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El Ocla, Hosam
Department of Computer Science and Communication Engineering, Graduate School of Information Science and Electrical Engineering, Kyushu University : Graduate Student

Tateiba, Mitsuo
Department of Computer Science and Communication Engineering, Graduate School of Information Science and Electrical Engineering, Kyushu University

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Angular Correlation Function of Waves Scattered from Conducting Targets in Random Media

Hosam EL OCLA* and Mitsuo TATEIBA**

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Abstract: We analyze numerically the angular correlation function for waves scattered from practical targets embedded in random media. We assume perfect conducting targets with simple and complicated surfaces to know exactly the parameters that affect the correlation of scattered waves. To obtain an expression of the correlation, we apply a method that uses current generator and Green's function. By this method we can solve the scattering problem as a boundary value problem. The numerical results suggest that the angular correlation function is affected only by random media irrespective of target parameters and incident wave polarization.

Keywords: Angular correlation, Random media, Scattering waves

1. Introduction

Correlations of scattered waves from random media and rough surface have been attracted considerable interest and have stimulated even more intensive research activities in recent years [1),2)]. Experimental studies of the correlation function have been conducted for targets in random media as in reference [3)]. Those studies have run the detection measurements on simple convex cylinders only buried in random media and also didn't consider the polarization of incident waves. In this regard, authors wanted to handle the correlation problem numerically and assume different circumstances of targets parameters and incident angle with putting into consideration the polarization of incident waves.

To investigate the angular correlation function (ACF) numerically of waves scattered from practical targets of finite size in random media, one of the authors has analyzed the ACF for circular conducting cylinders [4)] and showed the following result. The ACF does not depend on the radius of the circle and depends only on the random medium whose thickness is much larger than the radius. Above result has been shown only to a simple shape target and to an E-wave polarization. To make sure the result, we investigate the ACF for targets with different shapes and parameters in random media with taking account of incident wave polarization.

In this paper, our aim is to analyze numerically the ACF of waves scattered from conducting targets with different shapes and parameters in random media for E and H-wave incidences. In doing that, we will draw on our method [5),6)] which assumes current generator and Green's function to get the scattered waves in random media and therefore become able to calculate ACF numerically. In this case, we point out to that our handling of the angular correlation problem is based on the assumption that incident and scattered waves are not correlated. Therefore, on this assumption, we may express the fourth order moment of Green's functions as the products of the second order moments. The time factor $\exp(-i\omega t)$ is assumed and suppressed in the following section.

* Department of Computer Science and Communication Engineering, Graduate Student

** Department of Computer Science and Communication Engineering

2. Scattering Problem

Geometry of the problem is shown in **Fig. 1**. A random medium is assumed as a sphere of radius L around a target of the mean size $a \ll L$, and also to be described by the dielectric constant $\varepsilon(\mathbf{r})$, the magnetic permeability μ, and the electric conductivity σ. For simplicity $\varepsilon(\mathbf{r})$ is expressed as

$$\varepsilon(\mathbf{r}) = \varepsilon_0[1 + \delta\varepsilon(\mathbf{r})] \tag{1}$$

where ε_0 is assumed to be constant and equal to free space permitivity and $\delta\varepsilon(\mathbf{r})$ is a random function with

$$\langle \delta\varepsilon(\mathbf{r}) \rangle = 0, \qquad \langle \delta\varepsilon(\mathbf{r})\, \delta\varepsilon(\mathbf{r}') \rangle = B(\mathbf{r}, \mathbf{r}') \tag{2}$$

and

$$B(\mathbf{r}, \mathbf{r}) \ll 1, \qquad kl(\mathbf{r}) \gg 1 \tag{3}$$

Here, the angular brackets denote the ensemble average and $B(\mathbf{r},\mathbf{r})$, $l(\mathbf{r})$ are the local intensity and scale-size of the random medium fluctuation, respectively, and where $k = \omega\sqrt{\varepsilon_0\mu_0}$. Also μ and σ are assumed to be constant; $\mu = \mu_0$, $\sigma = 0$.

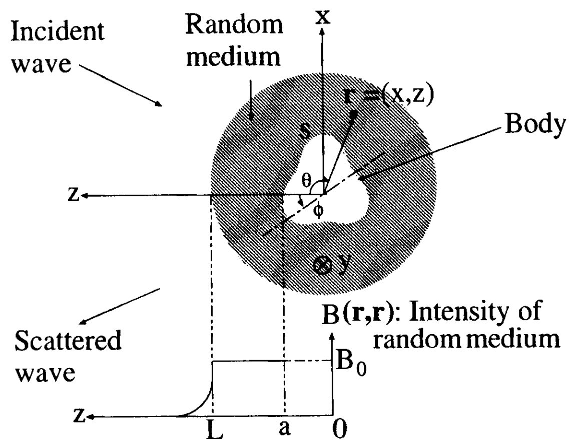


Fig.1 Geometry of the problem of wave scattering from a conducting cylinder in random media.

For practical turbulent media the condition (3) may be satisfied. Therefore, we can assume the forward scattering approximation and the scalar approximation [8]. Consider the case where a directly incident wave is produced by a line source $f(\mathbf{r}')$ distributed uniformly along the y axis. Then, the incident wave is cylindrical and becomes plane approximately around the target because the line source is very far from the target.

Here, let us designate the incident wave by $u_{in}(\mathbf{r})$, the scattered wave by $u_s(\mathbf{r})$, and the total wave by $u(\mathbf{r}) = u_{in}(\mathbf{r}) + u_s(\mathbf{r})$. The targets are assumed conducting cylinders of which cross-sections are expressed by

$$r = ab/\sqrt{(a\sin(\theta-\phi))^2 + (b\cos(\theta-\phi))^2} \qquad (4)$$

and

$$r = a[1 - \delta\ \cos 3(\theta-\phi)] \qquad (5)$$

Equations (4) and (5) are for convex and partially convex surfaces, respectively, where ϕ is the rotation index. In equation (4), a and b ($< a$) are the half lengths of the major and minor axices of the elliptic cylinder, respectively. In equation (5), a is the mean size and δ is the concavity index. We can deal with this scattering problem two-dimensionally under the condition (3); therefore, we represent $\mathbf{r}$ as $\mathbf{r} = (x, z)$. According as polarization of incident waves: E_y or H_y, where E_y, H_y are the y component of electric and magnetic fields, respectively, we can impose two types of boundary condition on wave fields on the cylinder surface S: the Dirichlet condition (DC) for E-wave incidence and the Neumann condition (NC) for H-wave incidence

$$u(\mathbf{r}) = \mathbf{0}, \qquad \text{for DC} \qquad (6)$$

$$\frac{\partial}{\partial n}u(\mathbf{r}) = \mathbf{0}, \qquad \text{for NC} \qquad (7)$$

where $\partial/\partial n$ denotes the outward normal derivative at $\mathbf{r}$ on S. In (6), $u(\mathbf{r})$ represents E_y while in (7) $u(\mathbf{r})$ represents H_y.

According to our method [5),6)], using the current generator $Y_{\mathrm{E}}, Y_{\mathrm{H}}$ and Green's function in random medium $G(\mathbf{r} \mid \mathbf{r}')$, we can express the scattered wave as

$$\begin{aligned} u_s(\mathbf{r}) &= \int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \left[G(\mathbf{r} \mid \mathbf{r}_2) Y_{\mathrm{E}}(\mathbf{r}_2 \mid \mathbf{r}_1) G(\mathbf{r}_1 \mid \mathbf{r}_t)\right] \\ &\quad \text{for E–wave incidence} \\ &= -\int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \\ &\quad \left[\left(\frac{\partial}{\partial n_2} G(\mathbf{r} \mid \mathbf{r}_2)\right) Y_{\mathrm{H}}(\mathbf{r}_2 \mid \mathbf{r}_1) G(\mathbf{r}_1 \mid \mathbf{r}_t)\right] \\ &\quad \text{for H–wave incidence} \end{aligned} \qquad (8)$$

Here, Y_{E} and Y_{H} are the operator that transforms incident waves into surface currents on S and depends only on the scattering target. The current generator can be expressed in terms of wave functions which satisfy Helmholtz equation and the radiation condition. That is, for E-wave incidence and H-wave incidence, the current generator is obtained as

$$Y_E(\mathbf{r} \mid \mathbf{r}') \simeq \mathbf{\Phi}^*_M(\mathbf{r}) A_E^{-1} \ll \mathbf{\Phi}^T_M(\mathbf{r}') \qquad (9)$$

$$Y_H(\mathbf{r} \mid \mathbf{r}') \simeq -\frac{\partial \mathbf{\Phi}^*_M(\mathbf{r})}{\partial n} A_H^{-1} \ll \mathbf{\Phi}^T_M(\mathbf{r}') \qquad (10)$$

Here, the basis functions $\mathbf{\Phi}_M$ are called the modal functions and constitute the complete set of wave functions satisfying the Helmholtz equation in free space and the radiation condition; $\mathbf{\Phi}_M = [\phi_{-N}, \phi_{-N+1}, \ldots, \phi_N]$, $M = 2N+1$ is the total mode number, $\phi_m(\mathbf{r}) = H_m^{(1)}(kr)\exp(im\theta)$, and A_E is a positive definite Hermitian matrix given by

$$A_E = \begin{pmatrix} (\phi_{-N}, \phi_{-N}) & \cdots & (\phi_{-N}, \phi_N) \\ \vdots & \ddots & \vdots \\ (\phi_N, \phi_{-N}) & \cdots & (\phi_N, \phi_N) \end{pmatrix} \quad (11)$$

in which its m, n element is the inner product of ϕ_m and ϕ_n:

$$(\phi_m, \phi_n) \equiv \int_S \phi_m(\mathbf{r}) \phi_n^*(\mathbf{r}) \mathrm{d}\mathbf{r} \quad (12)$$

A_H is A_E of (11) with (ϕ_m, ϕ_n) replaced by $(\partial\phi_m/\partial n, \partial\phi_n/\partial n)$. In (9) and (10), $\ll \mathbf{\Phi}_M^T$ denotes the operation (13) of each element of $\mathbf{\Phi}_M^T$ and the function u_{in} to the right of $\mathbf{\Phi}_M^T$

$$\ll \phi_m(\mathbf{r}), u_{in}(\mathbf{r}) \gg \equiv \phi_m(\mathbf{r}) \frac{\partial u_{in}(\mathbf{r})}{\partial n} - \frac{\partial \phi_m(\mathbf{r})}{\partial n} u_{in}(\mathbf{r}) \quad (13)$$

The Y_E and Y_H are proved to converge in the sense of mean on the true operator when $M \to \infty$.

Let us assume that source and observation points are on the same circle of r. Then $u_s(\mathbf{r})$ is expressed as $u(\theta_{i1}, \theta_{r1})$ and represents the scattered wave observed at the scattering angle θ_{r1} when the incident angle is θ_{i1}. Suppose that the incident angle is changed from θ_{i1} to θ_{i2} and the scattered wave $u(\theta_{i2}, \theta_{r2})$ is observed at θ_{r2} (see **Fig. 2**) . The correlation function is defined as

$$\Gamma(\theta_{i1}, \theta_{i2}, \theta_{r1}, \theta_{r2}) = \langle u_s(\theta_{i1}, \theta_{r1})\, u_s^*(\theta_{i2}, \theta_{r2}) \rangle \quad (14)$$

where $\theta_{r1} = \theta_{i1} + \psi$, $\theta_{i2} = \theta_{i1} + \theta_1$, $\theta_{r2} = \theta_{r1} + \theta_2$, and θ_1 and θ_2 are the deviation in both angles of reference incident and scattered waves.

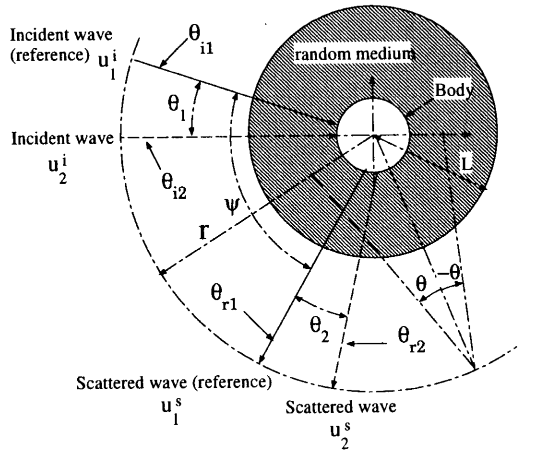


Fig.2 Angular correlation of waves scattered by a conducting cylinder in a random medium.

3. Numerical Results

Although the incident wave becomes sufficiently incoherent, we should pay attention to the spatial coherence function of the incident wave [7]. The degree of spatial coherence is expressed as a function of θ on the circle of r , shown in **Fig. 2**, and can be defined by

$$\Gamma(\theta) = \frac{\langle G(\theta \mid 0) G^*(-\theta \mid 0) \rangle}{\langle \mid G(0 \mid 0) \mid^2 \rangle} \quad (15)$$

Here, we assume $B(\mathbf{r}/2, -\mathbf{r}/2) = B_0 \exp(-r^2/l^2)$ and $kB_0L = 3\pi$; therefore the coherence attenuation index $k^2B_0Ll/4$ given in reference [6] is $15\pi^2$, $75\pi^2$, and $150\pi^2$ for $kl = 20\pi$, 100π, 200π, respectively, which means that the incident wave becomes sufficiently incoherent. The normalized spatial coherence length (SCL) is defined as $2k\rho$ at which $k\rho = kr \sin\theta$ where θ is defined as $\mid \Gamma(\theta) \mid = e^{-1} \simeq 0.37$ in **Fig. 3** where $kr = 6$ for convenience. Accordingly, in this case SCL is approximately equal to 3.2, 6.8, and 9.5, for $kl = 20\pi$, 100π, 200π, respectively. Under the condition that incident and scattered waves are not correlated, we may assume that the fourth order moment of Green's functions is expressed as the product of the second order moments.

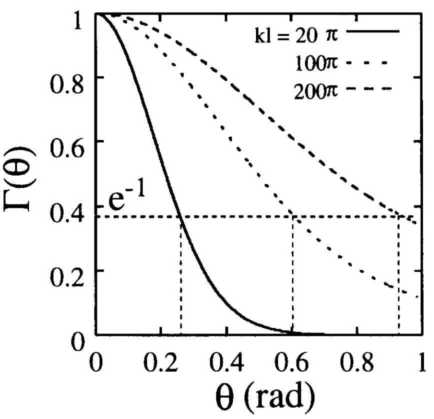


Fig.3 The degree of spatial coherence of an incident wave about the cylinder.

The correlation function of the scattered waves defined by equation (14) was calculated numerically as a function of the following parameters.

- Target shape: we handle different target shapes including circular, elliptic, and concave-convex surfaces. Also we will change the parameters of the different shapes. For elliptic cylinders we will change the axis ratio (b/a) where b and a are defined in

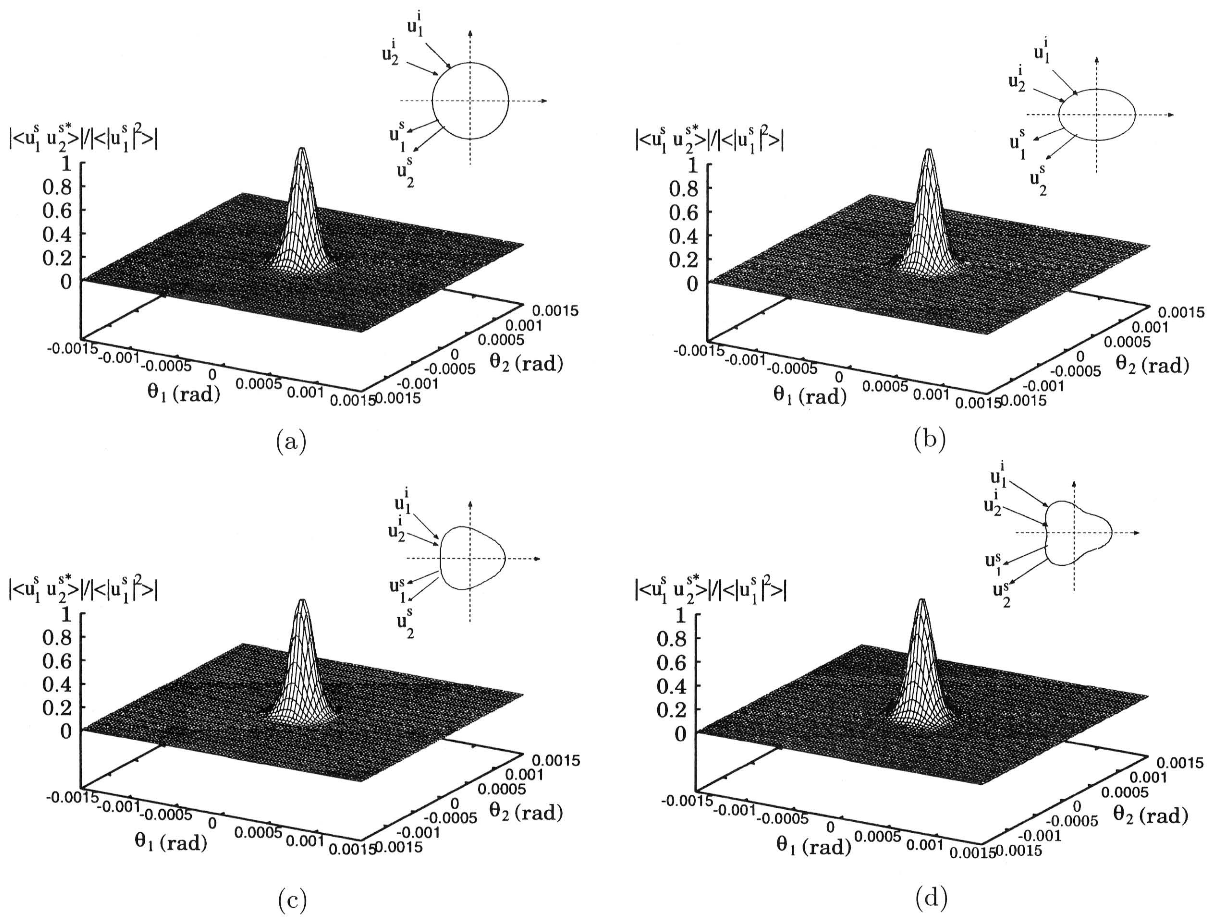


Fig.4 Normalized angular correlation function for E-wave incidence at which ka=1, ϕ=0, kl=20π and where (a) b/a=1, (b) b/a=0.6, (c) δ=0.1, (d) δ=0.2.

equation (4). For the concave-convex targets we will change δ defined in equation (5).

• Target size: We will deal with different sizes for targets by changing ka.

• Incident angle: Different incident angles will be handled by changing ϕ.

• Incident wave polarization: E and H-wave incidences.

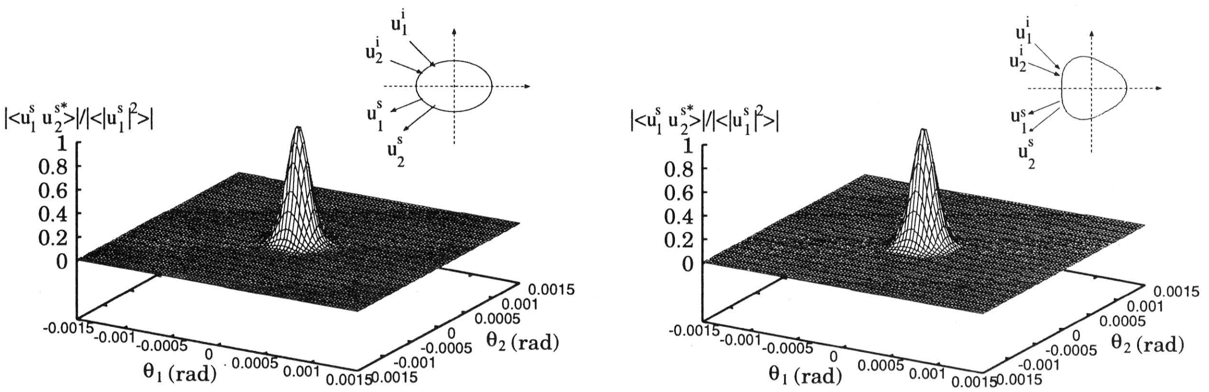
• Random medium parameter kl.

In our numerical results we will postulate θ_{i1} and ψ to equal 0.98π and 0.04 [rad], respectively, to keep the assumption that incident and scattered waves are not correlated. In **Fig. 4** we present numerical results for ACF for E-wave incidence on targets of different shapes where $ka = 1$, $\phi = 0$, and $kl = 20\pi$. From **Fig. 4** we observe that ACF doesn't change with shape of target. Afterwards we proceed in our study; in **Figs. 5** to **8** we change ka, ϕ, polarization, and kl, respectively, compared to previous case of **Fig. 4**.

From **Figs. 5** to **8**, we observe that ACF does not change with target size, incident wave angle, and polarization.

The ACF changes obviously with the random media. As kl increases, the SCL increases too as shown in **Fig. 3** and also there is another increase in the correlation width w defined as $2k\rho$ at which $\rho = r\sin\theta$ where θ is defined as the angle at which ACF$= e^{-1} \simeq 0.37$. In **Fig. 9** we draw a relationship between the square root of kl and w where a circular cylinder of $ka = 1$ is assumed; it is clear that w is almost directly proportional to the square root of kl. This fact shows also that the ACF depends on the random media.

Also ACF changes with incident angle and to clarify such change, we plotted ACF in two-dimensional graph as shown in **Fig. 10**. It is clear that as increasing θ_1 slightly the ACF decreases obviously which reveals that ACF is very sensitive to the incident angle deviation.



(a) (b)

Fig.5 Normalized angular correlation function for E-wave incidence at which ϕ=0, kl=20π and where (a) b/a=0.8, ka=2, (b) δ=0.1, ka=3.

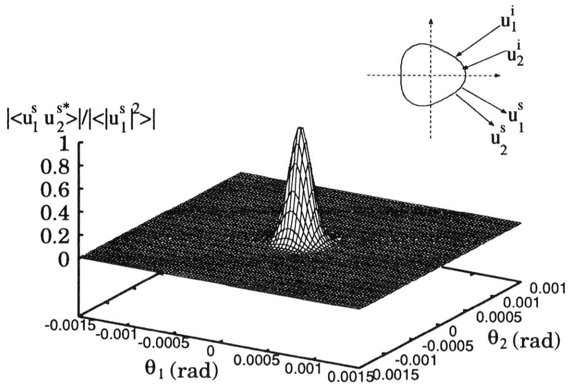


Fig.6 As **Fig. 4(c)**, but for $\phi = \pi$.

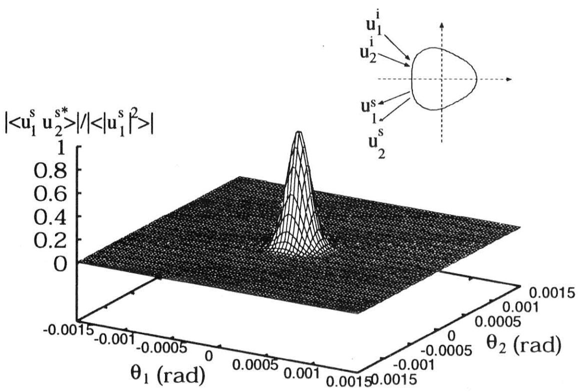
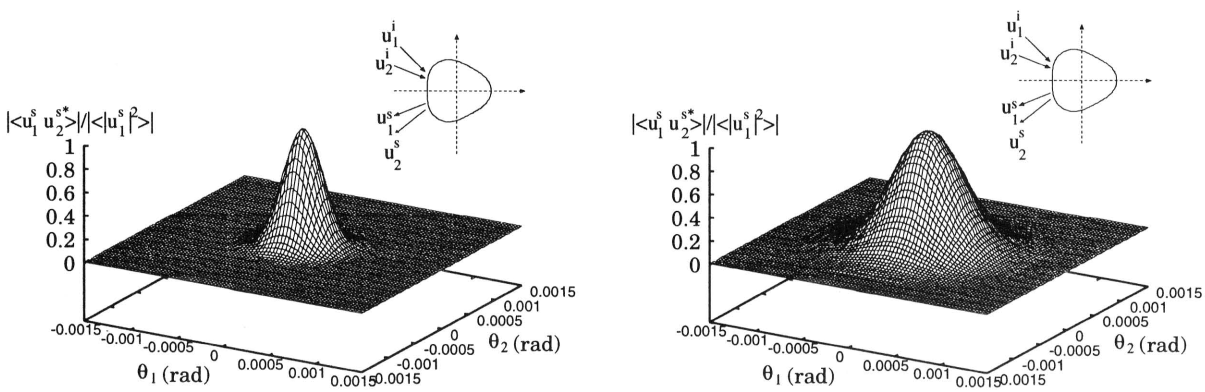


Fig.7 As **Fig. 4(c)**, but for H-wave incidence.



(a) (b)

Fig.8 Normalized angular correlation function for E-wave incidence at which ϕ =0, ka=1, δ=0.1 and where (a) kl=60π, (b) kl=200π.

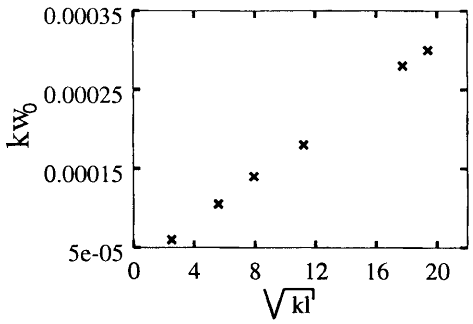


Fig.9 Relationship between scale size and correlation width of the scattered waves from circular cylinder in random medium.

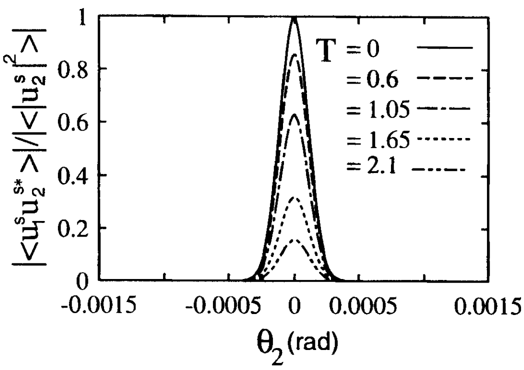


Fig.10 Clarification of angular correlation function at $kl = 20\pi$; $\theta_1 = T * 10^{-4}$ $[rad]$.

4. Conclusion

An angular correlation function (ACF) of scattered waves from complicated conducting targets in random media has been calculated and analyzed numerically for E and H-wave incidences. Numerical results suggest an important fact; the ACF depends only on the random media irrespective of targets parameters and incident wave polarization on the assumption that incident and scattered waves are not correlated. As a result, we could find that the correlation width of the ACF is directly proportional to the square root of scale-size of random media.

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