-2.713	490.8617
(2,2)	-2.763	-2.697	490.8625
(3,1)	-2.763	-2.697	490.8695

from fitted GARCH models. Notice the upward trajectory of returns volatility around 1986, 1987, 1990, 1998, and 2008. These are periods associated with significant global financial events from the late 1980s market crash to the GFC of 2007–2009.

On selecting the most appropriate models and assessing their adequacy and fit, the Akaike information criteria (AIC), Schwartz Bayesian criteria (SBC) and log-likelihood statistics were examined. Table 6 presents model selection criteria results of ARCH/GARCH models with GARCH (0, 5) having the highest log-likelihood, although it has more parameters. The three reasonable models from Table 5 seem to be the IGARCH, GARCH-M and EGARCH models. Not only does the EGARCH model capture leverage effect, it also fits the data better than the other models with additional explanatory variables and specifications.

However, as suggested by Bollerslev, *et al.* (1994) caution needs to be exercised in interpreting results from these information criteria as their statistical properties are largely unknown in the ARCH context. The maximised value of the log-likelihood function is larger for the EGARCH model compared with the GARCH or IGARCH models respectively. Meanwhile, the maximised fitted likelihood in respect of the EGARCH models is higher than that of the SV model even though the EGARCH model is less parsimonious. However, it should be noted that this conclusion is in relation to the simplest SV model that is assumed to be gaussian. The EGARCH models seem to have higher log-likelihood functions than the other models. The near-unity level of persistence with respect to returns is consistent with findings from the SV, GARCH and the finance literature.

Table 7 present random walk SV, ARMA (1, 1), basic SV model estimates and diagnostics comprising a Ljung-Box-(Q) test, a Jarque-Bera (JB) normality test and a Goldfeld–Quandt test for heteroscedasticity. The H-statistic defined as: $H(h)=\sum_{t=n-h+1}^{n} e_t^2 / \sum_{t=d+1}^{d+h} e_t^2$ with the standardized prediction errors denoted by e and d the number of initial diffuse elements, tests whether the variance of the first 117 elements of the residual is unequal to the variance of the last 117 elements of the residuals. The high value for the normality statistic (83.03) further confirms that

Table 7: Stochastic Volatility Model Estimates for Nikkei-225 Returns (1984:1-2013:4)

Parameter	RW SV	ARMA(1,1)	Basic SV
ϕ	-	-	0.8937*
			(0.0534)
$\hat{\sigma}^2_\varepsilon$	-	-	4.6762*
			(0.3443)
$\hat{\sigma}^2_\eta$	0.0045	-	0.0539*
	(0.0057)		(0.0038)
γ	-	-	-0.6055
			(0.3031)**
constant	-	-5.6995*	-
		(0.1144)	
AR(1)	-	-0.4199	-
		(0.7585)	
MA(1)	-	0.3656	-
		(0.7780)	
Log lik.	-798.853	-777.68	-776.9538
Q(29-2)	33.97(0.24)	35.109(0.415)	32.75 (0.169)
Normality	493.88 (0.000)	-	81.68 (0.000)
H (117)	1.55(0.018)	-	1.04
SBC	4.57	-	4.494
Observations	352	352	352

Notes: RW, Log lik., AIC and SBC stands for random walk, Log likelihood, Akaike and Schwarz Bayesian criteria. Estimation method for the SV models is BFGS while the ARMA (1, 1) model of Nikkei-225 returns was by the least squares Gauss-Newton technique. Monthly data from 1984:01 to 2013:04.

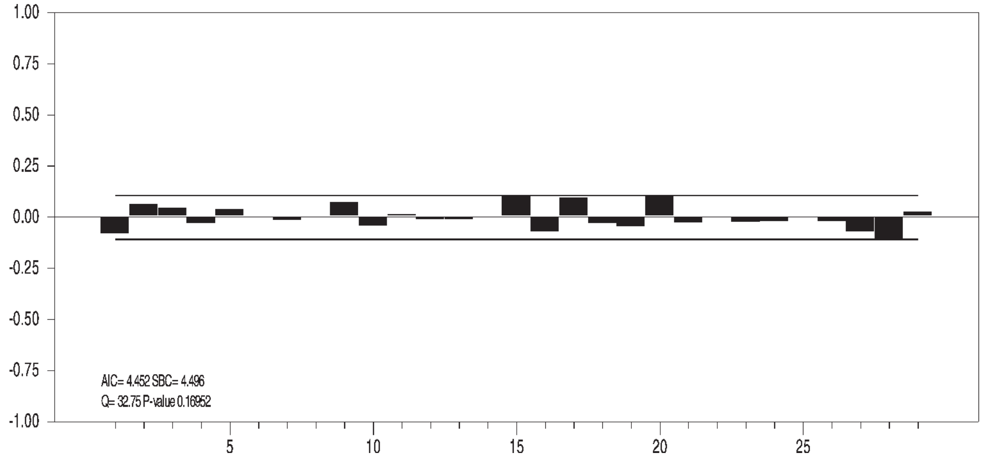
innovations of the SV model are not Gaussian. The normality statistic defined as $N = T\{\widehat{\text{skew}}^2/6 + (\widehat{\text{kurt}} - 3)^2/24\}$, tests whether the skewness and kurtosis of the distribution of the residuals comply with a normal distribution. Since 81.68 is greater than the critical value $\chi^2_{(3,0.05)} = 7.81$, the null hypothesis of normality for residuals is rejected.

Figures 5(a) and 5(b) present the graphs of autocorrelations and regression residuals of the estimated SV model. On checks for serial correlation of the innovation, the Q statistic based on the first 29 sample autocorrelation was Q (29)=32.75 and since this value is smaller than $\chi^2_{(29,0.05)} = 43.77$, evaluated as a whole the first 29 autocorrelations does not significantly deviate from zero (meaning that the null hypothesis of independence should be accepted). The two horizontal lines in the correlogram are the 95% confidence limits $\pm 2/\sqrt{T} = \pm 2/\sqrt{352} = \pm 0.1066$. The independence between random normally distributed errors reflects the fact that all autocorrelations [of which the first 29 are graphed in figure 5(a)] are very close to zero and do not exceed the confidence limits.

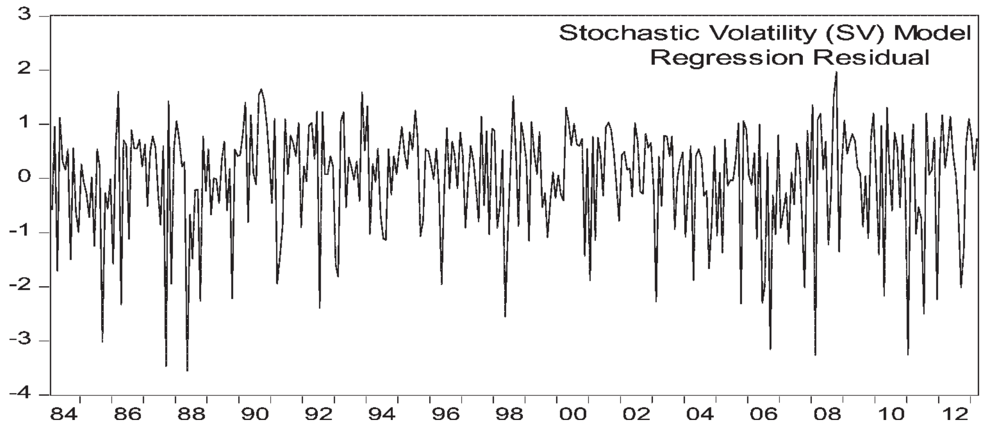
The clustering in periods with low and high volatility is also clearly visible in the returns residual series. We applied the QML approach to obtain parameter estimates through the

Figure 5: The Stochastic Volatility (SV) Model Diagnostics and Recursive Residuals

(a)



(b)

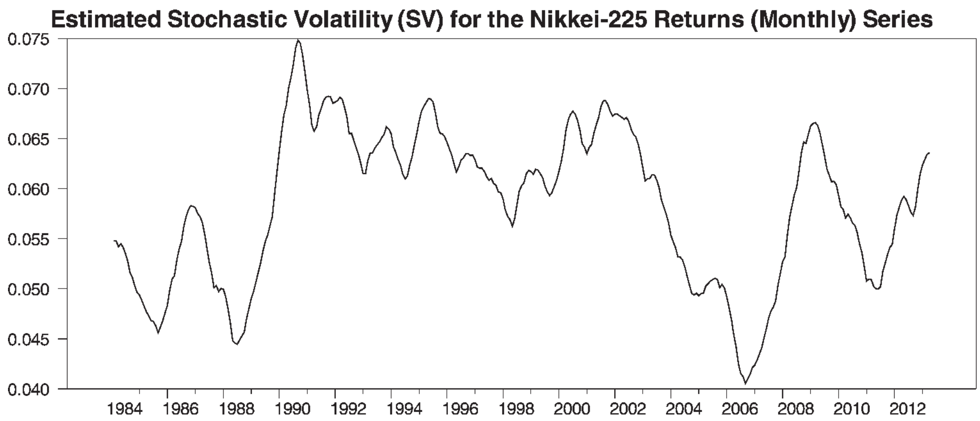


Kalman filter. Harvey *et al.* (1994) argued that the method works well for the sample sizes encountered in applications to financial time series. We employ Kalman filtering to estimate parameters $\theta_{SV}=(\phi,\ \sigma^2_\eta,\ \sigma^2_\varepsilon,\ \gamma)$. Results are presented in Table 7 (the estimated coefficients are reported with their standard errors). In handling the initial conditions, we use the flexible presample-ergodic option which analyses the transition matrix and determines its roots. The SV model was fitted to the difference of the logarithm of the Nikkei-225 index return with the mean subtracted to ensure that there are no returns identically equal to zero. The results and the estimated volatility indicate the salient features in the volatility dynamics of Japan's stock market [see Figures 6(a) and (b)]. The volatility increases in 1987, 1989, the slowdown in 2006, to another rise from 2006 to 2010 can be observed from the stochastic volatility trends.

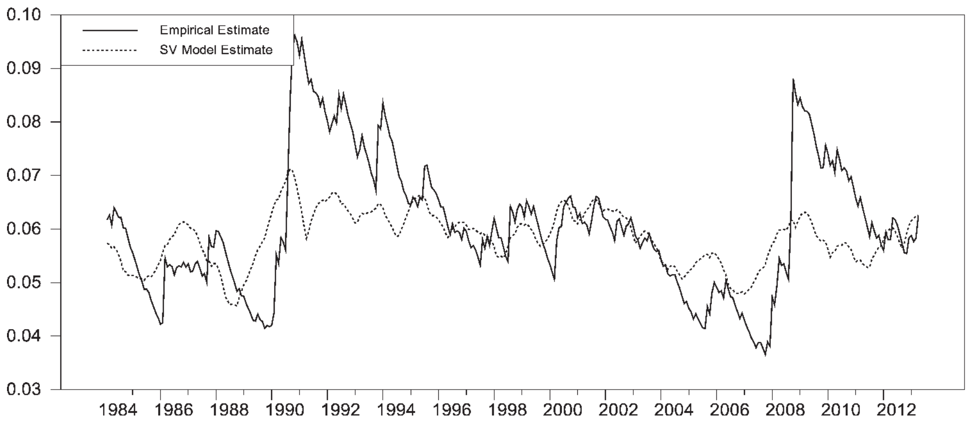
The major difference between the estimated volatilities of the GARCH and SV models is that trends are noisier in the case of the former, although the main patterns are similar and the estimated ϕ is 0.89. The $\widehat{\sigma}^2_\eta$ is 0.0539 and σ^2_ε is 4.68 which are both significant (referred to as hyperparameters). The log-likelihood of −776.95 is maximised using the Broyden-Fletcher-Goldfarb-Shannon (BFGS) (derivative-based) hill climbing technique. The resulting filtered and smoothed estimates in Figures 6 ((a) and (b)) show the smoothed estimates trend (broken line) with the empirical (filtered) estimates (solid line). The graph shows the expected feature of the filtered volatility and the smoothed volatility and somewhat having a similar trend. However, note the similarity in the trend of the estimated SV model (Figure 6(a)) with the IGARCH model (Figure 4(5)).

Figure 6: Nikkei-225 Stock Index Returns Volatility Dynamics (SV Model)-($exp(h_{t|T}/2)$)

(a)



(b)



6. Volatility Forecasting Methodologies and Prediction

6.1. GARCH Model Forecasts

From GARCH (1, 1) model (3) where $\varepsilon_t = z_t\sigma_t$ such that $z_t \sim iid(0, 1)$ with a symmetric distribution given $t=1, 2, 3, ..., T$. The optimal forecast of σ^2_{T+k} given information at time T is $E_T(\sigma^2_{T+k})$ and can be computed recursively. For $k=1$,

$$\begin{aligned} E_T(\sigma^2_{T+1}) &= \alpha_0 + \alpha_1 E_T(\varepsilon^2_T) + \beta_1 E_T(\sigma^2_T) \\ E_T(\sigma^2_{T+1}) &= \alpha_0 + \alpha_1 \varepsilon^2_T + \beta_1 \sigma^2_T \end{aligned} \tag{12}$$

Similarly, for $k=2$,

$$\begin{aligned} E_T(\sigma^2_{T+2}) &= \alpha_0 + \alpha_1 E_T(\varepsilon^2_{T+1}) + \beta_1 E_T(\sigma^2_{T+1}) \\ E_T(\sigma^2_{T+2}) &= \alpha_0 + (\alpha_1 + \beta) E_T(\sigma^2_{T+1}) \end{aligned} \tag{13}$$

Since $E_T(\varepsilon^2_{T+1}) = E_T(z^2{}_{T+1}\sigma^2_{T+1}) = E_T(\sigma^2_{T+1})$. In general, for $k \geq 2$

$$E_T(\sigma^2_{T+k}) = \alpha_0 + (\alpha_1 + \beta) E_T(\sigma^2_{T+k-1})$$

$$E_T(\sigma^2_{T+k}) = \alpha_0 \textstyle\sum_{i=0}^{k-1} (\alpha_1 + \beta)^i + (\alpha_1 + \beta)^{k-1} (\alpha_1 \varepsilon^2_T + \beta \sigma^2_T) \tag{14}$$

The forecasting algorithm (14) produces forecasts for the conditional variance σ^2_{T+k} and the forecasts for the conditional volatility σ_{T+k} is the square root of the forecast for σ^2_{T+k}.

6.2. Stochastic Volatility Model Forecasts

Given $y_t = \mu_t + \sigma_t\varepsilon_t$, $h_t = \phi h_{t-1} + \sigma_t\eta_t$, $\sigma^2_t = \sigma^{*2}\exp(h_t)$; ε_t, $\eta_t \sim NID(0, 1)$, the one-step ahead volatility forecast for the SV model defined above is computed as:

$$E(\sigma^2_{T+1|T}) = \exp(\ln \hat{\sigma}^{*2} + h_{T+1|T} + \frac{1}{2} P_{T+1|T}) \tag{15}$$

where $h_{T+1|T}$ is the estimator of h_{T+1} using all observations available at time T with estimation error variance $P_{T+1|T}$. Further, define $\sigma^2_{T+1,T+N}$ as the volatility over the period $T+1, ..., T+N$, then the N-step SV volatility forecast is given by:

$$\begin{aligned} E(\sigma^2_{T+1,T+N|T}) &= \textstyle\sum_{j=1}^{N} \exp(\ln \hat{\sigma}^{*2} + h_{T+j|T} + \frac{1}{2} P_{T+j|T}) \\ &= \hat{\sigma}^{*2} \exp(h_{T+1|T} + \frac{1}{2} P_{T+1|T}) + \\ &\quad \hat{\sigma}^{*2} \textstyle\sum_{j=2}^{N} \exp\left[\hat{\phi}^{j-1} h_{T+1|T} + \frac{1}{2} \left(\hat{\phi}^{2(j-1)} P_{T+1|T} + \sum_{i=0}^{N-2} \hat{\phi}^{2i} \hat{\sigma}^2_\eta \right) \right] \end{aligned} \tag{16}$$

where $\hat{\phi}$ and $\hat{\sigma}^2_\eta$ are the maximum likelihood estimates of ϕ and σ^2_η respectively.

6.3. Forecast Evaluation

Given that the forecast sample is $j = T+1, T+2, ..., T+h$, and denoting the actual and forecasted value in period t as y_t and $\hat{y}_t$, respectively. Table 8 presents the mean absolute error

(MAE), mean squared error (MSE), the root mean squared error (RMSE) and the Theil's U statistics are computed by the formulas given by:

$$RMSE = \sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h} \tag{17}$$

$$MAE = \sum_{t=T+1}^{T+h} |\hat{y}_t - y_t| / h \tag{18}$$

$$Theil's\ U = \frac{\sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h}}{\sqrt{\sum_{t=T+1}^{T+h} \hat{y}_t^2 / h} + \sqrt{\sum_{t=T+1}^{T+h} y_t^2 / h}} \tag{19}$$

where $\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2$ is the sum of forecast errors, y_{i0} is the naïve forecast, $\hat{y}_{it}$ is the forecast at step t from the ith Theil start and y_t is the actual value of the dependent variable. The RMSE and MAE depend on the scale of the dependent variable and are often used as relative measures to compare forecasts for the same series across different models. The model which produces the smallest values of the forecast evaluation statistics is judged to be the best model (Zivot, 2009). The Theil inequality coefficient lies between zero and one, with zero indicating a perfect fit. It allows for comparison with the random walk (naïve) model, and $U<1$ indicates that the model being used is better than the naïve model.

From Table 8, the forecast evaluation results for the Nikkei-225 returns reveals that among the

Table 8: Forecast Evaluation Results for the Nikkei-225 Return Series

Horizon	1	3	6	12
		ARCH		
MAE	0.04737	0.04758	0.04732	0.04732
MSE	0.00224	0.00226	0.00224	0.00224
RMSE	0.06165	0.06184	0.06160	0.06154
Theil's U	0.93984	0.94040	0.94034	0.93884
		GARCH (1, 1)		
MAE	0.04742	0.04763	0.04750	0.04736
MSE	0.00225	0.00226	0.00226	0.00224
RMSE	0.06164	0.06183	0.06168	0.06152
Theil's U	0.93681	0.93742	0.93744	0.93589
		EGARCH (1, 1)		
MAE	0.04732	0.04752	0.04726	0.04727
MSE	0.00224	0.00225	0.00223	0.00223
RMSE	0.06168	0.06187	0.06164	0.06157
Theil's U	0.94970	0.95011	0.95000	0.94881
		SV Model		
MAE	5.58394	5.60687	5.64245	5.71063
MSE	31.71487	31.8802	32.1469	32.6608
RMSE	5.63161	5.64626	5.66982	5.71496

Notes: The table reports the mean absolute error (MAE), mean squared error (MSE) the root mean squared error (RMSE) and the Theil's U statistics at forecast horizons of 1, 3, 6, and 12 months

GARCH models (without breaks), the EGARCH has a better forecast performance than the ARCH (1, 1) and GARCH (1, 1) models. In turn the GARCH models outperformed the basic SV model as the MAE is quite lower than those of the SV model at all the forecast horizons. One possible reason for the GARCH models outperforming the SV could be due to the behaviour of stock index returns for almost 20 years before the financial crisis of 2008 which was largely stable and not exhibiting substantial time variation. Further, volatility predictions over shorter term horizons for the GARCH models are generally less accurate than over longer term horizons, which is consistent with Heynan and Kat (1994).

7. Summary and Conclusion

We examine the volatility of Nikkei-225 stock returns within the framework of both ARCH, GARCH and SV models and estimates relevant parameters associated with the behaviour of returns over the periods. In this vein, we restricted our analysis to the estimation of simple ARCH, GARCH and basic SV models using the QML approach with parameter estimation carried out through the Kalman filter and smoother. SV models have strong foundation in the financial theory on option pricing and connection with the state space methods and they represent another approach to modelling time- varying volatility. In this approach, the conditional covariance matrix depends on an unobserved latent process and not on past observations as in the ARCH-GARCH models. The results of the GARCH parameters were compared with the estimation results obtained from their SV counterparts. The findings can be summarised as follows.

Firstly, the GARCH-M model revealed evidence of a weak and statistically insignificant negative relationship between risk, return and the volatility process. Secondly, we observe that the volatility persistence parameter in the SV model which indicates volatility clustering is comparable with the persistence measure of GARCH models. The stock market average experienced steep rises, declines and sustained periods of fluctuations since the late 1980s. We also found a similarity in the trend of the estimated SV model [Figure 6(a)] with that of the IGARCH model. The paper however has a number of shortcomings as the linear Gaussian techniques with respect to SV model applied in this paper only offer approximate maximum likelihood estimates of the relevant parameters. Nor does our analysis enable us to evaluate the role of expectations, value of the yen and policy actions of the BOJ and the Ministry of Finance on Japan's stock index returns.

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