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Identification of Lifted State-space Models for a Class of Dual-rate Systems from Input-ouput Data

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Abstract: This paper solves the problem of estimating lifted state-space models for a class of dual-rate systems in which input sampling period is an integer multiple of output sampling period from the input-output data directly. We first derive the lifted model of this class of dual-rate system to prove that the lifted model can be an indirect model for the original dual-rate system. Then based on the input-output data we estimate the lifted model using N4SID algorithms.

Keywords: Multirate system, Dual-rate system, Lifted state-space model, N4SID algorithms

1. Introduction

Due to sensors and actuators speed limits, some systems input sampling rate is different from output sampling rate. We call such a system a multirate system. For decades, the study of multirate systems has been very active, including control problem[1), fault detection and isolation problem[2)] and so on. M. Araki and K. Yamato solved the problems of state-space description, transfer characteristics, and Nyquist criterion of multivariable multirate system[3)].

In fact researchers always study dual-rate systems and generalize dual-rate systems into multirate system. For most conventional algorithms, they cannot be applied to dual-rate systems directly. F. Ding and T. Chen derived lifted state-space models for dual-rate systems using the lifting technique and estimated the lifted model parameters with the hierarchical identification algorithms in their related work[4)]. Although they got good results in their work, there are still some problems:

- Like F. Ding and T. Chen stated in the conclusion of their work, there still exists the problem that under what conditions the paramerer estimation by the hierarchical identification algorithms is convergent;
- The hierarchical identification algorithms still need precise priori information of original dual-rate systems to decide the orders of lifted state-space models .

Comparing with the hierarchical identification algorithms, numerical algorithms for subspace state space system identification (N4SID) algorithms[5)] can avoid a priori parametrization and can decide the order of the model through inspection of the dominant singular values of a matrix that is calculated during the identification. As an off-line method, N4SID algorithms do not suffer from no guaranteed convergence and sensitivity to initial estimates. There is no difference between zero and non-zero initial states for N4SID algorithms.

With lifting technique, we can derive a lifted state-space model for a dual-rate system. Under most conditions the lifted state-space models have to satisfy with causality constraints which means the input data in time t must be decided by the output data in time t and(/or) before time t. The qualification that the output matrices of lifted state-space models are lower block-triangular matrices ensures that lifted stated-space models correspond to causality. Although N4SID algorithms have many merits, it is difficult to ensure output matrices to be lower block-triangular matrices using N4SID algorithms directly.

But when the input sampling period is an integer multiple of the output sampling period the lifted state-space model for such a dual-rate system has no causality constraints. First we will prove it through deriving the lifted state-space model for such a dual-rate system in this paper.

Then we will solve the identification problems of lifted state-space models for the special dual-rate systems. In this paper we assume there is no priori information about original dual-rate systems and identify the lifted state-space models only from input-output data using N4SID algorithms.

This paper is organized as follows: The problem description can be found in section 2. The lifted state-space model is derived in section 3. Section

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4 describes the identification problems. Section 5 presents an illustrative example for the proposed algorithms in this paper. Finally conclusions are given in section 6.

2. Problem Formulation

The focus of this paper is dual-rate system with disturbance depicted in **Fig. 1**. As mentioned in section 1, in this paper the input of the dual-rate system updating period is an integer multiple of the output sampling period.

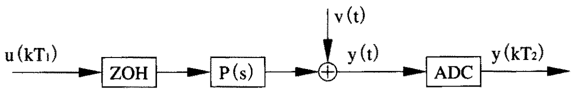

Fig. 1 A dual-rate system.

$u(kT_1)$ is the input of the dual-rate system. $P(s)$ is continuous linear time-invariant(LTI) process. The input $u(t)$ of $P(s)$ is the output of zero-order holder ZOH. The output of $P(s)$ is corrupted by the noise $v(t)$ and becomes $y(t)$. And then we can get the dual-rate system output by an A/D converter ADC.

In the paper, we assume $P(s)$ is a linear time-invariant (LTI) system with the following state-space representation:

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_1 u(t) \\ y(t) = C_1 x(t) + D_1 u(t) + v(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^r$ is the input vector, $y(t) \in R^m$ is the output vector and $v(t) \in R^m$ is the noise vector. A_1, B_1, C_1 and D_1 are matrices of appropriate dimensions.

In this paper we need solve two problems:

- Derive the lifted state-space model for the dual-rate system described above;
- Identify the lifted state-space model from input-output data using N4SID algorithms.

3. Lifted State-space Model

Basing on the lifting technique, we assume the lifted output vector $\underline{y}(kT_1) \in R^{m\times p}$ and the lifted noise vector $\underline{v}(kT_1) \in R^{m\times p}$:

$$\underline{y}(kT_1) = \begin{pmatrix} y(kT_1) \\ y(kT_1 + T_2) \\ \vdots \\ y(kT_1 + (p-1)T_2) \end{pmatrix} \tag{2}$$

$$\underline{v}(kT_1) = \begin{pmatrix} v(kT_1) \\ v(kT_1 + T_2) \\ \vdots \\ v(kT_1 + (p-1)T_2) \end{pmatrix} \tag{3}$$

With the lifted output vector and the lifted noise vector we can derive a lifted state-space model from (1):

$$\begin{cases} x((k+1)T_1) = A_2 x(kT_1) + B_2 u(kT_1) \\ \underline{y}(kT_1) = C_2 x(kT_1) + D_2 u(kT_1) + \underline{v}(kT_1), \end{cases} \tag{4}$$

In (4), $x(kT_1) \in R^n$,$u(kT_1) \in R^r$, $\underline{y}(kT_1) \in R^{m\times p}$ and $\underline{v}(kT_1) \in R^{m\times p}$.

To obtain the mapping relationship from (1) to (4) we need first assume a model

$$\begin{cases} x((k+1)T_2) = A_m x(kT_2) + B_m u(kT_2) \\ y(kT_2) = C_1 x(kT_2) + D_1 u(kT_2) + v(kT_2) \end{cases} \tag{5}$$

We can calculate A_m and B_m in (5) by the following

$$A_m = e^{A_1 T_2} \tag{6}$$

$$B_m = \int_0^{T_2} e^{A_1 t} dt B_1. \tag{7}$$

We can get the model by discretizing $P(s)$ with the sampling period T_1 and then the input $u(kT_2)$ will hold

$$\begin{aligned} u(kT_1) &= u(kT_1 + T_2) \\ &= \cdots = u(kT_1 + (p-1)T_2). \end{aligned} \tag{8}$$

Replace k in (5) with kp and note that $T_1 = pT_2$, we have

$$\begin{cases} x(kT_1 + T_2) = A_m x(kT_1) + B_m u(kT_1) \\ y(kT_1) = C_1 x(kT_1) + D_1 u(kT_1) + v(kT_1). \end{cases} \tag{9}$$

Hence it is not difficult to get

$$\begin{aligned} x((k+1)T_1) &= x(kT_1 + pT_2) \\ &= A_m^p x(kT_1) + A_m^{p-1} B_m u(kT_1) \\ &\quad + A_m^{p-2} B_m u(kT_1 + T_2) \\ &\quad + \cdots + B_m u(kT_1 + (p-1)T_2) \end{aligned} \tag{10}$$

We also can perform (10) as follows:

$$\begin{aligned} x((k+1)T_1) &= A_m^p x(kT_1) \\ &\quad + \textstyle\sum_{i=1}^{p} A_m^{i-1} B_m u(kT_1) \end{aligned} \tag{11}$$

Similarly, we have

$$\begin{aligned} & y(kT_1+iT_2) \\ &= C_1x(kT_1+iT_2)+D_1u(kT_1+iT_2) \\ &= C_1A_m^ix(kT_1+iT_2)+C_1A_m^{i-1}B_mu(kT_1)+ \\ &\quad C_1A_m^{i-2}B_mu(kT_1+T_2)+\cdots+ \\ &\quad C_1A_mB_mu(kT_1+(i-2)T_2)+ \\ &\quad C_1B_mu(kT_1+(i-1)T_2)+D_1u(kT_1+iT_2) \end{aligned} \tag{12}$$

Based on (2) and (12), we have

$$\begin{aligned} \underline{y}(kT_1) &= \begin{pmatrix} y(kT_1) \\ y(kT_1+T_2) \\ \vdots \\ y(kT_1+(p-1)T_2) \end{pmatrix} \\ &= \begin{pmatrix} C_1 \\ C_1A_m \\ C_1A_m^2 \\ \vdots \\ C_1A_m^{p-1} \end{pmatrix} x(kT_1+iT_2)+ \\ & \begin{pmatrix} D_1 \\ C_1B_m+D_1 \\ \vdots \\ C_1(A_m^{i-1}+A_m^{i-2}\cdots+A_m+I)B_m+D_1 \\ \vdots \\ C_1(A_m^{p-1}+A_m^{p-2}\cdots+A_m+I)B_m+D_1 \end{pmatrix} u(kT_1) \end{aligned} \tag{13}$$

Finally the mapping relationship from (1) to (4) will be

$$A_2 = A_m^p \tag{14}$$

$$B_2 = \sum_{i=1}^{p} A_m^{i-1}B_m \tag{15}$$

$$C_2 = \begin{pmatrix} C_1 \\ C_1A_m \\ C_1A_m^2 \\ \vdots \\ C_1A_m^{p-1} \end{pmatrix} \tag{16}$$

$$D_3 = \begin{pmatrix} D_1 \\ C_1B_m+D_1 \\ \vdots \\ C_1(\sum_{i=0}^{p-1}A_m^i)B_m+D_1 \end{pmatrix} \tag{17}$$

D_3 prove that the lifted state-space model for such a dual-rate system has no causality constraints. For convenience, we omit T_1 in (4)and get

$$\begin{cases} x(k+1) = A_2x(k)+B_2u(k) \\ \underline{y}(k) = C_2x(k)+D_2u(k)+\underline{v}(k). \end{cases} \tag{18}$$

4. Identification Problems

4.1 Fundamental Matrices Definition and Identification Framework

N4SID algorithms are based on concepts from system theory, linear algebra and statistics. Before describing the identification scheme we need denote some matrices to introduce. First we build Hankel matrices with the lifted model input and output data as

$$U_{0|i-1} := \begin{pmatrix} u(0) & u(1) & \cdots & u(j-1) \\ u(1) & u(2) & \cdots & u(j) \\ \cdots & \cdots & \ddots & \cdots \\ u(i-1) & u(i) & \cdots & u(i+j-1) \end{pmatrix} \tag{19}$$

$$Y_{0|i-1} := \begin{pmatrix} \underline{y}(0) & \underline{y}(1) & \cdots & \underline{y}(j-1) \\ \underline{y}(1) & \underline{y}(2) & \cdots & \underline{y}(j) \\ \cdots & \cdots & \ddots & \cdots \\ \underline{y}(i-1) & \underline{y}(i) & \cdots & \underline{y}(i+j-1) \end{pmatrix}. \tag{20}$$

We can build $U_{i|2i-1}$ and $Y_{i|2i-1}$ in the similar way. i and j are user-defined index which is large enough. i should at least be larger than the maximum order of the lifted state-space model. j is typically equal to $s-2i+1$ in which s means the number of all available data samples. In any case, j should be larger than $2i-1$.

Then we define the state sequence X_i as

$$X_i := (\, x(i) \; x(i+1) \; \cdots \; x(i+j-1) \,), \tag{21}$$

where $X_i \in R^{n\times j}$.

The extended observability matrix Γ_i is defined as

$$\Gamma_i = \begin{pmatrix} C_2 \\ C_2A_2 \\ C_2A_2^2 \\ \vdots \\ C_2A_2^{i-1} \end{pmatrix}. \tag{22}$$

Then we will define a lower block triangular Toeplitz matrix:

$$H_i = \begin{pmatrix} D_2 & 0 & \cdots 0 \\ C_2B_2 & D_2 & \cdots 0 \\ \cdots & \cdots & \ddots \cdots \\ C_2A_2^{i-2}B_2 & C_2A_2^{i-3}B_2 & \cdots D_2 \end{pmatrix}. \tag{23}$$

We define the matrices O_i and O_{i-1} as

$$O_i = Y_{i|2i-1} / \begin{pmatrix} U_{0|2i-1} \\ Y_{0|i-1} \end{pmatrix}, \tag{24}$$

$$O_{i-1} = Y_{i+1|2i-1} / \begin{pmatrix} U_{0|2i-1} \\ Y_{0|i} \end{pmatrix}, \tag{25}$$

where $A/B = AB^T(BB^T)^\dagger B$($\bullet^\dagger$ denotes the Moore-Penrose pseudo-inverse of the matrix $\bullet$). The row space of A/B means the projection of the row space of A onto the row space of B.

With the defined matrices, we can have

$$O_i = \Gamma_i X_i + H_i U_{i|2i-1}, \tag{26}$$
$$O_{i-1} = \Gamma_{i-1} X_{i+1} + H_{i-1} U_{i+1|2i-1}. \tag{27}$$

We can get the proof of (26) and (27) from the related work[5]. Since they are the linear combinations of the system matrices A_2, B_2, C_2 and D_2, the projections O_i and O_{i-1} can determine the lifted model. So we can start the N4SID identification scheme from the projections O_i and O_{i-1}.

In common, the identification framework is described as follows.

First determine the projections

$$\begin{aligned} O_i &= Y_{i|2i-1} / \begin{pmatrix} U_{0|i-1} \\ U_{i|2i-1} \\ Y_{0|i-1} \end{pmatrix} \\ &= (\, L_i^1 \; L_i^2 \; L_i^3 \,) / \begin{pmatrix} U_{0|i-1} \\ U_{i|2i-1} \\ Y_{0|i-1} \end{pmatrix} \end{aligned} \tag{28}$$

and

$$\begin{aligned} O_{i+1} &= Y_{i+1|2i-1} / \begin{pmatrix} U_{0|i} \\ U_{i+1|2i-1} \\ Y_{0|i} \end{pmatrix} \\ &= (\, L_{i+1}^1 \; L_{i+1}^2 \; L_{i+1}^3 \,) / \begin{pmatrix} U_{0|i} \\ U_{i+1|2i-1} \\ Y_{0|i} \end{pmatrix}. \end{aligned} \tag{29}$$

Then perform singular value decomposition

$$\begin{aligned} &(\, L_i^1 \; L_i^3 \,) \begin{pmatrix} U_{0|i-1} \\ Y_{0|i-1} \end{pmatrix} \\ &= (\, U_1 \; U_2 \,) \begin{pmatrix} \Sigma_i & 0 \\ 0 & 0 \end{pmatrix} V, \end{aligned} \tag{30}$$

and then we have

$$\Gamma_i = U_1 \Sigma_i^{1/2}. \tag{31}$$

Then estimate the state sequence

$$\hat{X}_i = \Gamma_i^\dagger (\, L_i^1 \; L_i^3 \,) \begin{pmatrix} U_{0|i-1} \\ Y_{0|i-1} \end{pmatrix} \tag{32}$$

and

$$\hat{X_{i+1}} = \Gamma_{i-1}^\dagger (\, L_{i+1}^1 \; L_{i+1}^3 \,) \begin{pmatrix} U_{0|i} \\ Y_{0|i} \end{pmatrix} \tag{33}$$

where $\hat{\bullet}$ means the estimate of $\bullet$ and Γ_{i-1} denotes the matrix Γ_i without the last $m \times p$ rows.

Finally, solve the least squares problem to determine the system matrices

$$\begin{pmatrix} \hat{X_{i+1}} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \begin{pmatrix} \hat{X}_i \\ U_{i|i} \end{pmatrix}. \tag{34}$$

4.2 Numerical Algorithms

In fact, the identification algorithms can be implemented in a numerically stable and efficient way.

To calculate the projection of the row space of the input-output data, we do LQ factorization as

$$\begin{pmatrix} U_{0|i-1} \\ U_{i|i} \\ U_{i+1|2i-1} \\ Y_{0|i-1} \\ Y_{i|i} \\ Y_{i+1|2i-1} \end{pmatrix} = \begin{pmatrix} L_{11} & 0 & 0 & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 & 0 & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} & 0 & 0 \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} & 0 \\ L_{61} & L_{62} & L_{63} & L_{64} & L_{65} & L_{66} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{pmatrix}. \tag{35}$$

The oblique projection can be obtained as

$$O_i = L_{U_p} L_{11} Q_1 + L_{Y_p} (\, L_{41} \; L_{42} \; L_{43} \; L_{44} \,) \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix}, \tag{36}$$

and we can get L_{U_p} and L_{Y_p} from

$$\begin{aligned} &(\, L_{U_p} \; L_{U_f} \; L_{Y_p} \,) \begin{pmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} \end{pmatrix} \\ &= \begin{pmatrix} L_{51} & L_{52} & L_{53} & L_{54} \\ L_{61} & L_{62} & L_{63} & L_{64} \end{pmatrix}. \end{aligned} \tag{37}$$

We can calculate O_{i-1} in the similar way:

$$O_{i-1} = L_{U_p}^{+}\begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix}\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} + L_{Y_p}^{+}\begin{pmatrix} L_{41} & L_{42} & L_{43} & L_{44} & 0 \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{pmatrix}\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix}, \tag{38}$$

where

$$\begin{pmatrix} L_{U_p}^{+} & L_{U_f}^{+} & L_{Y_p}^{+} \end{pmatrix}\begin{pmatrix} L_{11} & 0 & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} & 0 \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{pmatrix} = (\, L_{61} \; L_{62} \; L_{63} \; L_{64} \; L_{65} \,). \tag{39}$$

Then we perform singular value decomposition and delete the small singular values

$$L_{U_P}(\, L_{11} \; 0 \; 0 \; 0 \,) + L_{V_P}(\, L_{41} \; L_{42} \; L_{43} \; L_{44} \,) = (\, U_1 \; U_2 \,)\begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \simeq U_1 S_1 V_1. \tag{40}$$

Then the order of the lifted state-space model is equal to the number of remained singular values. The observability matrix Γ_i can be taken to be

$$\Gamma_i = U_1 S_1^{1/2}, \tag{41}$$

and the state sequence $\hat{X}_i$ will be

$$\hat{X}_i = \Gamma_i^{\dagger} O_i = S_1^{1/2} V_1 \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix}. \tag{42}$$

A shifted state sequence $\hat{X}_{i+1}$ can be obtained as

$$\hat{X}_{i+1} = \Gamma_{i-1}^{\dagger} O_{i-1}. \tag{43}$$

With $\hat{X}_i, \hat{X}_{i+1}, U_{i|i}$ and $Y_{i|i}$ we can solve a least squares problem to obtain the system matrices:

$$\begin{pmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{pmatrix} = \min \left\| \begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} - \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} \hat{X}_i \\ U_{i|i} \end{pmatrix} \right\|_F^2, \tag{44}$$

where $\| \bullet \|_F$ denotes the Frobenius-norm of a matrix.

5. Example

An example is given in this section. For the dual-rate system depicted in **Fig.** 1, we take the continuous process $P(s)$ as

$$P(s) = \frac{1}{2s^3 + 3s^2 + s + 1} \tag{45}$$

and $T_1 = 2s, T_2 = 1s$. So the lifted output vector and the lifted noise vector will be

$$\underline{y}(kT_1) = \begin{bmatrix} y(kT_1) \\ y(kT_1+1) \end{bmatrix} \tag{46}$$

$$\underline{v}(kT_1) = \begin{bmatrix} v(kT_1) \\ v(kT_1+1) \end{bmatrix}. \tag{47}$$

The input $u(kT_1)$ is a zero mean white signal (variance 1). $v(kT_2)$ is a zero mean white noise sequence. To show the performance of the N4SID method in the presence of considerable noise, the N4SID algorithm ($i = 15, j = 5000$) was implemented for 20 realizations of the measurement noise of NSR (noise to signal ratio)=20%. NSR was defined as the ratio of σ_v/σ_y, where σ_v and σ_y are the standard deviations of the measurement noise and of the noise-free output, respectively.

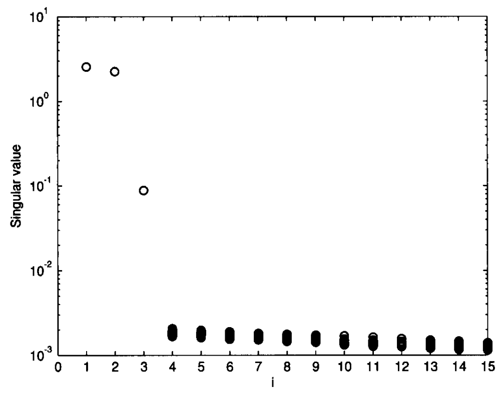

Fig. 2 Distribution of the singular values of 20 realizations.

We assume there is no priori information of $P(s)$. First we need to determine the order of the lifted state-space model through finding the number of dominant singular values. **Figure** 2 shows that the domain singular values achieved from the repeated simulations. From **Fig.** 2, it is obvious that the order of the lifted model is 3.

Figure 3 shows the estimated lifted step response together with the continuous-time step response.

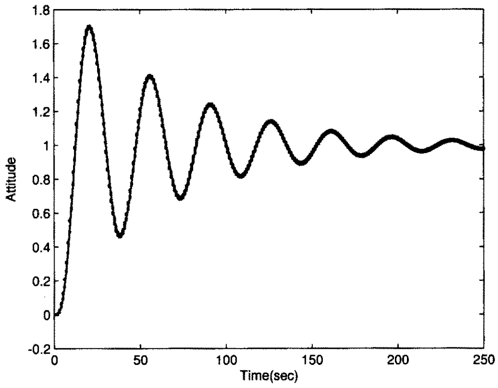

Fig. 3 The step response of the continuous-time process (solid line) and the estimated lifted step response of 20 realizations (dot).

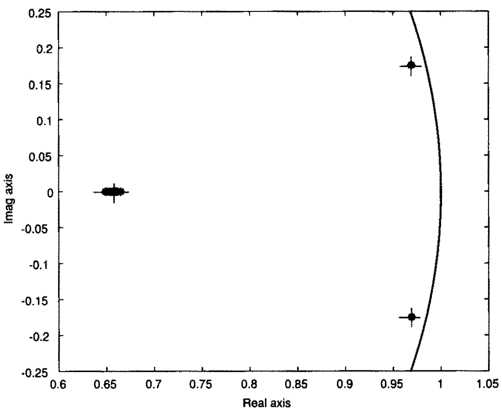

Fig. 4 Distribution of the poles of 20 realizations.

As expected, the points of the estimated lifted step response sit on that of the continuous-time process. The poles of the lifted state-space model of 20 realizations are presented in **Fig. 4**. It can be seen that the poles do not distribute dispersedly.

6. Conclusion

In this paper, the lifted state-space model for a class of dual-rate system is derived. Then the lifted state-space model is identified from the input-output data using N4SID algorithms. The lifted state-space model can be identified from input-output data directly using N4SID algorithms. Through inspecting the dominant singular values we can still identify the lifted state-space model in the case that the noise is colored noise.

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