Modified Bias Compensation Recursive Least-Squares Method for Noisy FIR Adaptive Filtering

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Abstract: In this paper a modified bias compensation recursive least-squares (MBCRLS) method is proposed to deal with the task of adaptive FIR filtering with noisy input-output data. This method is similar to the BCRLS method which is proposed by authors recently in terms of use of introducing an auxiliary estimator but a different form with that one in BCRLS method. Several modified points both in theoretical discussion and recursive computing aspects in the new MBCRLS method lead to a reduction in computing cost and simple, readable and understandable derivation. Simulation results are conductive to verify the discussions.

Keywords: Adaptive filters, Recursive least squares parameter estimation, Noisy FIR system, Bias compensation

1. Introduction

Adaptive finite impulse response (FIR) filtering, in which adjustable parameters are used, is widely applied in many fields such as control, signal processing and system identification[1]. Least mean squares (LMS) algorithm, which is based on the minimum mean squared error (MMSE) criterion, and recursive least squares (RLS) algorithm which is based on the least squares error criterion, are two widely used classes of adaptive filtering algorithms. It is known that if input measurement is noise-free, the LMS algorithm or the RLS algorithm can provide unbiased filter coefficients[2]. However, this condition is not satisfied in some practical situations. When both input and output are corrupted by additive noise, the LMS solution and the RLS solution are biased.

Total least squares (TLS) method is an unbiased estimation method when both input and output are corrupted by noises. Recently, several TLS-based algorithms have been proposed for FIR adaptive filtering in this noise environment[5][6][7]. More recently, under the assumption that the noise-free input is also a white Gaussian random process independent of input and output noise, the modified LMS (MLMS)[8] and the modified RLS (MRLS)[9] were proposed to give consistent FIR filtering coefficients with both input and output noise.

On the other hand, a technique based on the bias compensation principle (BCP) proposed by Sagara and Wada[3] is an efficient solution for bias problem reduced in RLS algorithm. The bias compensated recursive least squares (BCRLS) algorithm[4] which was derived based on the BCP, was proposed to provide consistent coefficients for FIR adaptive filtering with both input and output noises. Bias equation was given to illustrate that the bias reduced in RLS solution was caused only by the input noise whereas the output noise does not affect the estimate. In order to obtain the estimate of input noise variance, an auxiliary estimator was introduced to construct the cross-correlation equation of LS error and introduced auxiliary estimation error in BCRLS method. And with the help of computing the error cross-correlation, the estimate of the bias resulted from LS solution can be obtained. Then the bias of RLS can be compensated to result in a satisfactory parameter estimation for Noisy FIR adaptive filtering.

In this paper a modified algorithm of BCRLS method which can be called by MBCRLS algorithm is given. This new algorithm is similar to the BCRLS method in terms of introducing an auxiliary estimator but a different form with that one in BCRLS method. The new introduced auxiliary estimator in MBCRLS is formed by using directly the correlation matrix of noisy input data same to ones in LS estimator. It is exciting that this leads to no additional computing of another correlation matrix of noisy input data. Moreover, in aspect of estimation of error cross-correlation, a very simple, efficient approach is introduced to displace an expa-

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tiatory derivation in 4). These modified points both in theoretical discussions and recursive computing aspects in the new MBCRLS method lead to a reduction in computing cost and simple, readable and understandable derivation.

Computer simulations are carried out in two cases that noise-free input signal is white process and colored process and comparison with several other algorithms is also done. Numerical computation indicates that the new MBCRLS algorithm inherits all attractive properties of the former BCRLS algorithm and decrease more computation burden than the former BCRLS algorithm.

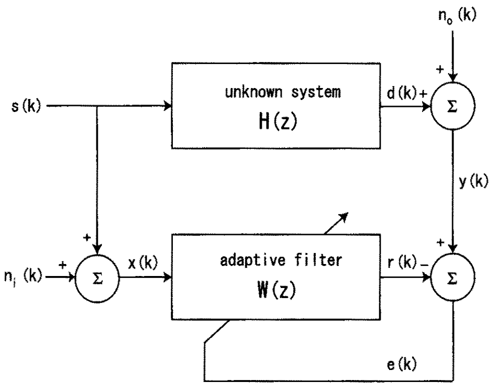

Fig. 1 System block diagram for system identification.

2. Problem Statement

Consider a noisy FIR filtering problem illustrated in **Fig. 1**. An unknown FIR system $H(z)$ with L-point impulse response vector $\boldsymbol{h}$ has an input $s(k)$ and an output $d(k)$. $x(k)$ and $y(k)$ are measured input and output corrupted by additive noise respectively. Our task is to estimate the parameter of $H(z)$ using adaptive FIR filter $W(z)$ from measured input $x(k)$ and output $y(k)$. Then the following relationships are given:

$$H(z^{-1}) = \sum_{i=0}^{L-1} h_i z^{-i} \tag{1}$$

$$W(z^{-1}) = \sum_{i=0}^{L-1} w_i z^{-i} \tag{2}$$

$$x(k) = s(k) + n_i(k) \tag{3}$$

$$y(k) = d(k) + n_o(k) \tag{4}$$

$$d(k) = \sum_{i=0}^{L-1} h_i s(k-i) \tag{5}$$

$$r(k) = \sum_{i=0}^{L-1} w_i x(k-i) \tag{6}$$

where input noise $n_i(k)$ and output noise $n_o(k)$ are assumed to be zero-mean white noise processes, independent of input $s(k)$ with variances σ_i^2 and σ_o^2 respectively. Let

$$\begin{aligned}
\boldsymbol{h} &= [\,h_0\, h_1 \ldots h_{L-1}\,]^T \\
\boldsymbol{y}(k) &= [\,y(k)\, y(k-1) \ldots y(k-L+1)\,]^T \\
\boldsymbol{x}(k) &= [\,x(k)\, x(k-1) \ldots x(k-L+1)\,]^T \\
\boldsymbol{n}_i(k) &= [\,n_i(k)\, n_i(k-1) \ldots n_i(k-L+1)\,]^T \\
\boldsymbol{w} &= [\,w_0\, w_1 \ldots w_{L-1}\,]^T,
\end{aligned}$$

then by equations (3) and (5), equation (4) can be written as:

$$y(k) = \boldsymbol{x}(k)^T \boldsymbol{h} + v(k) \tag{7}$$

where

$$v(k) = n_o(k) - \boldsymbol{n}_i(k)^T \boldsymbol{h}. \tag{8}$$

The error signal $e(k)$ is given by

$$\begin{aligned}
e(k) &= y(k) - r(k) \\
&= y(k) - \sum_{i=0}^{L-1} w_i x(k-i) \\
&= y(k) - \boldsymbol{x}(k)^T \boldsymbol{w}.
\end{aligned} \tag{9}$$

Minimization of the cost function $\sum_{k=1}^{N} e(k)^2$ with respect to the impulse response vector $\boldsymbol{w}$ leads to LS estimate $\widehat{\boldsymbol{w}}_{LS,N}$ as follows

$$\widehat{\boldsymbol{w}}_{LS,\,N} = \left(\sum_{k=1}^{N} \boldsymbol{x}(k)\boldsymbol{x}(k)^T \right)^{-1} \sum_{k=1}^{N} \boldsymbol{x}(k) y(k). \tag{10}$$

Substituting equation (7) into equation (10), yields

$$\widehat{\boldsymbol{w}}_{LS,\,N} = \boldsymbol{h} + P_N \sum_{k=1}^{N} \boldsymbol{x}(\boldsymbol{k}) v(k) \tag{11}$$

where

$$P_N = \left(\sum_{k=1}^{N} \boldsymbol{x}(k)\boldsymbol{x}(k)^T \right)^{-1}. \tag{12}$$

Taking the probability limit, we have

$$\operatorname*{plim}_{N\to\infty} \widehat{\boldsymbol{w}}_{LS,\,N} = \boldsymbol{h} + \boldsymbol{b}. \tag{13}$$

Here $\boldsymbol{b}$ is the asymptotic bias of $\widehat{\boldsymbol{w}}_{LS,\,N}$ described as

$$\boldsymbol{b} = R^{-1}\operatorname*{plim}_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} \boldsymbol{x}(\boldsymbol{k})v(k) \tag{14}$$

where

$$R = \operatorname*{plim}_{N\to\infty} \frac{1}{N} P_N^{-1}. \tag{15}$$

Based on the assumptions of input and output noises and from (3) and (8) equations, we have

$$\boldsymbol{b} = -\sigma_i^2 R^{-1}\boldsymbol{h}. \tag{16}$$

So if the estimate of asymptotic bias $\boldsymbol{b}$ is considered as

$$\widehat{\boldsymbol{b}}_N = -\sigma_i^2 N P_N \widehat{\boldsymbol{h}}_{N-1}, \tag{17}$$

then compensating the bias results in the consistent estimate of $\boldsymbol{h}$ as the following:

$$\widehat{\boldsymbol{h}}_N = \widehat{\boldsymbol{w}}_{LS,\,N} + \sigma_i^2 N P_N \widehat{\boldsymbol{h}}_{N-1}. \tag{18}$$

$\widehat{\boldsymbol{w}}_{LS}$ and P_N can be computed using the recursive LS algorithm form as

$$\widehat{\boldsymbol{w}}_{LS,N} = \widehat{\boldsymbol{w}}_{LS,N-1} + \frac{P_{N-1}\boldsymbol{x}(N)(y(N) - \boldsymbol{x}(N)^T\widehat{\boldsymbol{w}}_{N-1})}{1+\boldsymbol{x}(N)^T P_{N-1}\boldsymbol{x}(N)} \tag{19}$$

$$P_N = P_{N-1} - \frac{P_{N-1}\boldsymbol{x}(N)\boldsymbol{x}(N)^T P_{N-1}}{1+\boldsymbol{x}(N)^T P_{N-1}\boldsymbol{x}(N)}. \tag{20}$$

Above discussions indicate that in the case of both white input and output noises, the RLS estimate is biased, and the bias is caused only by the input noise whereas the output noise has no effect. If input noise variance σ_i is known or an estimate $\hat{\sigma}_i$ of it is available, then applying the BCP gives rise to the consistent estimate of $\boldsymbol{h}$. In usually practical cases, no prior knowledge about noises can be available, so the core lies in estimation of input noise variance σ_i^2.

In the next section, we will present the solution how to obtain the estimate of the input noise variance σ_i^2.

3. Estimation of input noise variance

In order to estimate input noise variance, we presented the estimation equation of the input noise variance σ_i^2 in 4) described as

$$\widehat{\sigma}_i^2 = \frac{g_N}{N\widehat{\boldsymbol{h}}^T\widehat{\boldsymbol{\varphi}}_N}, \tag{21}$$

where $\widehat{\boldsymbol{\varphi}}_N$ is an introduced auxiliary estimator and g_N is the error cross-correlation function of LS estimation error and the auxiliary estimation error. It is clear that the construction of $\widehat{\boldsymbol{\varphi}}_N$, the computations of $\widehat{\boldsymbol{\varphi}}_N$ and g_N become our research cares. In the following we shall discuss these key points respectively.

3.1 Introduction of Auxiliary Estimator

In the BCRLS algorithm[4], the auxiliary estimator

$$\widehat{\boldsymbol{\varphi}}_N = \left(\sum_{k=1}^{N} \boldsymbol{x}(k-1)\boldsymbol{x}(k-1)^T\right)^{-1} \sum_{k=1}^{N} \boldsymbol{x}(k-1)y(k)$$

was introduced to construct the auxiliary estimation error

$$\xi(k) = y(k) - \boldsymbol{x}(k-1)^T\widehat{\boldsymbol{\varphi}}_N$$

and the error cross-correlation function

$$g_N = \sum_{k=1}^{N} \varepsilon(k)\xi(k)$$

where LS error $\varepsilon(k) = y(k) - \boldsymbol{x}(k)^T\widehat{\boldsymbol{w}}_{LS,N}$ is orthogonal with $\boldsymbol{x}(k)$ shown as

$$\sum_{k=1}^{N} \boldsymbol{x}(k)\varepsilon(k) = \mathbf{o}. \tag{22}$$

$\widehat{\boldsymbol{\varphi}}_N$ can be computed similar to RLS algorithm as

$$\widehat{\boldsymbol{\varphi}}_N = \widehat{\boldsymbol{\varphi}}_{N-1} + \frac{Q_{N-1}\boldsymbol{x}(N-1)(y(N) - \boldsymbol{x}(N-1)^T\widehat{\boldsymbol{\varphi}}_{N-1})}{1+\boldsymbol{x}(N-1)^T Q_{N-1}\boldsymbol{x}(N-1)}$$

$$Q_N = Q_{N-1} - \frac{Q_{N-1}\boldsymbol{x}(N-1)\boldsymbol{x}(N-1)^T Q_{N-1}}{1+\boldsymbol{x}(N-1)^T Q_{N-1}\boldsymbol{x}(N-1)}$$

where

$$Q_N = \left(\sum_{k=1}^{N} \boldsymbol{x}(k-1)\boldsymbol{x}(k-1)^T\right)^{-1}.$$

It is obvious that appearance of another covariance matrix of data Q_N brings certainly more burden of computing. However the computation of $\widehat{\boldsymbol{\varphi}}_N$ is

inevitable so computation of Q_N is needed. As a result increase of computation load is also inevitable. If $\widehat{\varphi}_N$ is formed by directly using P_N not using Q_N, this disadvantage will be removed.
Now redefine $\widehat{\varphi}_N$ as

$$\widehat{\varphi}_N = \left(\sum_{k=1}^{N} \boldsymbol{x}(k)\boldsymbol{x}(k)^T\right)^{-1} \sum_{k=1}^{N} \boldsymbol{x}(k)y(k-1). \quad (23)$$

Comparing two forms of $\widehat{\varphi}_N$, it is obvious that the new form uses directly the covariance matrix of data P_N, so no additional computing of covariance matrix of data occurs. Therefore along with change of $\widehat{\varphi}_N$, the auxiliary estimation error $\xi(k)$ and the error cross-correlation function g_N are renewed respectively as

$$\xi(k) = y(k-1) - \boldsymbol{x}(k)^T\widehat{\varphi}_N \quad (24)$$

$$g_N = \sum_{k=1}^{N}\Big(y(k-1) - \boldsymbol{x}(k)^T\widehat{\varphi}_N\Big) \cdot \Big(y(k) - \boldsymbol{x}(k)^T\widehat{\boldsymbol{w}}_{LS,N}\Big) \quad (25)$$

which also has the useful orthogonal properties described as

$$\sum_{k=1}^{N} \boldsymbol{x}(k)\xi(k) = \mathbf{o}. \quad (26)$$

Finally the estimate of input noise variance can be obtained by equation (21) but the mentioned quantities will be renewed ones.

3.2 Recursive Algorithm for g_N

Firstly define matrix $\boldsymbol{\psi}_N$ as :

$$\boldsymbol{\psi}_N = \sum_{k=1}^{N} \begin{bmatrix} y(k) \\ \boldsymbol{x}(k) \end{bmatrix} \begin{bmatrix} y(k-1) & \boldsymbol{x}(k)^T \end{bmatrix}. \quad (27)$$

From equation (23) and (25), we have

$$\boldsymbol{\psi}_N \begin{bmatrix} -1 \\ \widehat{\varphi}_N \end{bmatrix} = \begin{bmatrix} -g_N \\ \mathbf{o} \end{bmatrix}. \quad (28)$$

By the definition of $\boldsymbol{\psi}_N$, we have

$$\boldsymbol{\psi}_N = \boldsymbol{\psi}_{N-1} + \begin{bmatrix} y(N) \\ \boldsymbol{x}(N) \end{bmatrix} \begin{bmatrix} y(N-1) & \boldsymbol{x}(N)^T \end{bmatrix}. \quad (29)$$

Post multiplying (29) by $[-1 \ \widehat{\varphi}_{N-1}^T]^T$ and using equation (28) yield

$$\boldsymbol{\psi}_N \begin{bmatrix} -1 \\ \widehat{\varphi}_{N-1} \end{bmatrix} = \begin{bmatrix} -g_{N-1} \\ \mathbf{o} \end{bmatrix} - \begin{bmatrix} y(N) \\ \boldsymbol{x}(N) \end{bmatrix} \left(y(N-1) - \boldsymbol{x}(N)^T\widehat{\varphi}_{N-1}\right). \quad (30)$$

Now let $s(N)$ and $\boldsymbol{t}_N$ be

$$s(N) = \sum_{k=1}^{N} y(k)\boldsymbol{x}(k)^T\boldsymbol{t}_N$$
$$\boldsymbol{t}_N = (\sum_{k=1}^{N} \boldsymbol{x}(k)\boldsymbol{x}(k)^T)^{-1}\boldsymbol{x}(N)$$

then we have

$$\boldsymbol{\psi}_N \begin{bmatrix} 0 \\ \boldsymbol{t}_N \end{bmatrix} = \begin{bmatrix} s(N) \\ \boldsymbol{x}(N) \end{bmatrix}. \quad (31)$$

Post multiplying by $\left(y(N-1) - \boldsymbol{x}(N)^T\widehat{\varphi}_{N-1}\right)$, and based on (26),(28), we can obtain

$$\boldsymbol{\psi}_N \begin{bmatrix} -1 \\ \widehat{\varphi}_{N-1} + \boldsymbol{t}_N(y(N-1) - \boldsymbol{x}(N)^T\widehat{\varphi}_{N-1}) \end{bmatrix} = \begin{bmatrix} -\left(g_{N-1} + (y(N) - s(N))(y(N) - \boldsymbol{x}(N)^T\widehat{\varphi}_{N-1})\right) \\ \mathbf{o} \end{bmatrix} \quad (32)$$

Hence the recursive forms of g_N and $\widehat{\varphi}_N$ are given by

$$g_N = g_{N-1} + (y(N) - s(N)) \cdot \left(y(N-1) - \boldsymbol{x}(N)^T\widehat{\varphi}_{N-1}\right) \quad (33)$$

$$\widehat{\varphi}_N = \widehat{\varphi}_{N-1} + \boldsymbol{t}_N\left(y(N-1) - \boldsymbol{x}(N)^T\widehat{\varphi}_{N-1}\right). \quad (34)$$

In addition, consider (10) and (12), $s(N)$ and $\boldsymbol{t}_N$ can be written as

$$s(N) = \widehat{\boldsymbol{w}}_{LS,N}^T\boldsymbol{x}(N)$$
$$\boldsymbol{t}_N = P_N\boldsymbol{x}(N).$$

So it is seen that

$$y(N) - s(N) = y(N) - \widehat{\boldsymbol{w}}_{LS,N}^T\boldsymbol{x}(N) = \varepsilon(N).$$

Based on the above, the estimate of input noise is summarized as follows.

$$\begin{aligned}
\boldsymbol{d}_N &= P_{N-1}\boldsymbol{x}(N) \\
f_N &= 1 + \boldsymbol{x}(N)^T \boldsymbol{d}_N \\
\boldsymbol{t}_N &= \boldsymbol{d}_N / f_N \\
e_p(N) &= y(N) - \boldsymbol{x}(N)^T \widehat{\boldsymbol{w}}_{LS,N-1} \\
\widehat{\boldsymbol{w}}_{LS,N} &= \widehat{\boldsymbol{w}}_{LS,N-1} + \boldsymbol{t}_N e_p(N) \\
e_r(N) &= y(N-1) - \boldsymbol{x}(N)^T \widehat{\boldsymbol{\varphi}}_{N-1} \\
\widehat{\boldsymbol{\varphi}}_N &= \widehat{\boldsymbol{\varphi}}_{N-1} + \boldsymbol{t}_N e_r(N) \\
P_N &= P_{N-1} - \boldsymbol{d}_N \boldsymbol{d}_N^T / f_N \\
g_N &= g_{N-1} - e_p(N) e_r(N) / f_N \\
\widehat{\sigma}_i^2 &= \frac{g_N}{N \widehat{\boldsymbol{h}}_{N-1}^T \widehat{\boldsymbol{\varphi}}_N}.
\end{aligned}$$

Consequently, the consistent estimate of noisy FIR filter parameter is obtained by computing

$$\widehat{\boldsymbol{h}}_N = \widehat{\boldsymbol{w}}_N + \frac{g_N}{\widehat{\boldsymbol{h}}_{N-1}^T \widehat{\boldsymbol{\varphi}}_N} P_N \widehat{\boldsymbol{h}}_{N-1}.$$

3.3 Alternative Recursive Algorithm for g_N

Although the above introduced recursive algorithm for g_N is very efficient, the derivation is too expatiatory to be hard readable. Here we will present a simple, readable derivation to obtain the recursive construction for computing g_N.
According to the orthogonal properties described as (22) and (26), g_N is also written by:

$$g_N = \sum_{k=1}^{N} y(k)(y(k-1) - \boldsymbol{x}(k)^T \widehat{\boldsymbol{\varphi}}_N) \quad (35)$$

$$= \sum_{k=1}^{N} y(k-1)(y(k) - \boldsymbol{x}(k)^T \widehat{\boldsymbol{w}}_{LS,N}). \quad (36)$$

Define

$$\begin{aligned}
c_{yy}(N) &= \sum_{k-1}^{N} y(k)y(k-1), \\
\boldsymbol{c}_{xy,1}(N) &= \sum_{k=1}^{N} \boldsymbol{x}(k)y(k), \\
\boldsymbol{c}_{xy,2}(N) &= \sum_{k=1}^{N} \boldsymbol{x}(k)y(k-1).
\end{aligned}$$

so we can obtain the alternative recursive algorithm for $\widehat{g}(N)$ as follows.

$$\begin{aligned}
c_{yy}(N) &= c_{yy}(N-1) + y(k)y(k-1), \\
\boldsymbol{c}_{xy,1}(N) &= \boldsymbol{c}_{xy,1}(N-1) + \boldsymbol{x}(k)y(k), \\
\boldsymbol{c}_{xy,2}(N) &= \boldsymbol{c}_{xy,2}(N-1) + \boldsymbol{x}(k)y(k-1), \\
g_N &= c_{yy}(N) - \boldsymbol{c}_{xy,1}(N)^T \widehat{\boldsymbol{\varphi}}_N,
\end{aligned}$$

or

$$g_N = c_{yy}(N) - \boldsymbol{c}_{xy,2}(N)^T \widehat{\boldsymbol{w}}_{LS,N}.$$

3.4 Relationship between the Two Algorithms of g_N

We shall make $g_N = c_{yy}(N) - \boldsymbol{c}_{xy,2}(N)^T \widehat{\boldsymbol{w}}_{LS,N}$ be a example to explore the relationship between the two algorithms for g_N.

$$\begin{aligned}
g_N &= c_{yy}(N) - \boldsymbol{c}_{xy,2}^T(N) \widehat{\boldsymbol{w}}_{LS,N} \\
&= c_{yy}(N-1) + y(N)y(N-1) \\
&\quad - \widehat{\boldsymbol{w}}_{LS,N}^T \boldsymbol{c}_{xy}(N-1) + \boldsymbol{x}(N)y(N-1)
\end{aligned}$$

Due to $\widehat{\boldsymbol{w}}_{LS,N} = \widehat{\boldsymbol{w}}_{LS,N-1} + \boldsymbol{t}_N e_p(N)$, we have

$$\begin{aligned}
g_N &= g_{N-1} + y(N)y(N-1) - \boldsymbol{c}_{xy,N-1}^T \boldsymbol{t}_N e_p(N) \\
&\quad - \widehat{\boldsymbol{w}}_{LS,N-1}^T \boldsymbol{x}(N) y(N-1) \\
&\quad - \boldsymbol{x}(N)^T \boldsymbol{t}_N e_p(N) y(N-1) \\
&= g_{N-1} + y(N-1)e_p(N) - \boldsymbol{c}_{xy,N-1}^T \boldsymbol{t}_N e_p(N) \\
&\quad + e_p(N) y(N-1) \boldsymbol{x}(N)^T \boldsymbol{t}_N \\
&= g_{N-1} + \frac{e_p(N)}{f_N}(y(N-1) \\
&\quad - \boldsymbol{x}(N)^T P_{N-1} \boldsymbol{c}_{xy,N-1} \\
&\quad + y(N-1)\boldsymbol{x}(N)^T \boldsymbol{d}_N - y(N-1)\boldsymbol{x}(N)^T \boldsymbol{d}_N) \\
&= g_{N-1} + \frac{e_p(N)}{f_N}(y(N-1) - \boldsymbol{x}(N)^T \widehat{\boldsymbol{\varphi}}_{N-1}) \\
&= g_{N-1} + e_p(N) e_r(N) / f_N.
\end{aligned}$$

It is found that the two recursive forms are identical substantively.

4. Simulation Result

The unknown FIR system is characterized by $\boldsymbol{h} = [-0.3; -0.9; 0.8; -0.7; 0.6]$. The input noise and output noise, $n_i(k)$ and $n_o(k)$, are independent white Gaussian random variables. The input and output signal-to-noise ratios, $SNR_i = \sigma_s^2/\sigma_i^2$ and $SNR_o = \sigma_s^2/\sigma_o^2$, are chosen as 5dB and 10dB.Computer simulations have been carried on in the following two cases respectively

Case 1:
The noise-free input $s(k)$, input noise $n_i(k)$ and output noise $n_o(k)$ are independent white Gaussian random processes. The variance of $s(k)$ is fixed to be unity, and the input and output signal-to-noise ratios, $SNR_i = \sigma_s^2/\sigma_i^2$ and $SNR_o = \sigma_s^2/\sigma_o^2$, are chosen 5dB and 10dB.

Case 2:
The noise-free input signal $s(k)$ is the AR process

$$s(k) + 0.9s(k-1) = g(k)$$

where $g(k)$ is a zero mean white Gaussian random variable. The variances of the input noise and output noise are $\sigma_i^2 = \sigma_o^2 = 0.25$.

Table 1 Simulation result in Case 1.

	LS method		MBCRLS method		True Values
N	3000	5000	3000	5000	
h_0	-0.1939	-0.1863	-0.3042	-0.2969	-0.3
h_1	-0.5692	-0.5766	-0.8956	-0.8997	-0.9
h_2	0.5061	0.5138	0.7974	0.8020	0.8
h_3	-0.4452	-0.4481	-0.7015	-0.6990	-0.7
h_4	0.3816	0.3855	0.6006	0.6010	0.6

Table 2 Simulation result in Case 2.

	LS method		MBCRLS method		True Values
N	3000	5000	3000	5000	
h_0	0.0183	0.0110	-0.2999	-0.2978	-0.3
h_1	-0.6355	-0.6321	-0.9028	-0.8980	-0.9
h_2	0.7246	0.7258	0.8009	0.8021	0.8
h_3	-0.6774	-0.6812	-0.6970	-0.6988	-0.7
h_4	0.6001	0.6052	0.5993	0.6016	0.6

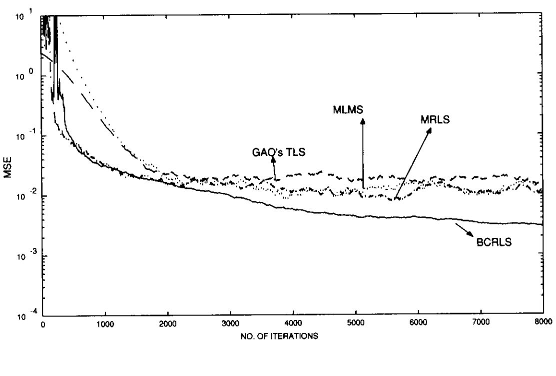

Fig. 2　MSE of Four Algorithms in Case 1.

Comparisons between RLS algorithm and MBCRLS algorithm in both cases of input signal are shown in **Table 1** and **Table 2** respectively. Results in two tables are averages of 50 independent trials with data length of 3000 and 5000. Comparisons are also made with the MLMS algorithm[8], the MRLS algorithm[9] and the GAO's TLS algorithm[7] in Case 1 and the mean square errors (MSE) of them are shown in **Fig. 2**. The results are averages of 50 independent trials with a data length of 5000. It is shown by simulation results that the new MBCRLS algorithm reduce computing time whereas no any loss. The new MBCRLS algorithm inherits all merits of the former BCRLS algorithm such as better accuracy, no strict assumption, wider application and so on.

5. Conclusions

In this paper, the modified BCRLS algorithm has been proposed for adaptive FIR filter parameter estimation in the presence of the input and output noise. Several modified points both in theoretical discussion and recursive computing in the new MBCRLS method lead to a reduction in computing cost and simple, readable and understandable derivation. Numerical computation indicates that the MBCRLS has more attractive property since it can decrease more computation burden than the former BCRLS algorithm. It also has been shown by simulation results that the modified algorithm inherits all merits of the former such as better accuracy, no strict assumption, more wide application etc.

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